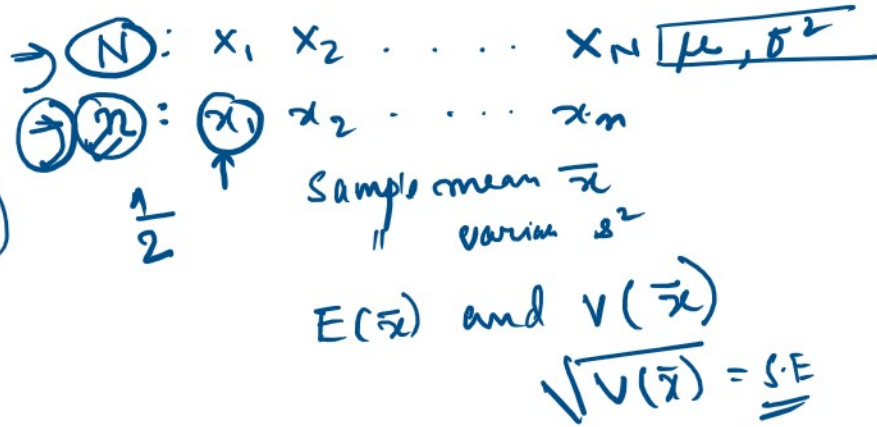


Random Sampling

(i) SRSWR (with Replacement)



Probability = $\frac{1}{N}$

pop mean = $\mu = \frac{1}{N} \sum_{i=1}^N x_i$
 pop varian = $\sigma^2 = \frac{\sum_{i=1}^N (x_i - \mu)^2}{N}$

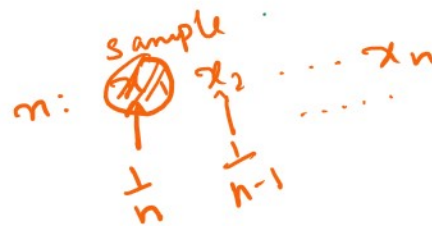
Sample mean, $\bar{x} = \frac{\sum x_i}{n}$

Mean of sample mean
 $E(\bar{x}) = \mu$

$Var(\bar{x}) = \frac{\sigma^2}{n}$
 $SE(\bar{x}) = \frac{\sigma}{\sqrt{n}}$

$Cov(x_i, x_j) = 0$
 $V(x_1 + x_2) = V(x_1) + V(x_2) + 2Cov(x_1, x_2)$
 $= \sigma^2 + \sigma^2 + 2 \cdot 0 = 2\sigma^2$
 $V(x_1 + x_2 + \dots + x_n) = n\sigma^2$

(ii) SRSWOR



$Var(\bar{x}) = \frac{1}{n^2} V(\sum x_i)$
 $= \frac{1}{n^2} \{ \sum V(x_i) + \sum_{i \neq j} 2Cov(x_i, x_j) \}$
 $= \frac{1}{n^2} \{ n\sigma^2 + \sum_{i \neq j} 2Cov(x_i, x_j) \}$

$E(\bar{x}) = \mu = \text{population mean}$

$V(\bar{x}) = \frac{\sigma^2}{n} \left[\frac{N-n}{N-1} \right]$

$N = 5$

$\{ \underline{20}, \underline{24}, \underline{20}, \underline{30}, \underline{26} \}$ (marks of 5 students)
 $n = 2$

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$(20, 24, 26, 30)$
 SRSWOR of size $(2) \leftarrow n=2$

Samples of size 2	Sample values	Sample mean, \bar{x}
	(20, 24)	22
	(20, 20)	20
	(20, 30)	25
	(20, 26)	23
	(24, 20)	22
	(24, 24)	24
	(24, 30)	25
	(24, 26)	25
	(20, 30)	23
	(20, 26)	23
	(30, 26)	28

Sampling distribution

value of \bar{x}	probability	$\bar{x} - \mu = (\bar{x} - 24) \times P$
20	$\frac{1}{10}$	$(-4)^2 = \frac{16}{10}$
22	$\frac{2}{10}$	$(-2)^2 = \frac{8}{10}$
23	$\frac{2}{10}$	$(-1)^2 = \frac{2}{10}$
25	$\frac{3}{10}$	$(1)^2 = \frac{3}{10}$
27	$\frac{1}{10}$	$(3)^2 = \frac{9}{10}$
28	$\frac{1}{10}$	$(4)^2 = \frac{16}{10}$
	$\sum P = \frac{10}{10} = 1$	$\sum (\bar{x} - \mu)^2 P = \frac{54}{10}$

