25 August 2023 09:38 PM

Random Sampling

$\rightarrow N: X_1 \; X_2 \; \cdots \; X_N \quad \boxed{\mu, \sigma^2}$

$\rightarrow n: x_1 \; x_2 \; \cdots \; x_n$

$\frac{1}{2}$

Sample mean $\bar{x}$

" varian s^2

$E(\bar{x})$ and $V(\bar{x})$

$\sqrt{V(\bar{x})} = S.E$

(i) SRSWR (with Replacement)

Probability $= \frac{1}{N}$

pop mean $= \mu = \frac{1}{N}\sum_{r=1}^{N} X_r$

pop varian $= \sigma^2 = \sum_r (X_r - \mu)^2 / N$

sample mean, $\bar{x} = \sum_i x_i / n$

Mean of sample mean $\boxed{E(\bar{x}) = \mu}$

$\boxed{Var(\bar{x}) = \frac{\sigma^2}{n}}$

$\boxed{SE(\bar{x}) = \sigma/\sqrt{n}}$

$Cov(x_i, x_j) = 0$

$V(x_1 + x_2) = V(x_1) + V(x_2) + 2\,Cov(x_1, x_2)$

$x_1 + x_2 + \cdots x_n$

$V(x_1) + V(x_2) \cdots V(x_n)$

$Var(\bar{x}) = \frac{1}{n^2} V(\sum x_i) = \frac{1}{n^2}\left\{\sum V(x_i) + \sum\sum Cov(x_i, x_j)\right\}$

(ii) SRSWOR

sample

$n: x_1 \; x_2 \; \cdots \; x_n$

$\frac{1}{n} \quad \frac{1}{n-1}$

$E(\bar{x}) = \mu =$ population mean

$V(\bar{x}) = \frac{\sigma^2}{n}\left[\frac{N-n}{N-1}\right]$

Q. $20, 24, 20, 30, 26.$ (marks of 5 students) $\leftarrow N = 5$

... size (n) $\rightarrow n = 2$

4 (20, 24, ...

SRSWOR of size 2 ← n=2

Samples of size 2	Sample values	Sample mean, $\bar{x}$
	(20, 24)	22
	(20, 20)	20
	(20, 30)	25
	(20, 26)	23
	(24, 20)	22
	(24, 24)	27
	(24, 30)	25
	(24, 26)	25
	(20, 30)	23
	(20, 26)	
	(30, 26)	28

Sampling distribution

value of $\bar{x}$	probability	$\bar{x}-\mu=\bar{x}-24$	$\times P$
20	1/10	$(-4)^2=16$	16/10
22	2/10	$(-2)^2=4$	8/10
23	2/10	$(-1)^2=1$	2/10
25	3/10	$(1)^2=1$	3/10
27	1/10	$(3)^2=9$	9/10
28	1/10	$(4)^2=16$	16/10
	$\Sigma p=\frac{10}{10}=1$	$\Sigma(\bar{x}-\mu)^2 P=\frac{54}{10}$	

$[20+22\times2+23\times2 \; ... +28]$

$$E(\bar{x}) = \frac{1}{10}\Sigma \bar{x}\cdot p_i = \frac{1}{10}\left[20 + 22\times 2 + 23\times 2 + \cdots + 28\right]$$

$$= 24 \text{ (ans)}.$$

$\mu = E(\bar{x}) = 24$ (population mean).

$$V(\bar{x}) = E(\bar{x} - \mu)^2$$

$$= \Sigma (\bar{x} - \mu)^2 p$$

$$= \frac{54}{10} = 5.4$$

$$\therefore S.E(\bar{x}) = \sqrt{V(\bar{x})} = \sqrt{5.4}$$

$\overset{\mu}{V(\bar{x}) = E(\bar{x} - E(\bar{x}))}$

$V(x) = E(x - E(x))^2$

$V(\bar{x}) = E(\bar{x} - E(\bar{x}))^2$

$= E(\bar{x} - \mu)^2$

$$\boxed{SE(\bar{x}) = \sigma/\sqrt{n}\left[\frac{N-n}{N-1}\right]}$$

20, 24, 26, 20, 30

$N = 5 \quad \mu = 24.$

population variance

$$\sigma^2 = \frac{1}{N}\Sigma (x_i - \mu)^2 = \frac{1}{5}\left[(20-24)^2 + (24-24)^2 + (20-24)^2 + (26-24)^2 + (30-24)^2\right]$$

