Trigonometry

Wednesday, May 17, 2023 12:00 PM

$\frac{45}{2}$) 45 ($22\frac{1}{2}$
44
1

$\sin(10\sqrt{3})$

$x \to ?$

$\sin x$,

$1^\circ = \frac{\pi}{180}$ radian

$\pi/180 \longrightarrow 1^\circ$

$1 \longrightarrow \frac{180}{\pi}^\circ$

$\pi/8 \longrightarrow \left(\frac{180}{\pi} \times \frac{\pi}{8}\right)^\circ =$

$\mathbb{N} = \{1, 2, 3, \ldots\}$

$W = \{0, 1, 2, 3, \ldots\}$

$\mathbb{Q} = \{$ set of all fractions in p/q form, where $q \neq 0$, $p, q \in \mathbb{Z}\}$

$\mathbb{Z} = -\mathbb{N} \cup \{0\} \cup \mathbb{N}$

$\mathbb{Q}^c = \{$not rational$\}$

$\mathbb{R} = \mathbb{Q} \cup \mathbb{Q}^c$

$\mathbb{C}$ = Complex number

sin, cos, tan
real-valued functions

$f(x) = \frac{1}{1-x}$

Angles of a triangle are in AP. $\frac{\text{least angle (degree)}}{\text{greatest angle (radian)}} = \frac{60}{\pi}$. Find angles.

$(a-d)^\circ, a^\circ, (a+d)^\circ$ $a = 60^\circ$. greatest $= 60+d = \frac{\pi}{180}(60+d)$ rad

$\therefore \frac{60-d}{\pi/180\,(60+d)} = \frac{60}{\pi} \Rightarrow \frac{60-d}{60+d} = \frac{1}{3} \Rightarrow 180 - 3d = 60 + d$
$\Rightarrow d = 30^\circ$

Angles of a quadrilateral are in AP, greatest angle is 120°. Find the $\angle$s (in rad)

$a-3d, a-d, a+d, a+3d$; $\therefore a = 90^\circ$ (sum of 4 angles of a quadrilateral $= 360^\circ$)

$\Rightarrow 90 + 3d = 120^\circ \Rightarrow d = 10^\circ$

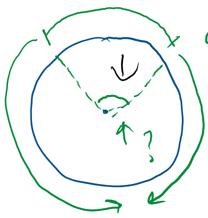
$\therefore$ 4 angles $\longrightarrow 60^\circ, 80^\circ, 100^\circ$ & 120° . In radians $\rightarrow \frac{\pi}{3}, \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{2\pi}{3}$

The minute hand of a clock is 10 cm long. How far does the tip of the hand move in 20 mins?

$2\pi(10)$ cm distance covered in 60 mins .

60 mins $\longrightarrow 20\pi$ cm
20 mins $\longrightarrow \frac{20\pi}{3}$ cm

Circle with radius 21 cm. Has an arc that subtends 60° angle at the center. Find arc length.



Sol 1

$\frac{1}{6}$ ($360^\circ \longrightarrow 42\pi$ cm) $\frac{1}{6}$
$60^\circ \longrightarrow 7\pi$ cm

Arc length $= \frac{22}{7} \times 7$ cm $= 22$ cm

Sol 2

$s = r\theta$, where s = arc length, r = radius, θ = angle at center by the arc , (in rad)

$s = \left(21 \times \pi/3\right)$ cm $= 7\pi$ cm $= 22$ cm

$\sin^{-1} x$, $(\sin x)^{-1} = \frac{1}{\sin x} = \operatorname{cosec} x$

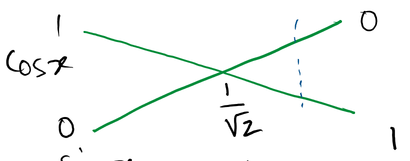
$\downarrow$

$\sin^{-1} x = a$

$\Rightarrow x = \sin a$

$\boxed{\begin{array}{l} |\sin x| \leq 1 \\ |\cos x| \leq 1 \end{array}}$ $\longrightarrow$

$\sin^2 x + \cos^2 x = 1$



$\downarrow$

$\pi/4 = 45^\circ$

Logarithm

Thursday, May 25, 2023 2:00 PM

(1) $\log_a a = 1$ (2) $\log_a 1 = 0$

(3) $\log_a b \cdot \log_b a$ let $\log_b a = x \Rightarrow b^x = a$

$\Rightarrow \log_a b^x = \log_a a = 1$

$\log_a b^x = 1$ — (A)

Using in A $\longrightarrow x \log_a b = 1$

$\Rightarrow \log_a b \cdot \log_b a = 1$

(4) $\log_a b^x = x \log_a b$

(5) $\log_b a = \log_b c \cdot \log_c a$ (base changing formula)

(6) $\log_a (bc) = \log_a b + \log_a c$ (7) $\log_a\left(\frac{b}{c}\right) = \log_a b - \log_a c$

