04 August 2023 06:42 PM

Production and Cost Revision :

Production function (relation between inputs and total output produced)

- **Short-run** ⇒ Production depends on variable inputs and fixed inputs
- **Long-run** ⇒ Production depends only on variable inputs

What are fixed and variable inputs ?

fixed inputs are those inputs which cannot be changed instantaneously with market demand. It is fixed or constant.
Ex : capital

variable inputs ⇒ Those inputs which can be changed instantaneously with market demand. Example: Labour.

∴ **Short-run production function** can be written as

$$Q = f(L)$$

Long-run production function can be written as

$$Q = f(L, K).$$

① Total productivity of Labour (TP_L) ⇒ total production using Labour

i.e., $TP_L = Q$

② Average productivity of Labour (AP_L) ⇒ Total production per unit Labour

i.e., $AP_L = \frac{TP_L}{L} = \frac{Q}{L}$

③ Marginal productivity of Labour (MP_L) = $\frac{\text{change in total production } (Q)}{\text{change in Labour}}$

i.e., $MP_L = \frac{\Delta Q}{\Delta L}$.

initial output is Q_0 with Labour L_0

initial output is Q_0 with Labour L_0

final output is Q_1 with Labour L_1

ie change in production or total output, ΔTP_L or $\Delta Q = Q_1 - Q_0$

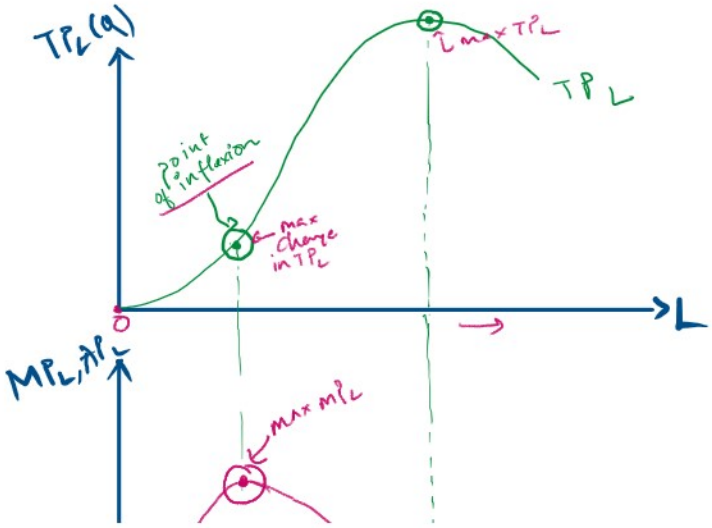
ie change in Labour $\Delta L = L_1 - L_0$

$$MP_L = \frac{\Delta Q}{\Delta L} = \frac{Q_1 - Q_0}{L_1 - L_0}$$

Amount of Labour (L)	Capital (K)	Total output (Q)	Average product ($AP_L = Q/L$)	Marginal Product ($MP_L = \frac{\Delta Q}{\Delta L}$)
0	10	0		
1	10	10	10/1 = 10	$\frac{10-0}{1-0} = \frac{10}{1} = 10$
2	10	30	30/2 = 15	$\frac{30-10}{2-1} = \frac{20}{1} = 20$
3	10	60	60/3 = 20	$\frac{60-30}{3-2} = \frac{30}{1} = 30$ (max MP_L)
4	10	80	80/4 = 20	$\frac{80-20}{4-3} = \frac{20}{1} = 20$
5	10	95	95/5 = 19	$\frac{95-80}{5-4} = \frac{15}{1} = 15$
6	10	108	108/6 = 18	$\frac{108-95}{6-5} = \frac{13}{1} = 13$
7	10	112	112/7 = 16	$\frac{112-108}{7-6} = \frac{4}{1} = 4$
8	10	112 (max TP_L)	112/8 = 14	$\frac{112-112}{8-7} = 0$ (MP_L)
9	10	108	108/9 = 12	$\frac{108-112}{9-8} = -4$
10	10	100	100/10 = 10	$\frac{100-108}{10-9} = -8$

