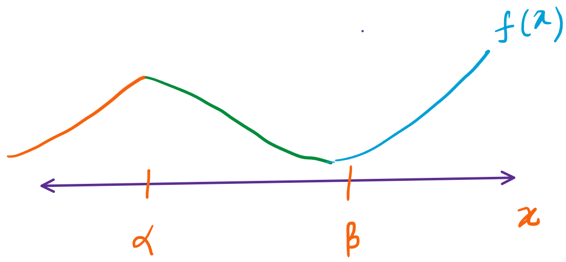
$f(x) = x^3 + ax^2 + bx + c$.

$f'(x) = 3x^2 + 2ax + b$.

Case II: $D_{f'(x)} > 0 \Rightarrow f'(x)$ has 2 real & unequal roots, say $\alpha, \beta\ [\alpha, \beta]$

$f'(x) = 3(x-\alpha)(x-\beta)$.

Shape of $f(x)$



If $x < \alpha$, $\underbrace{\ }_{<0}$ $\underbrace{\ }_{<0}$ $\Rightarrow f'(x) > 0$.

If $x > \beta$, $\underbrace{\ }_{>0}$ $\underbrace{\ }_{>0}$ $\Rightarrow f'(x) > 0$.

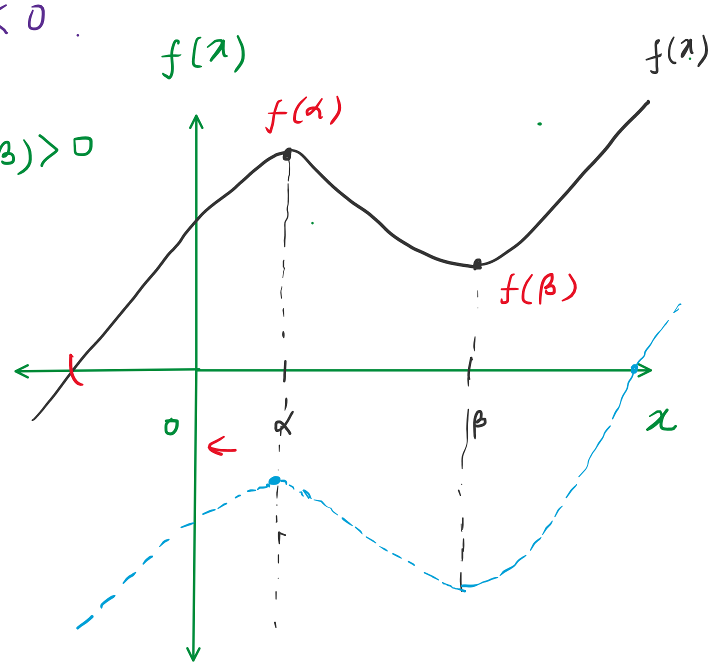
If $\alpha < x < \beta$, $\underbrace{\ }_{>0}$ $\underbrace{\ }_{<0}$ $\Rightarrow f'(x) < 0$.

Subcase I: If $f(\alpha) > 0$, $f(\beta) > 0$

One Real Root.

If $f(\alpha) < 0$, $f(\beta) < 0$.

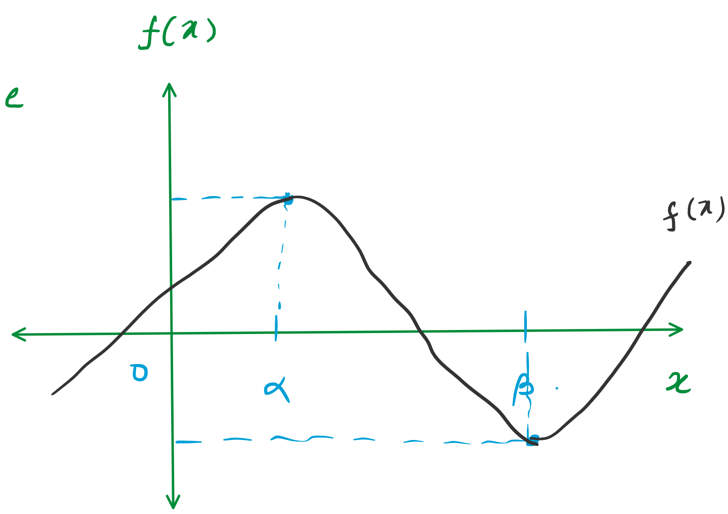
One real Root.



Subcase II: $f(\alpha)$ and $f(\beta)$ have opposite signs.

$f(\alpha) > 0$, $f(\beta) < 0$.

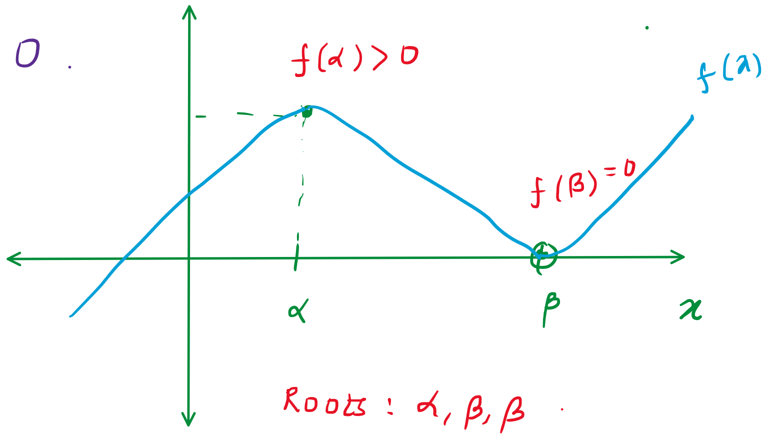
3 Real Roots.



