Random Variables Questions

Thursday, September 7, 2023 6:53 PM

Random var – II

9062395723

Random Variables ...

Random Variables ...

Negative Binomial ..

n = 10 → Head nBmf

Head ⇒ 6 n = ??

JAVAN → 500 crore → 1000 ... 1000 ... Profit ??

HH HT TH TT

Heads → 0, 1, 2 $R_x \to \{0, 1, 2\}$

finite → countable → X → discrete Random var.

1. I toss a fair coin twice, and let X be defined as the number of heads I observe. Which of the following is correct ? ($P_x(k)$ denotes the PMF)

(A) $P_x(0) = 1/2$ ✗ (B) $P_x(1) = 2/3$ ✗

(C) $P_x(2) = 3/4$ ✗ (D) $P_x(2) = 1/4$ ✓

$P_x(0) = P(X=0) = P(TT) = 1/4$

$P_x(1) = \frac{1}{4} + \frac{1}{4} = 1/2$

$P_x(2) = 1/4$

2. Suppose you have the following information about the cdf of a random variable X, which takes one of 4 possible values:

Value of X	1	2	3	4
Cdf	0.25	0.4	0.8	

Which of the following is/are true?

(A) Pr(X = 2) = 0.4

(B) E(X) = 2.5

(C) Pr(X = 4) = 0.2

(D) None of the above

3. Let X be a discrete random variable with values x = 0, 1, 2 and probabilities P(X = 0) = 0.25, P(X = 1) = 0.50, and P(X = 2) = 0.25, respectively.

Find $E(X^2)$

(A) 1.5

(B) 1

(C) 1.3

(D) 0

(C) 1.3 (D) 0

$\sum x^2 \cdot p(x)$

4. Let X be a discrete random variable that is the value shown on a single roll of fair die. What is the expected value of X?

(A) 4.5
(B) 3.5
(C) 2.5
(D) None of these

$E(X) = \sum x \cdot p(x) = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6} = 3.5$

5. Let X be a continuous random variable whose probability density function is

$$f(x) = \begin{cases} 2x & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

calculate the probability than X falls between 0 and 1/2.

(A) 1 (B) 0.5

(C) 0.25 (D) None of the above

6. Let the binary random variable X have pdf $f(x) = p^x(1-p)^{1-x}$ for $x = 0,1$ Let $X_1,\ldots,X_n$ be independent discrete (0,1) random variables each with probability density function f(x). The random variable $B = X_1 + X_2 + \ldots + X_n$ has a binomial distribution with parameters n and p. The values of B are b = 0, ..., n and represent the number of "successes" (i.e., $X_i = 1$) in n independent trials of an experiment, each with probability p of success. Calculate variance of B using the rules of expected values and variances.

(A) np
(B) np(1 + p)
(C) np(1 − p)
(D) None of the above

$f(x) = p^x(1-p)^{1-x}$

$E(x) = \sum x f(x) = 0 \cdot f(0) + 1 \cdot f(1) = p^1(1-p)^{1-1} = p$

$V(x) = \sum (x-p)^2 f(x) = \sum (x-p)^2 p^x (1-p)^{1-x}$

$= (-p)^2 \cdot p^0(1-p)^{1-0} + (1-p)^2 \cdot p^1(1-p)^{1-1}$

$= p^2(1-p) + (1-p)^2 \cdot p$

$= p(1-p)(p + 1 - p) = p(1-p)$

$E(B) = E(x_1 + x_2 + \ldots + x_n) = E(x_1) + E(x_2) + \ldots + E(x_n)$

$= p + p \ldots + p = np$

$V(x) = V(x_1 + \ldots) = p(1-p) \ldots \; n$ times $= np(1-p)$

7. Let $X_1,X_2,\ldots,X_n$ be independent random variables which all have the same probability distribution, with mean μ and variance σ². Let

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

then which of the following is true?

