

1. Bayes Theorem

2. Minors - Cofactors

3. Input-Output analysis.

Matrix and Determinant

$M_{31} (-1)^{3+1} = C_{31} = 63 (-1)^4 = 63 \checkmark$
 $M_{32} (-1)^{3+2} = C_{32} = 51 (-1)^5 = -51 \checkmark$
 $M_{33} (-1)^{3+3} = C_{33} = -15 (-1)^{3+3} = -15 \checkmark$

Minors:

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$
 $M_{11} = \begin{bmatrix} a_{22} \end{bmatrix}$
 $M_{12} = \begin{bmatrix} a_{21} \end{bmatrix}$
 $M_{21} = \begin{bmatrix} a_{12} \end{bmatrix}$
 $M_{22} = \begin{bmatrix} a_{11} \end{bmatrix}$

$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$A = \begin{bmatrix} 13 & 19 \\ 19 & 15 \end{bmatrix}$

13, 19

$M_{22} = (-1)^{2+2} (a_{11})_{1 \times 1} = +ve C_{22}$

$M_{21} = (a_{12})_{1 \times 1}$

$M_{12} = (a_{21})_{1 \times 1}$

$M_{pp} = (a_{pp})_{1 \times 1}$

$M_{11} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

$M_{21} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{33} \end{vmatrix}$

$M_{12} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$

$M_{22} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$

$A = \begin{bmatrix} 5 & 1 & 3 \\ 4 & 6 & 7 \\ 3 & 8 & 2 \end{bmatrix}$

$M_{11} = \begin{vmatrix} 6 & 7 \\ 8 & 2 \end{vmatrix} =$

$M_{12} = \begin{vmatrix} 4 & 7 \\ 3 & 2 \end{vmatrix} =$

$M_{13} = \begin{vmatrix} 4 & 6 \\ 3 & 8 \end{vmatrix} =$

$\dots = 1 \cdot 15 - 1 \cdot -2 \quad 4 \cdot 6 - 3 \cdot 8 \quad + 3 \cdot 4 - 2 \cdot 9$

$$|A| = 1 \begin{vmatrix} 5 & 1 \\ 8 & 9 \end{vmatrix} - 2 \begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + 3 \begin{vmatrix} 7 & 8 \\ 7 & 8 \end{vmatrix}$$

$$A = \begin{bmatrix} 9 & 11 & 4 \\ 3 & 2 & 7 \\ 6 & 10 & 4 \end{bmatrix}$$

Bayes theorem:

Conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$

S → Stripped
R → Red

$$P(S/R) = \frac{P(S \cap R)}{P(R)}$$

$$= \frac{4/10}{6/10} = 4/6 = 2/3$$

Joint / compound probability

$$P(A \cap B) = P(A/B) \times P(B)$$

$$P(A \cap B) = P(B/A) \times P(A)$$

$$P(R) = 6/10$$

$$P(S \cap R) = P(S/R) \cdot P(R)$$

$$= 2/3 \times 6/10$$

$$P(S/R) = 2/3 \quad P(R) = 6/10$$

$P(S \cap R) = P(R)$

$$P(S/R) = \frac{2}{3} \quad P(R) = \frac{6}{10}$$

$$P(S \cap R) = P(S/R) \times P(R)$$

$$P(A \cap B) = P(A) \times P(B) \rightarrow \text{indep.} = \frac{2}{3} \times \frac{6}{10} = \frac{4}{10}$$