DIFFERENTIAL EQUATION 7 12 23

Thursday, December 7, 2023 7:34 PM

11:15PM - 1:45AM

9062395123

$$f\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2} \ldots \frac{d^ny}{dx^n}\right) = 0$$

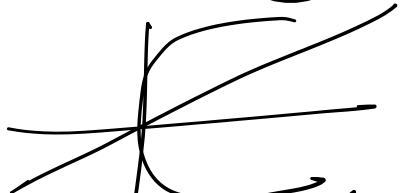
$y = \alpha + \beta x$

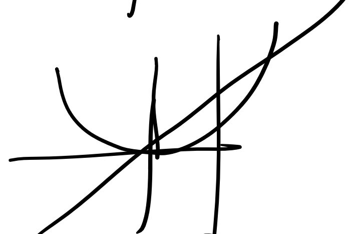
$y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$

$\frac{dy}{dx}$ $\frac{d^2y}{dx^2}$

Future for a movie
① 12th fail
② Jawan
③ Sam Bahadur
④ Animal

Cumulative Fast motion





$y = 2x^4 + 7$

$\frac{dy}{dx} = 8x^3 + 7$ = Slope

$\frac{d^2y}{dx^2} = 24x^2 + 7$ = Curvature

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$y = x^2$

$\frac{dy}{dx} = 2x$

$\frac{d^2y}{dx^2} = 2$

$y = \alpha + \beta x$

ODE ①
PDE 2, 3, 4 ...

$$\frac{dy}{dx} = 2\left(\frac{d^ny}{dx^2}\right)^1 + 4$$

① Order
② Degree

Linear vs none linear

If the function is linear function of the variables.

iff the function is linear function of the variables.

NLDE

$$P_0 \frac{d^n y}{dx^n} + P_1 \frac{d^{n-1} y}{dx^{n-1}} + \cdots + P_n y = Q$$

$Q = 0$ Homogeneous

$Q \neq 0$ non-homogeneous

$y_1 = 2x^2 + 3xy + 7y^2$ ①

$= \frac{4x^2y^2}{x+y}$ ② N/A

③

$= \frac{4(\lambda x)^2(\lambda y)^2}{\lambda x + \lambda y}$

$= \frac{\lambda^4 \cdot 4x^2y^2}{\lambda(x+y)} = \lambda^3 f(x,y)$

$y_1 = 2x^2 + 3x + 7x^3 + 8y^4$

$\lambda^2 \quad \lambda \quad \lambda^3 \quad \lambda^4$

Non homogeneous

$\lambda(2\lambda x^2 + 3x + 7\lambda^2 x^3 + 8\lambda^3 y^4)$

$y = \frac{x^2}{z^2} = \frac{\lambda^2}{\lambda^2}\frac{x^2}{z^2} = \lambda^0$

Order Degree | Method to study | PI | Clairaut's form

6 ways to solve

① variable separation form (VSF)

1. Variable separation form (VSF)
2. Homogeneous eq
3. Reducible to VSF
4. Reducible to H. eq
5. LDE of first order
6. Reduce ~

$$\int f(x)\,dx + \int h(y)\,dy = C$$

$ax + by + c = z$

$$\frac{dy}{dx} = f(\underbrace{ax+by+c}_{z})$$

$$a + b\frac{dy}{dx} = \frac{dz}{dx}$$

$$\frac{dy}{dx} = \left(\frac{dz}{dx} - a\right)\frac{1}{b}$$

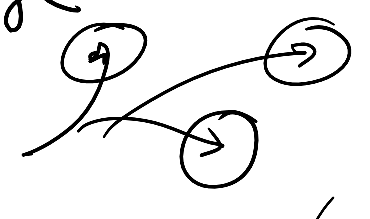
$$\frac{1}{b}\left(\frac{dz}{dx} - a\right) = f(z)$$

$$\boxed{\frac{dz}{dx} = a + b f(z)}$$

Homo → Similar

$$x^4 + y^4 + z^4 + pqrs^2$$

Differential eqn in Real Life



Sick → Medicine

Sick → Medicine

$A(t) \to$ medicine in the bloodstream

will decline at a rate of proportional to the amount

$$\frac{dA}{dt} = \lambda A$$

$\lambda =$ elimination const

$\boxed{\lambda > 0}$

IP
BP
USP

The Critical Point   qualitative behaviours

IP   BP   USP

Korean

Critical Point

ODE

$$\frac{dy}{dx} = y(y-1)(y-2) = 0$$

$y=0,\ y=1,\ y=2$

$y(0) = 0.5 \quad y\downarrow$

$y(0) = 1.2 \quad y \uparrow$

$y(0) = 1.5 \quad y$ UB

$y(0) < 0 \quad y$ BB

$y<0$

$-\infty<y<0$	$0<y<1$	$1<y<2$	$y>2$
✓	✗	✓	✗

$\frac{dy}{dx} < 0 \qquad \frac{dy}{dx} > 0$

If $y \in (-\infty, 0) \cup (1,2)$

$y \in (0,1) \cup (2, \infty)$

$\therefore$ If $y(0) \in (-\infty, 0) \cup (1,2)$

$Y(x)$ is decreasing function

$y(0) \in (0,1) \cup (2, \infty)$ inc function

$y(0) \in (0,1) \cup (2,\infty)$

$(y-1)(y-1.5)(y+2.3)(y+9784.5)(y+e^{7803}) \geq 0$

$y_1 = 1$, $y_2 = 1.5$, $y_3 = -2.3$, $y = -9784.5$, $y_4 = -e^{7803}$

$y=0$

$y < -e^{7803}$	$e^{-7803} < y < -9784.5$	$-9784.5 < y < -2.3$	$-2.3 < y < 1$	$1 < y < 1.5$	$y > 1.5$
×	✓	×	✓	×	✓