(A) $E(\bar{X}) = \mu$
(B) $E(\bar{X}) = \mu - 1$
(C) $E(\bar{X}) = \mu + 1$
(D) $E(\bar{X}) = \mu / 2$

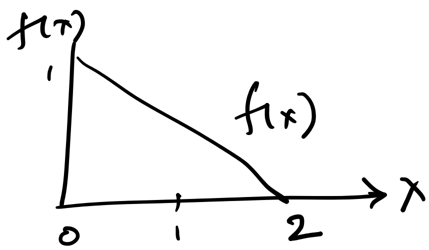
$E(\bar{x}) = E\left[\frac{1}{n}(x_1 + x_2 + \ldots + x_n)\right] = \frac{1}{n} E(x_1) + E(x_2) + \ldots E(x_n)$

(C) $E(\overline{X}) = \mu + 1$ (D) $E(\overline{X}) = \mu / 2$

$E(\bar{x}) = E\left[\frac{1}{n}(x_1 + x_2 + \dots + x_n)\right] = \frac{1}{n} E(x_1) + E(x_2) + \dots E(x_n)$

$= \frac{1}{n}(\mu + \mu + \dots + \mu)$

$= \frac{n\mu}{n} = \mu$



$x = 0.5 \quad f(0.5) = 0.75$

$P(X \le 0.5) = 1 - P(X > 0.5)$

$= 1 - \frac{1}{2} \times 1.5 \times 0.75$

$= 0.4375$

8. Let X be a continuous random variable with probability density function given by

$f(x) = -0.5x + 1, \qquad 0 \le x \le 2$

Find $P(X \le 0.5)$.

(A) 1 (B) 1.254

(C) 0.1287 (D) None of the above

9. Consider the discrete random variable Y that takes the values y = 1, 2, 3, and 4 with probabilities 0.1, 0.2, 0.3, and 0.4, respectively.
If we take a random sample of size N = 3 from this distribution, what are the mean and variance of the sample mean, $\bar{Y} = (Y_1 + Y_2 + Y_3)/3$?

(A) $\text{var}(\bar{Y}) = 3, E(\bar{Y}) = \frac{1}{3}$

(B) ~~$E(\bar{Y}) = 3, \text{var}(\bar{Y}) = \frac{1}{3}$~~

(C) $E(\bar{Y}) = 0.3, \text{var}(\bar{Y}) = \frac{1}{3}$

(D) $E(\bar{Y}) = 1, \text{var}(\bar{Y}) = 3$

$E(y) = 1(0.1) + 2(0.2) + 3(0.3) + 4(0.4) = 3$

$\text{var}(y) = \sum (1-3)^2 \cdot 0.1 + (2-3)^2\, 0.2 + (3-3)^2\, 0.3 + (4-3)^2 \cdot 0.4 = 1$

$E(\bar{y}) = E\left(\frac{y_1 + y_2 + y_3}{3}\right) = \frac{3}{3} + \frac{3}{3} + \frac{3}{3} = 3$

$V(\bar{y}) = \frac{1}{9} \cdot 1 + \frac{1}{9} \cdot 1 + \frac{1}{9} \cdot 1 = \frac{1}{3}$

$\int_0^3 cx^2 dx = 1 \Rightarrow 9c$

$c = 1/9$

10. Find the constant c such that the function

$$f(x) = \begin{cases} cx^2 & 0 < x < 3 \\ 0 & \text{otherwise} \end{cases}$$

is a density function.

(A) c = 1/5

(B) c = 2/9

(C) c = 1/9

(D) None of these

11. A random variable X has the density function $f(x) = c/(x^2 + 1)$. where $-\infty < x < \infty$. Find the probability that X^2 lies between 1/3 and 1.

(A) 1/6 (B) 1/3

(C) 4/3 (D) 1/9

12. The distribution function for a random variable X is

$$F(x) = \begin{cases} 1-e^{-2x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Find (a) the density function,

(A) $f(x) = \begin{cases} e^{-2x} & x > 0 \\ 0 & x < 0 \end{cases}$

(B) $f(x) = \begin{cases} 2e^{-2x} & x \geq 0 \\ 0 & x < 0 \end{cases}$

(C) $f(x) = \begin{cases} 2e^{-2x} & x > 0 \\ 0 & x < 0 \end{cases}$

(D) None of the above

$f(x) = \frac{d}{dx} F(x)$

$= 2e^{-3x} \quad x > 0$

$0 \quad x < 0$

$x = \sqrt[4]{u-1}$ $u = 2, 17, 82 \ldots$

Real +ve Root is taken

Regard Root for U is $g(u) = 2^{-\sqrt[4]{u-1}}$ $u = 2, 17, 82$

$= 0$ Others

13. The probability function of a random variable X is

$$f(x) = \begin{cases} 2^{-x} & x = 1,2,3,\ldots \\ 0 & \text{otherwise} \end{cases}$$

Find the probability function for the random variable $U = X^4 + 1$.