pt of inflexion

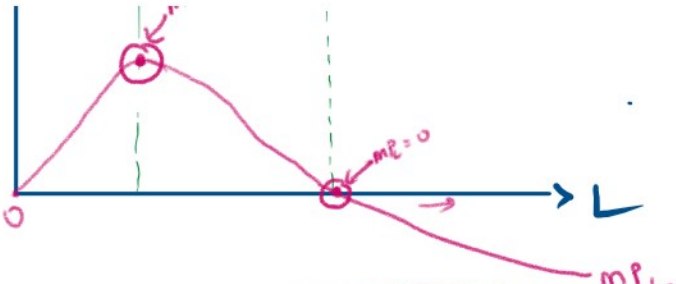
Diminishing MP_L



Summarise the relation between TPL and MPL :

a) when TP_L increasing at increasing rate MP_L is +ve and upward sloping.

b) when TP_L is change



upward slop... I

b) when TP_L is change is maximum at point of inflexion, MP_L reaches maximum

c) after point of inflexion TP_L increases at a decreasing rate and MP_L is +ve but downward sloping.

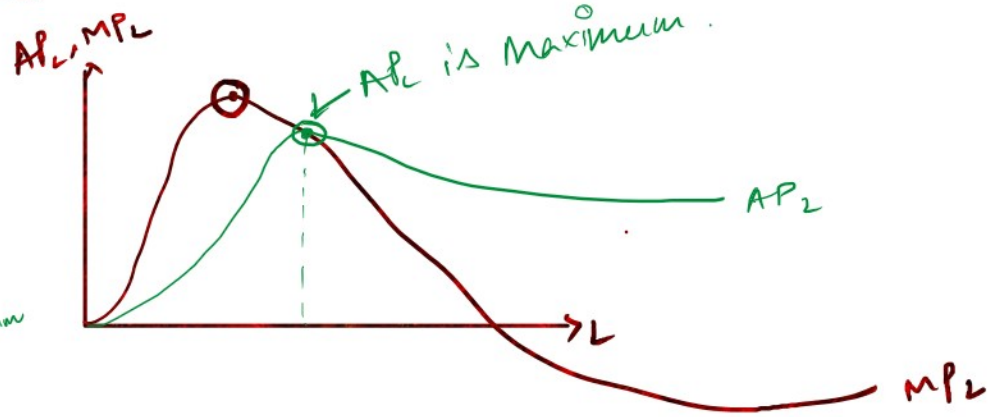
d) TP_L is at maximum, then $MP_L = 0$

e) TP_L is falling ⇒ MP_L is negative

Point to Note: very imp

1. at point of inflexion of TP_L i.e., point of maximum change ⇒ MP_L = maximum.
2. at point of maximum TP_L ⇒ $MP_L = 0$.

Relation between AP_L and MP_L:



1. when AP_L is increasing, $MP_L > AP_L$
2. when AP_L is at maximum $AP_L = MP_L$
3. when AP_L is falling $MP_L < AP_L$

COST CURVES

Short-run cost curves:

1. Tot variable cost (TVC) ⇒ cost of variable inputs (like Labour). Depends on output production. when $Q=0$ ⇒ $TVC=0$.
2. Tot Fixed cost (TFC) ⇒ cost of fixed inputs (like capital) It does not depends on output.
3. Total cost $(TC) = TVC + TFC$ ⇒ summation of TVC & TFC. $TC = TFC$ when $Q=0$