$$\sigma^2 = \frac{72}{5}$$

Verify using formula: $V(\bar{x}) = \frac{\sigma^2}{n}\left[\frac{N-n}{N-1}\right]$

$$= \frac{72}{5\times 2}\left[\frac{5-2}{5-1}\right]$$

$$= \frac{72}{10}\times\frac{3}{4} = 5.4$$

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Ratio Estimator:

$$\tau_y = \sum_{i=1}^{N} y_i \qquad \tau_x = \sum_{i=1}^{N} x_i$$

$$\boxed{r = \frac{\tau_y}{\tau_x} = \frac{\bar{y}}{\bar{x}} = \frac{\sum y_i}{\sum x_i}}$$

$$\boxed{\tau_y = \frac{\mu_y}{\mu_x} \cdot \tau_x}$$

Ratio estimator $\boxed{\hat{\tau}_r = \frac{\bar{y}}{\bar{x}} \cdot \tau_x}$

$$\boxed{\hat{\mu}_r = \frac{\bar{y}}{\bar{x}} \mu_x}$$

MSE of $V(\hat{\mu}_r)$

$$\rightarrow Var(\hat{\mu}_r) \simeq \frac{N-n}{N} \cdot \frac{\sigma_r^2}{n}$$

$$\Rightarrow \sigma_r^2 = \frac{1}{N-1} \sum_{i=1}^{N} \left(y_i - \frac{\tau_y}{\tau_x} \cdot x_i\right)^2$$

$$s_r^2 = \frac{1}{n-1} \sum_{1}^{n} \left(y_i - \frac{\bar{y}}{\bar{x}} \cdot x_i\right)^2$$

$$Var(\bar{y}) = \left(\frac{N-n}{N}\right) \cdot \frac{\sigma^2}{n}$$

An approximate $100(1-\alpha)\%$ CI for μ_y is

$$\hat{\mu}_r \pm t_{n-1,\alpha/2} \sqrt{\hat{Var}(\hat{\mu}_r)} = \text{value.}$$

$t_{14, 0.05}$

$\mu_x - t_{n-1,\alpha/2} \cdots$

$*\ \hat{\tau}_r = N\hat{\mu}_r = \dfrac{\bar{y}}{\bar{x}} \cdot \tau_x$

$*\ \hat{Var}(\hat{\tau}_r) = N \cdot (N-n) \dfrac{s_r^2}{n}$

Apples	(Juice weight) y	(Apple weight) x	$y - \hat{r}x$	$(y_i - rx_i)^2$
1	0.16	0.22		
2	0.15	0.26		
⋮	⋮	⋮		
15	0.22	0.35		

$\Sigma y = 2.85$ $\quad \Sigma x = 4.32$

$\bar{y} = 0.19$ $\quad \bar{x} = 0.288$

$r = \dfrac{\bar{y}}{\bar{x}}$

Ratio estimate of total weight

$\hat{\tau}_r = r\,\tau_x = \dfrac{0.190}{0.288} \times 2000$

$= 1319.44$

$s_r^2 = \dfrac{1}{n-1} \Sigma (y_i - r x_i)^2$

$\hat{Var}(\hat{\tau}_r) = \tilde{N}(\hat{N} - n) \dfrac{s_r^2}{n}$

$= \dfrac{\tau_x}{\ } \left(\dfrac{\tau_x}{\ } - n \right) \dfrac{(\quad)}{\ }$

$$= \frac{\hat{\tau}_x}{\bar{x}} \left(\frac{\hat{\tau}_x}{\bar{x}} - n \right) \frac{(\qquad)}{n}$$

$$\hat{S \cdot D} \, (\hat{\tau}_x) = \sqrt{\hat{V}(\hat{\tau}_x)}$$

95% CI :