(A) $g(u) = \begin{cases} 2^{-\sqrt[4]{u-1}} & u = 2,17,82,\ldots \\ 0 & \text{otherwise} \end{cases}$ (B) $g(u) = \begin{cases} 2^{-\sqrt{u-1}} & u = 2,17,82,\ldots \\ 0 & \text{otherwise} \end{cases}$

(C) $g(u) = \begin{cases} 2^{\sqrt[4]{u+1}} & u = 2,17,82,\ldots \\ 0 & \text{otherwise} \end{cases}$ (D) None of the above

$u = \frac{1}{3}(12 - x)$

$x = 12 - 3u$

1 to 1 Relation

now, $\psi'(u) = \frac{dx}{du} = -3$

$X \to$ CRV

$U = \phi(x)$ then $x = \psi(u)$

$g(u)\, du = f(x)\, dx$

$g(u) = f(x) \left| \frac{dx}{du} \right| = f[\psi(u)]\psi'(u)$

then the density function for U is

$g(u) = (12 - 3u)^2 / 27 \quad 2 < u < 5$

$= 0$, others

$\int_2^5 f(x)\,dx = \int_{-1}^{2} \frac{x^2}{3}\,dx = \left(\frac{x^3}{9}\right)_{-1}^{2} = 8/9 + 1/9 = 1$

14. The probability function of a random variable X is given by

$$f(x) = \begin{cases} x^2/81 & -3 < x < 6 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability density for the random variable $U = \frac{1}{3}(12 - X)$.

(A) $g(u) = \begin{cases} (12+3u)^2/27 & 2 < u < 5 \\ 0 & \text{otherwise} \end{cases}$ (B) $g(u) = \begin{cases} (12-3u)^2/27 & 2 < u < 5 \\ 0 & \text{otherwise} \end{cases}$

(C) $g(u) = \begin{cases} (12+3u^2)/27 & 2 < u < 5 \\ 0 & \text{otherwise} \end{cases}$ (D) None of the above

(A) $g(\cdot) \begin{cases} & \\ 0 & \text{otherwise} \end{cases}$ (B) $g(\cdot) \begin{cases} & \\ 0 & \text{otherwise} \end{cases}$

(C) $g(u) = \begin{cases} (12 + 3u^2)/27 & 2 < u < 5 \\ 0 & \text{otherwise} \end{cases}$ (D) None of the above

15. If the following function is a probability density function :

$$f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2 \\ 0, & \text{elsewhere} \end{cases}$$

find $P(0 < X \leq 1)$.

(A) 1/3 (B) 2/5

(C) 1/9 (D) None of the above

$\int_{-1}^{2} f(x)dx$

16. If X has the probability density

$$f(x) = \begin{cases} k.e^{-3x} & \text{for } x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

find k.

(A) 1 (B) 2

(C) 4 (D) 3

17. Find a probability density function for the random variable whose distribution function is given by

$$F(x) = \begin{cases} 0 & \text{for} \quad x \le 0 \\ x & \text{for} \quad 0 < x < 1 \\ 1 & \text{for} \quad x \ge 1 \end{cases}$$

(A) $f(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ 2 & \text{for} \quad 0 < x < 1 \\ 0 & \text{for} \quad x > 1 \end{cases}$

(B) $f(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ 1 & \text{for} \quad 0 < x < 1 \\ 0 & \text{for} \quad x > 1 \end{cases}$

(C) $f(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ 1 & \text{for} \quad 0 < x < 1 \\ 1 & \text{for} \quad x > 1 \end{cases}$

(D) None of the above

18. The probability distribution function of a random variable X is

$$f(x)=\begin{cases} x & \text{for } 0<x\le 1 \\ 2-x & \text{for } 1\le x<2 \\ 0 & \text{for } x\ge 2 \end{cases}$$

Compute the cumulative distribution function of X.