Subcase III: $f(\alpha)\cdot f(\beta) = 0$.

$\therefore$ Suppose $f(\alpha) > 0$, $f(\beta) = 0$.

3 Real Roots [with one as a Repeated Root].



Roots : α, β, β.

Q. The eqn: $5x^3 - 5x^2 + 2x - 1$ has:

(a) all Roots b/w 1 & 2 (b) all −ve Roots.

(c) a root b/w 0 & 1 (d) all roots > 2.

$$f(x) = 5x^3 - 5x^2 + 2x - 1$$

$$f'(x) = \underset{>0}{15}x^2 - 10x + 2.$$

$$D_{f'(x)} = (-10)^2 - 4\cdot(15)(2) = 100 - 120 = -20 < 0.$$

Roots of $f'(x)$ are complex, $\Rightarrow$ sign of $f'(x) > 0$.

$\Rightarrow$ $f(x)$ is monotone increasing $\Rightarrow$ one Real Root.

$f(0) = -1$, $f(1) = 1$, a Root exists.

Q. If $a > 1$, then roots of eqn: $(1-a)x^2 + 3ax - 1 = 0$ are

(a) Both +ve (c) Opposite sign

(b) Both −ve (d) None.

Q. $f(x) = Ax^2 + Bx + C$, $A, B, C \in \mathbb{R}$. If $f(x)$ is an integer whenever 'x' is an integer, then:

Q. $f(x) = Ax + Bx + C$, $A, B, C \in \mathbb{R}$. If $f(x)$ is an integer whenever 'x' is an integer, then:

(a) $2A$, $(A+B)$ are integers, C is non-integer.

(b) $(A+B)$, C are integers, $2A$ is non-integer.

(c) $2A$, $(A+B)$, C are integers.

(d) None.

$$f(1) = (\underbrace{A+B}_{\text{integer}} + C) \Rightarrow \text{integer}$$

$$f(0) = C \Rightarrow \text{integer}$$

$$f(-1) = A - B + C.$$

$$f(1) + f(-1) = \underbrace{2A}_{\text{integer}} + \underbrace{2C}_{\text{integer}}$$

$$f(2) = 4A + 2B + C.$$

$$= 2A + 2B + 2A + C$$

$$= 2(A+B) + 2A + C$$

Q. $f(x) = 8x^3 - 6x + 1$, $x \in [-1, 1]$. Then:-

(a) $f(x)$ has one root in $[-1, -\frac{1}{2}]$.

(b) $f(x)$ has all roots in $[-1, -\frac{1}{2}]$.

(c) $f(x)$ has one root in $[\frac{1}{2}, 1]$.

(d) $f(x)$ has all roots in $[-1, 1]$.

$$f(x) = 8x^3 - 6x + 1.$$

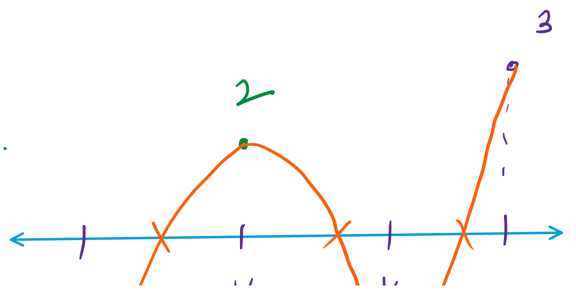
$f(1) = 3$, $f(-1) = -1$, $\exists$ atleast 1 Root b/w $[-1, 1]$.

$$f'(x) = 24x^2 - 6.$$

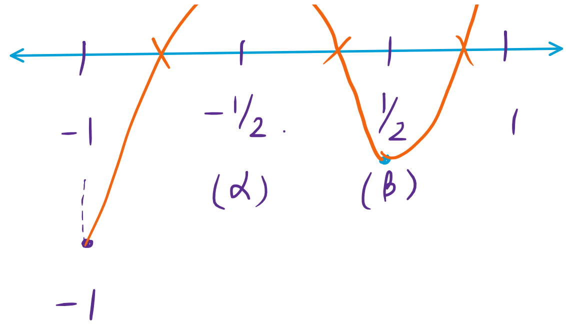
$$D_{f'(x)} = -4(24)(-6) > 0.$$

Roots of $f'(x)$ are real & unequal.

$$f'(x) = 0 \Rightarrow 24x^2 - 6 = 0.$$



$$f'(x) = 0 \Rightarrow 24x^2 - 6 = 0.$$
$$6(4x^2 - 1) = 0$$
$$x = \pm\frac{1}{2}.$$



$$f\left(\tfrac{1}{2}\right) = 8\left(\tfrac{1}{2}\right)^3 - 6\left(\tfrac{1}{2}\right) + 1$$
$$= 1 - 3 + 1 = -1.$$
$$f\left(-\tfrac{1}{2}\right) = 8\left(-\tfrac{1}{2}\right)^3 - 6\left(-\tfrac{1}{2}\right) + 1$$
$$= -1 + 3 + 1 = 2.$$

HW.

Q. If $a < b$ then $(x-a)(x-b) - 1 = 0$ has :

(a) Both Roots in (a, b)

(b) Both Roots $< a$

(c) Both Roots $> b$.

(d) None.