$(1) \times [20 + 22 \times 2 + 23 \times 2 + \dots + 28]$

$$E(\bar{x}) = \frac{1}{10} \sum \bar{x} \cdot P_i = \left(\frac{1}{10} \right) [20 + 22 \times 2 + \dots + 28]$$

$$= 24 \text{ (ans)}$$

$$\mu = E(\bar{x}) = 24 \text{ (population mean)}$$

$$V(\bar{x}) = E(\bar{x} - \mu)^2$$

$$= \sum (\bar{x} - \mu)^2 P$$

$$= \frac{54}{10} = 5.4$$

$$V(\bar{x}) = E(\bar{x} - \mu)^2$$

$$V(x) = E(x - E(x))^2$$

$$V(\bar{x}) = E(\bar{x} - E(\bar{x}))^2$$

$$= E(\bar{x} - \mu)^2$$

$$SE(\bar{x}) = \frac{\sigma}{\sqrt{n}} \left[\frac{N-n}{N-1} \right]$$

20, 24, 26, 28, 30

$N = 5$ $\mu = 24$

$\therefore S.E(\bar{x}) = \sqrt{V(\bar{x})} = \sqrt{5.4}$

Population variance

$$\sigma^2 = \frac{1}{N} \sum (x_i - \mu)^2 = \frac{1}{5} [(20-24)^2 + (24-24)^2 + (26-24)^2 + (28-24)^2 + (30-24)^2]$$

$$\sigma^2 = \frac{72}{5}$$

Verify using formula:

$$V(\bar{x}) = \frac{\sigma^2}{n} \left[\frac{N-n}{N-1} \right]$$

$$= \frac{72}{5 \times 2} \left[\frac{5-2}{5-1} \right]$$

$$= \frac{72}{10} \times \frac{3}{4} = 5.4$$

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Ratio Estimator:

$$\tau_y = \sum_{i=1}^N y_i$$

$$\tau_x = \sum_{i=1}^N x_i$$

$$r = \frac{\tau_y}{\tau_x} = \frac{\bar{y}}{\bar{x}} = \frac{\sum y_i}{\sum x_i}$$

$$\tau_y = \frac{\mu_y}{\mu_x} \cdot \tau_x$$

Ratio estimator

$$\hat{\tau}_x = \frac{\bar{y}}{\bar{x}} \cdot \tau_x$$

$$\hat{\mu}_x = \frac{\bar{y}}{\bar{x}} \mu_x$$

mse of $v(\hat{\mu}_x)$

$$\rightarrow \text{Var}(\hat{\mu}_x) \approx \frac{N-n}{N} \cdot \frac{\delta_r^2}{n}$$

$$\Rightarrow \delta_r^2 = \frac{1}{N-1} \sum_{i=1}^N \left(y_i - \frac{\tau_y}{\tau_x} \cdot x_i \right)^2$$

$$\delta_r^2 = \frac{1}{n-1} \sum_{i=1}^n \left(y_i - \frac{\bar{y}}{\bar{x}} \cdot x_i \right)^2$$

$$\text{Var}(\bar{y}) = \left(\frac{N-n}{N} \right) \cdot \frac{\sigma^2}{n}$$

An approximate $100(1-\alpha)\%$ v. ce for μ_y is

$$\hat{\mu}_x \pm t_{n-1, \alpha/2}^{t_{14, 0.05}} \sqrt{\widehat{\text{Var}}(\hat{\mu}_x)} = \text{value.}$$

✓
n

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$$\mu_1 = \dots, \sigma^2 = \dots$$

$$\hat{Y}_N = N \hat{\mu}_1 = \frac{\bar{y}}{\bar{x}} \cdot T_x$$

$$\text{Var}(\hat{Y}_N) = N \cdot (N-n) \frac{s_y^2}{n}$$

Apples	(Juli weight) y	(Apple weight) x	$y - \lambda x$	$(y - \lambda x)^2$
1	0.16	0.22		
2	0.15	0.26		
⋮	⋮	⋮		
15	0.22	0.35		

$$\Sigma y = 2.85$$

$$\bar{y} = 0.19$$

$$\Sigma x = 4.32$$

$$\bar{x} = 0.288$$

$$\lambda = \frac{\bar{y}}{\bar{x}}$$

Ratio estimate of total weight

$$\hat{Y}_N = \lambda T_x = \frac{0.190}{0.288} \times 2000$$

$$= 1319.44$$

$$s_y^2 = \frac{1}{n-1} \Sigma (y_i - \lambda x_i)^2$$

$$\text{Var}(\hat{Y}_N) = \tilde{N} (\tilde{N} - n) \frac{s_y^2}{n}$$

$$= \frac{T_x}{\bar{x}} \left(\frac{T_x}{\bar{x}} - n \right) \left(\frac{\quad}{\quad} \right)$$

$$= \frac{\bar{Y}_x}{\bar{x}} \left(\frac{\bar{Y}_x}{\bar{x}} - n \right) \frac{1}{n}$$

$$\hat{S.D.}(\hat{\bar{Y}}_x) = \sqrt{v^{\wedge}(\hat{\bar{Y}}_x)}$$

95% CI :