(A) $F(x)=\begin{cases} 0 & ,x\le 0 \\ 1 & ,0<x<1 \\ 2x-\frac{1}{2}x^2-1 & ,1<x\le 2 \\ 1 & ,x\ge 2 \end{cases}$

(B) $F(x)=\begin{cases} 1 & ,x\le 0 \\ \frac{1}{2} & ,0<x<1 \\ 2x-\frac{1}{2}x^2-1 & ,1<x\le 2 \\ 1 & ,x\ge 2 \end{cases}$

(C) $F(x)=\begin{cases} 0 & ,x\le 0 \\ \frac{1}{2} & ,0<x<1 \\ 2x-\frac{1}{2}x^2-1 & ,1<x\le 2 \\ 1 & ,x\ge 2 \end{cases}$

(D) None of the above

19. The probability mass function of a r.v X is given as follows

x:	0	1	2	3	4	5
f(x):	k^2	$\frac{k}{4}$	$\frac{5k}{2}$	$\frac{k}{4}$	$2k^2$	k^2

Find the value of k.

(A) 1 (B) 1/2

(C) 1/3 (D) 1/4

20. Let the distribution function of X be

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & 0 \le x \le 1 \\ 1 & x > 1 \end{cases}$$

Find P(2X + 3 $\leq$ 3.6).

(A) 1.1

(B) 0.2

(C) 0.3

(D) 0.5

$$F(x) = \int_{-\infty}^{x} f(t)\,dt = \int_{0}^{x} 2t\,dt$$

$$= \left[t^2\right]_0^x = x^2 - 0$$

$$= x^2$$

21. If a continuous r.v X has the pdf

$$f(x) = \begin{cases} 2x & \text{if } 0 < x \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the distribution function F(X)

(A) ✓ $F(x) = \begin{cases} 0 & , x < 0 \\ x^2 & , 0 \le x < 1 \\ 1 & , x > 1 \end{cases}$

(B) $F(x) = \begin{cases} 0 & , x < 0 \\ x & , 0 \le x < 1 \\ 1 & , x > 1 \end{cases}$

(C) $F(x) = \begin{cases} 0 & , x < 0 \\ x^3 & , 0 \le x < 1 \\ 1 & , x > 1 \end{cases}$

(D) None of the above

$$F(x) = \begin{cases} 0 & x < 0 \\ x^2 & 0 \le x < 1 \\ 1 & x > 1 \end{cases}$$

22. For the function

$$f(x)=\begin{cases}\frac{1}{16}(3+x)^2, & -3\le x\le -1\\ \frac{1}{16}(6-2x^2), & -1\le x\le 1\\ \frac{1}{16}(3-x)^2, & 1\le x\le 3\end{cases}$$

where the variable X is in the range (–3, 3).

which of the following is true?

(A) It is a probability mass function

(B) It is a probability density function

3 times separately integrate..

22(B)

(C) It is a cumulative distribution function

(D) None of the above

23. The function

$$f(x) = \frac{1}{\pi} \times \frac{1}{1+x^2}, -\infty \le x \le \infty \text{ is a}$$

(A) probability density function
(B) probability mass function
(C) cumulative distribution function
(D) None of the above

$0 \le F(t) \le 1$ for all t

$F(-\infty) = 0 \quad F(\infty) = 1$

$F(t)$ is non-decreasing function

+ right continuous for all t.

Hence its distribution function

24. The

$$F(t) = \begin{cases} 0, & t < 0 \\ t^2, & 0 \le t < 1/2 \\ 1 - 3(1 - t^2), & 1/2 \le t < 1 \\ 1, & t \ge 1 \end{cases}$$

is a

(A) probability mass function

(B) probability density function

(C) distribution function

(D) None of these

25. The following function

$$f(x) = \begin{cases} \frac{1}{2x} & \text{for } x = 1, 2, \\ 0 & \text{elsewhere} \end{cases}$$ is a

(A) a pdf

(B) not a pdf

(C) cdf

(D) None of the above

26. If the pdf of a r.v is given as $f(x) = \begin{cases} \frac{x}{15} & \text{if } x = 1, 2, 3, 4, 5 \\ 0 & \text{elsewhere} \end{cases}$ Find the distribution function of X?