P C ECO 2 Fence Math Stat

Clearing Equations

$M\,dx + N\,dy = 0$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Exact — Just differentiate & nothing else

Sol^n $\int M\,dx + \int (\text{terms } N \text{ without } x)\,dy = c$

IF → × long thus → to move to exact

IF → X

5 RULES OF IF

Real Analysis 20-25% DE

Integration $x_1 - x_{11}$

Rai & Singhania

$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$f'(x)$

Rule: 1 $M\,dx + N\,dy = 0$ homogeneous

then $\frac{1}{Mx+Ny}$ IF

250-300

$x^2 y\,dx - (x^3+y^3)\,dy = 0$

$IF = \frac{1}{Mx+Ny} = \frac{1}{x^3y - x^3y - y^4} = -\frac{1}{y^4}$

Rule: 2 $M\,dx + N\,dy$ is of the form $f_1(xy)\,y\,dx + f_2(xy)\,x\,dy = 0$

$\frac{1}{Mx-Ny}$ IF

$Mx - Ny \neq 0$

$(1+xy)\,y\,dx + (1-xy)\,x\,dy = 0$

$\frac{1}{2x^2y^2} = IF$

Mood Swing

5:15AM

The Happiness Project

Snooze

2-4 8AM

12-4

9-10-11-12

5:30 6:30 7:30 8:30 9:30

Rule: 3 $\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}$ is $f(x)$

then $e^{\int f(x)\,dx}$ IF

f(x,y)

then $e^{\int}$

N $\boxed{\text{IF}}$

but if $f(x,y)$

$$\left(y+\frac{y^3}{3}+\frac{x^2}{2}\right)dx+\left(\frac{1}{4}\right)(x+xy^2)dy=0$$

M $\qquad$ N

$$\frac{1}{N}\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)=\frac{4}{\cancel{4}\,\underline{(x+xy^2)}}\left(1+y^2-\frac{1}{4}-\frac{y^2}{4}\right)$$

M

$$=\cancel{\frac{3}{4}(1+y^2)}\ \frac{4}{x\cancel{(1+y^2)}}=\frac{3}{x}$$

$$e^{\int \frac{3}{x}dx}=x^3$$

Rule 4

$$Mdx+Ndy=0$$

$$\frac{\dfrac{\partial N}{\partial x}-\dfrac{\partial M}{\partial y}}{M}\Rightarrow f(y)$$

$$e^{\int f(y)dy}$$

$\cancel{e^{\int}}$

$$(xy^3+y)dx+2(x^2y^2+x+y^4)dy=0$$

$$\frac{1}{M}\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)$$

$f(y)\to N$

$f(x)\to M$

∂M

$$\frac{1}{N}\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)=f(y)$$

$$\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = f(x)$$

$$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = f(y)$$

$$\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = f(x)$$

Rule:5

** $x^a y^b (my\,dx + nx\,dy) + x^r y^s (py\,dx + qx\,dy) = 0$

$$\frac{a+h+1}{m} = \frac{b+k+1}{n}, \quad \frac{r+h+1}{p} = \frac{s+k+1}{q}$$

2020

$$\frac{dy}{dx} = y^2 \qquad y(0) = 1$$

don't need

$a^x \ldots [0, a] \qquad a > 1$

don't read

(a) $x \in (-\infty, 1]$ (b) $x \in [0, a]$ $a > 1$

(c) $x \in (-\infty, \infty)$ (d) $x \in [1, a]$ $a > 1$

$$\frac{dy}{dx} = y^2 \qquad y(0) = 1$$

$$\frac{dy}{y^2} = dx$$

$$\frac{y^{-2+1}}{-1} = x + C$$

$$\boxed{-\frac{1}{y} = x + C}$$

$$C = -1$$

$$\boxed{y = \frac{1}{1-x}} \qquad x = 1$$

$$\left(\cancel{\frac{d^2y}{dx^3}}\right)$$

multiple integr 2 / 3

or → 2

dy 1 $\left(\frac{d^2y}{dx^2}\right)^{3}$ = fun 3

order Degree

Ex. 6. *The degree of the equation* $(d^3y/dx^3)^{2/3} + (d^3y/dx^3)^{3/2} = 0$ *is*

(*a*) 3 (*b*) 5 (*c*) 4 (*d*) 9.

$$\left(\frac{d^3y}{dx^3}\right)^{2/3} = -\left(\frac{d^3y}{dx^3}\right)^{3/2}$$

9

or → 3

$$\left(\frac{d^3y}{}\right)^4 \qquad \left(\frac{d^3y}{}\right)$$

$$\left(\frac{d^3y}{dx^3}\right)^4 = -\left(\frac{d^3y}{dx^3}\right)^4$$ or $\to 3$

D $\to 4$