17 November 2023 07:35 PM

Pure ~~math~~ math both

General

1. Henderson and quandt (Math eco)
2. Simon and blume (mathemetics for economists)
3. Sydsaeter and Hammond.
4. Mehta - Madnani (Maths for economists)

Rank correlation (with ties)

$$r_R = \frac{\frac{n^2-1}{12} - \frac{T_u+T_v}{2} - \frac{1}{2n}\sum d_i^2}{\sqrt{\frac{n^2-1}{12} - T_u}\sqrt{\frac{n^2-1}{12} - T_v}}$$

Let us arrange no. of students by reference to examination marks having ties.

If k individuals are allocated same rank, then we have a tie of length k. When the k individuals having same rank follow r other individuals in the ranking then each of them is given the rank

$$\frac{1}{k}\left[(r+1)+(r+2)+(r+3)+\dots+(r+k)\right]$$

$$= \frac{1}{k}\left[rk + \frac{k(k+1)}{2}\right]$$

$$K \qquad = \frac{1}{K}\left[rK + \frac{K(K+1)}{2}\right]$$

$$= r + \frac{K+1}{2}$$

Sum of square of their ranks

$$S_1 = (r+1)^2 + (r+2)^2 + (r+3)^2 + \cdots + (r+k)^2$$

$$= kr^2 + 2r(1+2+3+\cdots+k) + (1^2+2^2+3^2+\cdots+k^2)$$

$$= kr^2 + K\frac{(K+1)}{\cancel{2}}\cdot\cancel{2}r + \frac{1}{6}K(K+1)(2K+1)$$

$$\boxed{S_1 = kr^2 + K(K+1)r + \frac{1}{6}K(K+1)(2K+1)}$$

whereas, the sum of square of tied ranks

$$S_2 = K\left[r + \frac{K+1}{2}\right]^2$$

$$= K\left[r^2 + (K+1)\cdot r + \frac{(K+1)^2}{4}\right]$$

$$S_2 = Kr^2 + K(K+1)r + \frac{1}{4}K(K+1)^2$$

$$S_1 - S_2 = \left\{\cancel{kr^2} + \cancel{k(k+1)r} + \frac{1}{6}k(k+1)(2k+1)\right\} - \left\{\cancel{kr^2} + \cancel{k(k+1)r} + \frac{1}{4}k(k+1)^2\right\}$$

$$= \frac{1}{6}k(k+1)(2k+1) - \frac{1}{4}k(k+1)^2$$

$$= \frac{1}{2}k(k+1)\left[\frac{1}{3}(2k+1) - \frac{1}{2}(k+1)\right]$$

$$= \frac{1}{2}k(k+1)\left[\frac{2(2k+1) - 3(k+1)}{6}\right]$$

$$= \frac{k(k+1)}{2}\left[\frac{4k+2-3k-3}{6}\right]$$

$$= \frac{k(k+1)}{2}\left[\frac{k-1}{6}\right]$$

$$= \frac{k(k^2-1)}{12} = \frac{k^3-k}{12}$$

Suppose there are l ties of length $\underbrace{k_1, k_2, \ldots, k_l}$ in first ranking and m ties of length $\underbrace{k_1', k_2' \ldots k_m'}$ in second ranking.

$$T = \frac{1}{12}\sum_{i=1}^{l}(k_i^3 - k_i) \quad \text{and} \quad T' = \frac{1}{12}\sum_{i=1}^{m}(k_i'^3 - k_i')$$

$\therefore\ T_u = \frac{1}{12n}\sum_{i=1}^{l}(k_i^3 - k_i)$ and $T_v = \frac{1}{12n}\sum_{i=1}^{m}(k_i'^3 - k_i')$

$\therefore$ variances should be

$$S_u^2 = \frac{n^2-1}{12} - T_u \quad \text{and} \quad S_v^2 = \frac{n^2-1}{12} - T_v$$

Again $\frac{1}{n}\sum_{i=1}^{n} d_i^2 = S_u^2 + S_v^2 - 2\,cov(u,v)$

or, $\frac{1}{n}\sum d_i^2 = \frac{n^2-1}{12} - T_u + \frac{n^2-1}{12} - T_v - 2\,cov(u,v)$

or, $2\,cov(u,v) = \frac{2(n^2-1)}{12} - (T_u + T_v) - \frac{1}{n}\sum_{i=1}^{n} d_i^2$

or, $cov(u,v) = \frac{n^2-1}{12} - \frac{(T_u+T_v)}{2} - \frac{1}{2n}\sum d_i^2$

$$\therefore\ r_R = \frac{cov(u,v)}{\sqrt{V(u)}\sqrt{V(v)}} = \frac{\frac{n^2-1}{12} - \frac{T_u+T_v}{2} - \frac{1}{2n}\sum d_i^2}{\sqrt{\frac{n^2-1}{12} - T_u}\ \sqrt{\frac{n^2-1}{12} - T_v}}$$

Q In an examination 9 students obtained the following marks in English and

The following marks in English and Mathematics. Find Spearman's rank correlation coefficient.

	5.5	1	8	5.5	8	8	2	3	4
Marks in Eng	45	60	32	45	32	32	58	56	47
" " maths	51	51	38	54	54	38	62	58	38
	5.5	5.5	8	3.5	3.5	8	1	2	8

Eng	5.5	1	8	5.5	8	8	2	3	4
maths	5.5	5.5	8	3.5	3.5	8	1	2	8

In the first (eng) ranking, there are two ties of length 2 and 3

$$\therefore T_u = \frac{1}{12n}\left[(2^3-2)+(3^3-3)\right]$$

$$= \frac{1}{12\times 9}\left[6+24\right) = \frac{\cancel{30}\ 10}{12\times\cancel{9}\ 3} = 0.277$$

In second ranking we have three ties of length 2, 3, 2.

$$\therefore T_v = \frac{1}{\ }\left[(2^3-2)+(3^3-3)+(2^3-2)\right]$$

$$\therefore T_v = \frac{1}{12n}\left[(2^3-2)+(3^3-3)+(2^3-2)\right]$$

$$= \frac{1}{\cancel{12}\times 9}\left[\cancel{36}\right] = \frac{1}{3} = 0.33$$

$$\frac{n^2-1}{12} = \frac{9^2-1}{12} = \frac{81-1}{12} = \frac{80}{12} = 6.66$$

$$\frac{1}{2n}\Sigma d_i^2 = \frac{1}{2\times 9}\left[0^2 + \cdots\cdots\right]$$

$$= \frac{64.75}{18} = 3.47$$

$$\therefore r_R = \frac{\frac{n^2-1}{12} - \frac{T_u+T_v}{2} - \frac{1}{2n}\Sigma d_i^2}{\sqrt{\frac{n^2-1}{12} - T_u}\sqrt{\frac{n^2-1}{12} - T_v}}$$

$$= \frac{2.888}{\sqrt{40.4634}}$$

$$= 0.45 \text{ (ans)}.$$