(A) $F(x) = \begin{cases} 0 & , \quad x < 1 \\ \frac{1}{15} & , \quad 1 \le x < 2 \\ 6 & , \quad 3 \le x < 1 \\ \frac{10}{15} & , \quad 4 \le x < 5 \\ 1 & , \quad x \ge 5 \end{cases}$ ✗

(B) $F(x) = \begin{cases} 0 & , \quad x < 1 \\ 1 & , \quad 1 \le x < 2 \\ \frac{6}{15} & , \quad 3 \le x < 1 \\ \frac{10}{15} & , \quad 4 \le x < 5 \\ 1 & , \quad x \ge 5 \end{cases}$ ✗

$F(x) = P(X \le x)$

$F(x) = \begin{cases} 0 & \text{if } x < 1 \\ 1/15 & \text{if } 1 \le x < 2 \\ 3/15 & 2 \le x < 3 \\ 6/15 & 3 \le x < 4 \\ 10/15 & 4 \le x < 5 \\ 1 & x \ge 5 \end{cases}$

(C)

(C) $F(x)=\begin{cases} 0 & , \quad x<1 \\ \frac{1}{15} & , \quad 1 \le x < 2 \\ \frac{6}{15} & , \quad 3 \le x < 1 \\ \frac{10}{15} & , \quad 4 \le x < 5 \\ 1 & , \quad x \ge 5 \end{cases}$

(D) None of the above

$f(x) = 0$

$$F(x) = \begin{cases} 1/8 & x=0 \\ 3/8 & x=1 \\ 3/8 & x=2 \\ 1/8 & x=3 \end{cases}$$

27. Obtain the distribution function of the total number of heads occuring in 3 tosses of an unbiased coin.

(A) $F(x)=\begin{cases}1 & ,x<0\\ \frac{1}{8} & ,0\le x<1\\ \frac{4}{8} & ,1\le x<2u\\ \frac{7}{8} & ,2\le x<3\\ 1 & ,\ge 3\end{cases}$

(B) $F(x)=\begin{cases}0 & ,x<0\\ \frac{1}{8} & ,0\le x<1\\ 4 & ,1\le x<2u\\ \frac{7}{8} & ,2\le x<3\\ 1 & ,\ge 3\end{cases}$

(C) $F(x)=\begin{cases}0 & ,x<0\\ \frac{1}{8} & ,0\le x<1\\ \frac{4}{8} & ,1\le x<2u\\ \frac{7}{8} & ,2\le x<3\\ 1 & ,\ge 3\end{cases}$

(D) None of these

$F(x) = P(X \le x)$ will be

0

1/8

4/8

7/8

1

Randomness

0, 1, 3, 4, - - .

$-\infty, -67, -843^{45}, e^{x^{3^4}}, + 843^{844}$

So, the differentiated similarities of the Random Variables were differ.

Randomness is a relative concept with respect to the given analysis..

28. Suppose the life in weeks of a certain kind of computers has the pdf:

$$f(x)=\begin{cases}\frac{100}{x^2} & \text{when } x\ge 100\\ 0 & \text{when } x<100\end{cases}$$

What is the probability that none of three such computers will have to be replaced during the first 150 weeks of operation?

(A) 1/27 (B) 1/28

(C) 1/29 (D) 1/33

(A) 1/27

(B) 1/28

(C) 1/29

(D) 1/33

29. The amount of sugar (in kg), a certain shop is able to sell is given with the probability density:

$$f(x) = \begin{cases} \frac{1}{25}x & 0 \le x < 5 \\ \frac{10-x}{25} & 5 \le x < 10 \end{cases}$$

Find the probability that the sales is between 2.5 kg and 7.5 kg.

(A) 0.45

(B) 0.75

(C) 0.55

(D) 0.33

30. If the distribution function of X is given by

$$F(b) = \begin{cases} 0 & b < 0, \\ 1/3 & 0 \le b < 1, \\ 2/3 & 1 \le b < 2 \\ 1 & b \ge 2 \end{cases}$$

Calculate the probability mass function of X.