(8) $\log_{(a^q)} b = \frac{1}{q} \log_a b$ (9) Combining 4 & 8 $\log_{a^q} b^p = \frac{p}{q} \log_a b$

(10) $a^{\log_a b} = b$ $\underbrace{\log_a b = x}$ $\Rightarrow a^x = b \longrightarrow a^x = b$

(11) $\log a \longrightarrow \log_{10} a$, $\ln a \longrightarrow \log_e a$

(1) $4^{3/2} = 8$ (2) $(2\sqrt{2})^{-2/3} = 1/2$ (3) $\log_{5\sqrt{5}} 5 = ?$ (4) $\log_{100} 0.1 = ?$

$\log_2 4^{3/2} = \log_2 8$

$\Rightarrow 3/2 \log_2 4 = 3$

$\Rightarrow \log_2 4 = 2$

$\Rightarrow \log_2 2^2 = 2$

$\Rightarrow 2 \times 1 = 2$

True

$(2 \cdot 2^{1/2})^{-2/3} = 1/2 = 2^{-1}$

$\Rightarrow 2^{-2/3} \cdot 2^{-1/3} = 2^{-1}$

$\Rightarrow \log_2\left(2^{-2/3} \cdot 2^{-1/3}\right) = -1$

$\Rightarrow \log_2 2^{-2/3} + \log_2 2^{-1/3} = -1$

$\Rightarrow -2/3 + -1/3 = -1$ (True)

$\log_{5 \cdot 5^{1/2}} 5$

$= \log_{5^{(1+1/2)}} 5$

$= \log_{5^{3/2}} 5$

$= 2/3$

$\log_{100} 0.1 = \log_{10^2} 10^{-1} = -1/2$

(1) $3^{-1/2 \log_3 9}$ (2) $2^{2-\log_2 5}$

(1) $3^{\log_3 9^{-1/2}} = 3^{\log_3 1/\sqrt{9}} = 3^{\log_3 1/3} = 1/3$

$\left[\begin{array}{l} a^m \cdot a^n \\ = a^{m+n} \end{array}\right]$

(2) $2^{2-\log_2 5} = 2^2 \cdot 2^{-\log_2 5} = 4 \; 2^{\log_2 5^{-1}} = 4 \cdot 1/5 = \frac{4}{5}$

$a^{m+n} = a^m \cdot a^n$

$a^{m-n} = a^m \cdot a^{-n}$

(3) $10^{\log m + \log n} = mn$

(4) $2^{\log_{2\sqrt{2}} 15} = 2^{\log_{2^{3/2}} 15} = 2^{2/3 \log_2 15} = 2^{\log_2 15^{2/3}} = (15)^{2/3}$

Trigonometry

Wednesday, May 31, 2023 12:30 PM

$s^2+c^2=1$,

$\cos^2 x = 1-\sin^2 x$

$\cos x = \sqrt{1-\sin^2 x}$

$\tan x = \frac{\sin x}{\cos x}$

$= \frac{\sin x}{\sqrt{1-\sin^2 x}}$

$1+\tan^2 x = \sec^2 x$

$1+\cot^2 x = \operatorname{cosec}^2 x$

$-1 \le \sin x \le 1$
$\cos x$

$\sin(-x)$
$= -\sin x$ odd

$f(-x) = -f(x)$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

$\sin x = \frac{1}{2}$

$\operatorname{cosec} x = \frac{1}{1/2} = 2$

$\cos(-x)$
$= \cos x$ even

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$

$\sec x, \operatorname{cosec} x$

$(-\infty,-1] \cup [1,\infty)$

$f(-x) = f(x)$

$\tan x, \cot x$

$f(x) = x^2$

$f(-x) = (-x)^2 = x^2$

$\sqrt{\frac{1-\sin a}{1+\sin a}} = \sec a - \tan a$

LHS $\sqrt{\frac{(1-\sin a)(1-\sin a)}{1-\sin^2 a}} = \sqrt{\frac{(1-\sin a)^2}{\cos^2 a}} = \frac{1-\sin a}{\cos a} = \sec a - \tan a$

$= RHS$

$\sqrt{\frac{1+\cos a}{1-\cos a}} = ?$ $\quad \sqrt{\frac{(1+\cos a)^2}{1-\cos^2 a}} = \sqrt{\frac{(1+\cos a)^2}{\sin^2 a}} = \frac{1+\cos a}{\sin a}$

multiplying numerator & denominator by $(1+\cos a)$

$= \operatorname{cosec} a + \cot a$

$(1-\cos a)(1+\cos a) \longrightarrow (a+b)(a-b) = a^2-b^2$

$\sec^4 a(1-\sin^4 a) - 2\tan^2 a = ?$

$\sec^4 a(1-\sin^2 a)(1+\sin^2 a) - 2\tan^2 a$

$= \sec^4 a \cdot \cos^2 a(1+\sin^2 a) - 2\tan^2 a$

$= \sec^2 a + \tan^2 a - 2\tan^2 a = \sec^2 a - \tan^2 a = 1$

Blank

Tuesday, May 30, 2023 12:00 PM