discrete random variable

Q 1 pmf $f(x)$ of a random variable x is 0, except at the points $x = 0, 1, 2$

(i) $f(x) \geq 0$ for all x.

(ii) $\sum_x f(x) = 1$

if (i) and (ii) are satisfied, the $f(x)$ is a pmf.

and $f(0) = c$
$f(1) = 2c - 3c^2$
$f(2) = 4c - 1$

(i) Determine the value of c.
(ii) find $P(x > 0 | x < 2)$
(iii) Find expectation and variance of x

Ans

We know $f(x)$ is a pmf of x.

$\therefore \sum_x f(x) = 1$

or, $\sum_{x=0}^{2} f(x) = 1$

or, $f(0) + f(1) + f(2) = 1$

or, $c + 2c - 3c^2 + 4c - 1 = 1$

or, $3c^2 - 7c + 2 = 0$

or, $3c^2 - 6c - c + 2 = 0$

or, $3c(c-2) - 1(c-2) = 0$

or, $(3c-1)(c-2) = 0$

$\therefore \boxed{c = 1/3 \text{ or } 2}$

But $c \neq 2$ because $f(x) < 0$ for $x = 1$

$\therefore$ value of $c = 1/3$ (ans).

$\therefore f(0) = c = 1/3$

$f(1) = 2c - 3c^2 = 2 \times \frac{1}{3} - 3 \times \left(\frac{1}{3}\right)^2 = \frac{2}{3} - \frac{3}{9} = \frac{1}{3}$

$f(2) = 4c - 1 = 4 \times \frac{1}{3} - 1 = \frac{1}{3}$.

$f(2) = 4c-1 = 4\times\frac{1}{3}-1 = \frac{1}{3}$

$\therefore f(0) = f(1) = f(2) = \frac{1}{3}$

(ii) $P(X>0 / X<2) = \frac{P[X>0 \cap X<2]}{P(X<2)} = \frac{P(X=1)}{P(X=0)+P(X=1)} = \frac{1/3}{\frac{1}{3}+\frac{1}{3}}$

$= \frac{1}{3}\times\frac{3}{2} = \frac{1}{2}$ ans.

(iii) $E(x) = \sum_{x} x \cdot f(x)$

$= \sum_{x=0}^{2} x f(x) = 0\cdot f(0) + 1\cdot f(1) + 2\cdot f(2)$

$= 0 + 1\times\frac{1}{3} + 2\times\frac{1}{3} = \frac{3}{3} = 1$

$\therefore E(x) = 1.$

$V(x) = E(x^2) - \{E(x)\}^2$

Now $E(x^2) = \sum_{x=0}^{2} x^2 f(x) = 0^2 f(0) + 1^2 f(1) + 2^2 f(2)$

$= 0 + 1\times 1/3 + 4\times\frac{1}{3} = 5/3$

$V(x) = \frac{5}{3} - 1^2 = \frac{5-3}{3} = 2/3$ (ans).

$X \sim$ continuous r.v.
(i) $f(x) \geq 0 \;\forall x$.
(ii) $\int f(x) = 1$

② Is the following a pdf ?

$$f(x) = \begin{cases} 2x, & 0 < x \leq 1 \\ 4-2x, & 1 < x \leq 2 \\ 0, & \text{elsewhere} \end{cases}$$

Solution for $f(x)$ to be a pdf

(i) $f(x) > 0$ for all values of x.

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_0^1 2x\,dx + \int_1^2 (4-2x)\,dx$$

$$= \cancel{2}\left[\frac{x^2}{\cancel{2}}\right]_0^1 + 4[x]_1^2 - \cancel{2}\left[\frac{x^2}{\cancel{2}}\right]_1^2$$

$$= 1 + 4 - 3$$

$$= 5 - 3$$

$$= 2 \neq 1$$

$\because \int f(x)\,dx \neq 1 \quad \therefore f(x)$ is not a pdf.