(A) 1/2 (B) 1/3

(C) 3/2 (D) 2/3

SECTION-(B) MULTIPLE SELECT QUESTIONS (MSQ)

1. Let $X \sim N(\mu, \sigma^2)$. If $\sigma^2 = \mu^2$, $(\mu > 0)$, then the value of $p(X < -\mu \mid X < \mu)$ in terms of cumulative function of N(0, 1) is

(A) $2[1 - p(z \le 1)]$ (B) $2[1 - p(z \le 2)]$

(C) $[1 - p(z \le 1)]$ (D) $[1 - p(z \le 2)]$

$$\frac{P(X < -\mu \cap X < \mu)}{P(X < \mu)} = \frac{P(X < -\mu)}{P(X < \mu)} \quad (\text{here } \mu > 0)$$

or, $$\frac{P\left(\frac{X-\mu}{\sigma} < -\frac{2\mu}{\sigma}\right)}{P\left(\frac{X-\mu}{\sigma} < \frac{\mu-\mu}{\sigma}\right)} \Rightarrow \frac{P\left(\frac{X-\mu}{\sigma} < -2\right)}{P\left(\frac{X-\mu}{\sigma} < 0\right)} \quad (\mu^2 = \sigma^2)$$

or $$\frac{P\left(\frac{X-\mu}{\sigma} < -2\right)}{P\left(\frac{X-\mu}{\sigma} < 0\right)} \Rightarrow \frac{P(Z < -2)}{P(Z < 0)} = \frac{P(Z > 2)}{P(Z < 0)}$$

$$\Rightarrow \frac{P(Z > 2)}{\frac{1}{2}} = 2P(Z > 2)$$

$$\Rightarrow 2[1 - P(Z \le 2)]$$

2. If X_1 and X_2 are independent cauchy variates with parameters (λ_1, μ_1) and (λ_2, μ_2) respectively then $X_1 + X_2$ is

(A) a normal variate $N(\lambda_1 + \lambda_2, \mu_1 + \mu_2)$.

(B) a normal variate $N(\lambda_1 + \lambda_2, \mu_1^2 + \mu_2^2)$.

(C) a cauchy variate with parameters $(\lambda_1 + \lambda_2, \mu_1 + \mu_2)$.

(D) a cauchy variate with parameters $(\lambda_1 - \lambda_2, \mu_1 - \mu_2)$.

c.f. $\phi_{X_1}(t) = e^{i[\mu_1 t + \lambda_1 (t)]}$

(D) a cauchy variate with parameters $(\lambda_1 - \lambda_2, \mu_1 - \mu_2)$.

c.f. $\phi_{X_1}(t) = e^{i[\mu_1 t + \lambda_1 |t|]}$

now, $\phi_{X_1+X_2}(t) = e^{i[(\mu_1+\mu_2)t + (\lambda_1+\lambda_2)t]}$

$= e^{i[\mu_1 t - \lambda_1 |t|]} e^{i[\mu_2 t - \lambda_2 |t|]}$

$= e^{i[\mu_1 t - \lambda_1 |t|]} e^{i[\lambda_2 t - \lambda_2 |t|]}$

$= \phi_{X_1} + \phi_{X_2}$ $+(-)$

the function is being unique so, (X_1+X_2) is a Cauchy variate with parameters $(\lambda_1+\lambda_2)$ $(\mu_1+\mu_2)$ ...

$i[(\mu_1+\mu_2)t + (\lambda_1+\lambda_2)t$

$= i[\mu_1 t + \mu_2 t + \lambda_1 t + \lambda_2 t]$

$= i[(\mu_1+\lambda_1)t + (\mu_2+\lambda_2)t]$

3. If $P(A) = \frac{3}{5}$ and $P(B) = \frac{2}{3}$, then which of the following option (S) is/are correct ?

(A) $P(A \cup B) \geq \frac{2}{3}$

(B) $\frac{4}{15} \leq P(A \cap B) \leq \frac{3}{5}$

(C) $\frac{2}{5} \leq P(A/B) \leq \frac{9}{10}$

(D) $P(A \cap \bar{B}) \leq \frac{1}{3}$

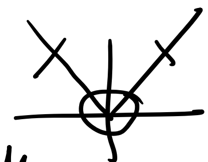
Multiple Regression

Cont →
Dum → Cont

90623-95723

L | R

y = |x|



4. A continues r.v X has the pdf $f(x) = 3x^2$, $0 \le x \le 1$, find a and b such that

1. $P(X \le a) = P(X > a)$
2. $P(X > b) = 0.05$

not desirable

$y = \alpha + \beta x + \epsilon_i$

TSS = ESS + RSS

(A) $a = \sqrt[3]{\frac{1}{2}}$

(B) $b = 1$

(C) $a = 0.95$

(D) $b = 0.98$

5. Random variable X is distributed with the following pdf :

$$f_x(u) = \begin{cases} \sin(x) & 0 \le x \le A \\ 0 & \text{otherwise} \end{cases}$$

Then which of the following is/are correct ?

(A) $A = \frac{\pi}{2}$

(B) $Var(X) = \pi - 3$

(C) $E|X| = 1$

(D) All of these

6. A random variable X has the following probability distribution :

x:	0	1	2	3	4	5	6	7
p(x):	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

then which of the following are true?

(A) $k = 1/10$

(B) $P(X > 6) = 0.45$

(C) $P(X < 6) = 0.81$

(D) All of the above

7. If $P(x) = \begin{cases} \frac{x}{15}; & x = 1,2,3,4,5 \\ 0, & \text{elsewhere} \end{cases}$

then which of the following options are correct?

(A) P{X = 1 or 2} = 1/7

(B) P{X = 1 or 2} = 1/5

(C) $P\left\{\frac{1}{2} < X < \frac{5}{2} \mid X > 1\right\} = 1/5$

(D) $P\left\{\frac{1}{2} < X < \frac{5}{2} \mid X > 1\right\} = 1/7$

8. The diameter of an electric cable, say X, is assumed to be a continuous random variable with p.d.f $f(x) = 6x(1 - x)$, $0 \leq x \leq 1$. Then which of the following are correct?

(A) f(x) is a cdf

(B) f(x) is a pmf

(C) f(x) is a pdf

(D) b = 0.5

9. Let X be a continuous random variate with p.d.f.

$$f(x)=\begin{cases} ax, & 0 \le x \le 1 \\ a, & 1 \le x \le 2 \\ -ax+3a, & 2 \le x \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

then which of the following are correct?

(A) $a = 0.5$

(B) $P(X \le 1\cdot5) = 1$

(C) $a = 5$

(D) $P(X \le 1\cdot5) = 0.5$

10. A random variable X has the probability law :

$$dF(x) = \frac{x}{b^2} . e^{-x^2/2b^2} \, dx, \quad 0 \le x < \infty$$

then which of the following holds?

(A) the ratio of the distance between the quartiles the standard deviation of X is independent of the parameter 'b'

(B) the distance between the quartiles is $2b^2$

(C) the distance between the quartiles is 2b

(D) None of these

SECTION-(C) NUMERICAL ANSWER TYPE QUESTIONS (NAT)

1. Let the events A and B be defined as follows :

   A : One waits between 0 to 2 minutes inclusive :

   B : One waits between 0 to 3 minutes inclusive

then $P(\overline{A} \cap \overline{B}) =$ ________ (correct upto two decimal places)

2. The amount of bread (in hundreds of pounds) X that a certain bakery is able to sell in a day is found to be a numerical valued random phenomenon, with a probability function specified by the probability density function f(x), given by

$$f(x) = \begin{cases} A.X, & \text{for } 0 \le x < 5 \\ A(10 - x), & \text{for } 5 \le x < 10 \\ 0, & \text{otherwise} \end{cases}$$

The value of A such that f(x) is a probability density function is ______.

3. The mileage C in thousands of miles which car owner get with a certain kind of tyre is a random variable having probability density function

$$f(x)=\begin{cases}\frac{1}{20}e^{-x/20}, & \text{for } x>0\\ 0, & \text{for } x\le 0\end{cases}$$

The probability that one of these tyres will-last at most 10,000 miles is _______. (correct upto 4 decimal places)

4. Suppose that the time in minutes that a person has to wait at a certain station for a train is found to be a random phenomenon, a probability function specified by the distribution function,

$$F(x) = \begin{cases} 0, & \text{for } x \le 0 \\ \frac{x}{2}, & \text{for } 0 \le x < 1 \\ \frac{1}{2}, & \text{for } 1 \le x < 2 \\ \frac{x}{4}, & \text{for } 2 \le x < 4 \\ 1, & \text{for } x \ge 4 \end{cases}$$

Then the probability that a person will have to wait more than 3 minutes is _______.