③ A continuous random variable x has a density function given by

$$f(x) = \frac{1}{2} - ax, \quad 0 \le x \le 4$$
$$= 0, \quad \text{elsewhere}$$

Find (i) value of 'a'
(ii) $P(1 < x < 2)$ (iii) $P(2x+3 > 5)$ (iv) $E(x)$

(i) Since $f(x)$ is a pdf

$$\therefore \int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_0^4 \left(\frac{1}{2} - ax\right) dx = 1$$

$$\Rightarrow \frac{1}{2}[x]_0^4 - a\left[\frac{x^2}{2}\right]_0^4 = 1$$

$$\Rightarrow \frac{4}{2} - \frac{a}{\cancel{2}} \times \cancel{16}^{\,8} = 1$$

$$\Rightarrow 2 - 8a = 1$$

$$\Rightarrow$$

$$\Rightarrow \quad 2 - 8a = 1$$

$$\Rightarrow \quad 8a = 2 - 1$$

$$\Rightarrow \quad \boxed{a = 1/8} \text{ (ans)} \qquad \therefore \boxed{f(x) = \frac{1}{2} - \frac{x}{8}}$$

(ii) $$P(1 < X < 2) = \int_1^2 f(x)\,dx$$

$$= \int_1^2 \left(\frac{1}{2} - \frac{x}{8}\right) dx$$

$$= \frac{1}{2}[x]_1^2 - \frac{1}{8\times 2}[x^2]_1^2$$

$$= \frac{1}{2} - \frac{1}{16}[3]$$

$$= \frac{5}{16} \text{ (ans)}$$

(iii) $$P(2x+3>5) = P(2x>2)$$

$$= P(\underline{x>1}) = \int_1^\infty f(x)\,dx = \int_1^4 \left(\frac{1}{2} - \frac{x}{8}\right) dx$$

$$= \frac{1}{2}[x]_1^4 - \frac{1}{8\times 2}[x^2]_1^4$$

$$= \frac{3}{2} - \frac{1}{16}(15)$$

$$= \frac{9}{16} \text{ (ans)}$$

(iv) $$E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_0^4 x\left(\frac{1}{2} - \frac{x}{8}\right) dx$$

$$= \int_0^4 \frac{x}{2}\,dx - \frac{1}{8}\int_0^4 x^2\,dx$$

$$= \int_0 \frac{1}{2} \quad \frac{}{8}\int_0$$

$$= \frac{1}{2}\times\frac{1}{2}\left[x^2\right]_0^4 - \frac{1}{8\times 3}\left[x^3\right]_0^4$$

$$= \frac{1}{4}[16] - \frac{1}{\cancel{24}\ 6\ 3}\left[\cancel{64}\right] \cancel{16}\ 8$$

$$= 4 - \frac{8}{3} = \frac{4}{3} \text{ (ans)}$$

$$(x+y)^2$$
$$x^2 + y^2 + 2xy$$
$$V(x) + V(y) + 2\,Cov(x,y)$$
$$(x-y)^2$$
$$x^2+y^2-2xy$$

$$Cov(X,Y) = E(XY) - E(X)E(Y)$$

$$V(X) = E(x^2) - (E(x))^2$$

$$\boxed{V(X+Y) = V(X) + V(Y) + 2\,Cov(X,Y)}$$

if X and Y are independent $\Rightarrow Cov(X,Y)=0$
then $V(X+Y) = V(X)+V(Y)+0 = V(X)+V(Y)$
$V(X-Y) = V(X)+V(Y) - 2Cov(X,Y) = V(X)+V(Y)$

$\therefore$ if X and Y are independent
then $V(X+Y) = V(X-Y)$
$= V(X)+V(Y)$

$$Cov[(x+y),(x-y)]$$
$$Cov[x^2 - xy + xy - y^2]$$
$$Cov(x^2-y^2) = V(x) - V(y)$$

$$Cov[(x+y),(x+z)]$$
$$Cov[x^2 + xz + yx + yz]$$
$$V(x) + Cov(x,z)$$
$$+ Cov(y,x)$$
$$+ Cov(y,z)$$

$$(x+y)(x-y)$$
$$x^2 - y^2$$

$$V(2x) = 4V(x)$$

$$V(2x) = 4V(x)$$

$$V(2x+y) = 4V(x) + V(y) + 2\,\mathrm{Cov}(2x, y).$$