5. A petrol pump is supplied with petrol once a day. If its daily volume X of sales in thousands of litres is distributed by

$$f(x) = 5(1 - x)^4, \quad 0 \le x \le 1,$$

then the capacity of its tank in order that the probability that its supply will be exhausted in a given day shall be 0.01 is ________.

6. Let k > 0 be a constant, and

$$f(x) = \begin{cases} kx(1-x), & 0 < x < 1, \\ 0, & \text{otherwise.} \end{cases}$$

then f defines a pdf if k = ______.

7. If the probability density function of a *continuous random variable X is*

$$f(x) = \begin{cases} 0.5x & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

then, P(1 ≤ x ≤ 1.5) is _______. *(correct upto 4 decimal places)*

8. If the probability density function of a continuous random variable X is

$$f(x) = \begin{cases} kx^2 & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

then the value of k is ________. (correct upto 3 decimal places)

9. The probability density function of X, the lifetime of a certain type of electronic device (measured in hours), is given by,

$$f(x) = \begin{cases} \frac{10}{x^2} & x > 10 \\ 0 & x < 10 \end{cases}$$

P(X > 20) = ________.

10. If X has a density function given by

$$f(x) = \begin{cases} c(1-x^2) & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

E(X) = ________.

11. The lifetime in hours of an electronic tube is a random variable having a probability density function given by

$$f(x) = xe^{-x} \qquad x > 0$$

The expected value lifetime of such a tube is equal to _______.

12. Trains headed for destination A arrive at the train station at 15-minute intervals starting at 7 AM, whereas trains headed to destination B arrive at 15-minute intervals starting at 7:05 AM.

If a certain passenger arrives at the station at a time uniformly distributed between 7 and 8 AM and then gets on the first train that arrives, th proportion of time does he or she go to destination A is ________. (correct upto two decimal places)

13. If 65 percent of the population of a large community is in favor of a proposed rise in school taxes, approximate probability that a random sample of 100 people will contain at least 50 who are in favor of the proposition is _________. (correct upto 4 decimal places)

14. For the random variable Y in

$$P_Y(y) = \begin{cases} (1-p)^{y-1}p & \text{for } y = 1, 2, 3, \ldots \\ 0 & \text{otherwise} \end{cases}$$

If $p = \frac{1}{2}$, then $P(2 \leq Y < 5)$ = ________. (correct upto 4 decimal places)

15. Suppose that the random variable X is equal to the number of hits obtained by a certain baseball player in his next 3 at bats. If P(X –1) – 0.3, P(X –2) – 0.2 and P(X –0) – 3P(X – 3). Then E(X) = ________. (correct upto 3 decimal places)

16. A continuous random variable X has pdf

$$f(x) = \begin{cases} kx(1-x)^{10}, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

then k equals to ________.

17. For the pdf $f(x)=\begin{cases} cx(1-x)^3 & ,0<x<1 \\ 0 & ,\text{elsewhere} \end{cases}$

then the constant c equals to _______.

18. The random vector (X, Y) is said to be uniformly distributed over a region R in the plane if, for some constant c, its joint density is

$$f(x,y)=\begin{cases} c & \text{if } (x,y)\in R \\ 0 & \text{otherwise} \end{cases}$$

Then the probability that (X, Y) lies in the circle of radius 1 centered at the origin (i.e., $P(X^2 + Y^2 < 1)$ is _______. (correct upto 3 decimal places)

19. Let X be a random variable with image Im(X) = {0, 1, 2, 3}.

x	0	1	2	3
$p_x(x)$	0.5	0.25	0.1	_

$p_x(x = 3) =$ ________.

20. The cumulative distribution function of random variable X is

$$F_X(x) = \begin{cases} 0 & x < -1, \\ (x+1)/2 & -1 \le x < 1, \\ 1 & x \ge 1. \end{cases}$$

Then the value of a such that $P[X \le a] = 0.8$ is ______.