$(AVC) = TVC/Q$

③ Total cost (TC) =

④ Average variable cost (AVC) = TVC/Q

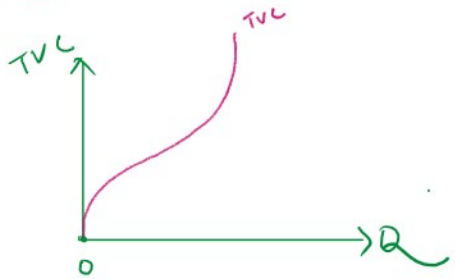
⑤ Average fixed cost (AFC) = TFC/Q

⑥ Average total cost (ATC) = AVC + AFC

or, $\frac{TC}{Q} = \frac{TVC}{Q} + \frac{TFC}{Q}$

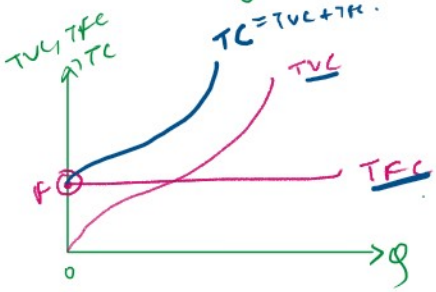
⑦ Marginal cost $(MC) = \frac{\Delta TC}{\Delta Q}$ or $\frac{TC_1 - TC_0}{Q_1 - Q_0}$

① TVC ⇒ starts from origin and upward sloping.



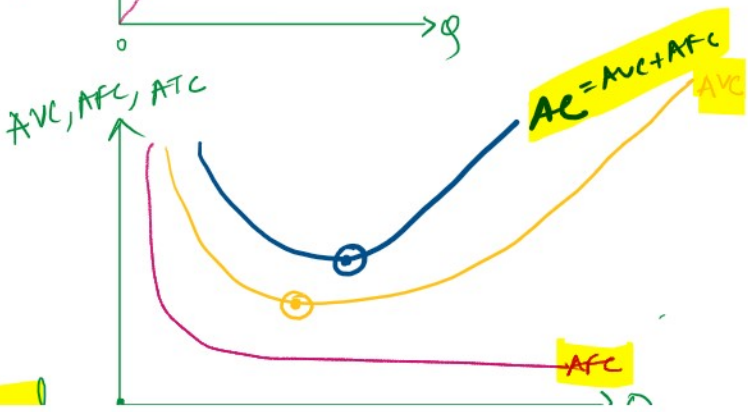
② TFC ⇒ fixed and do not depend on output (Horizontal to output-axis)

③ TC = TVC + TFC ⇒ Never starts from origin because when Q = 0 TVC = TFC.



④ $AVC = \frac{TVC}{Q}$ ⇒ U-shaped curve

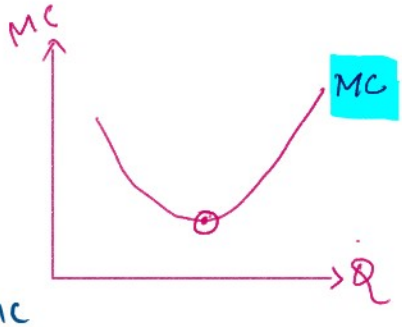
⑤ $AFC = \frac{TFC}{Q}$ ⇒ Rectangular Hyperbola.



(5) AFC = $\frac{TFC}{Q}$ ⇒ Hyperbola.

(6) ATC = AVC + AFC ⇒ U-Shaped curve.

(7) MC = $\frac{\Delta TC}{\Delta Q}$ ⇒ U-shaped.

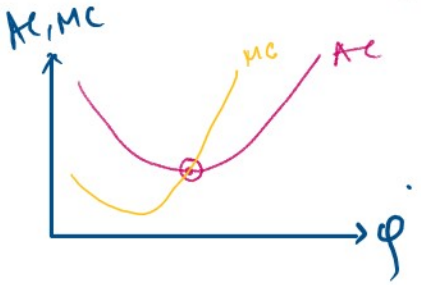


Relation between AC and MC.

(1) When AC is falling
⇒ MC below AC
i.e. MC < AC.

(2) When AC is at minimum
⇒ MC = AC

(3) When AC is increasing
⇒ MC is above AC i.e. MC > AC



Numericals:

(1) A firm's average total cost is ₹60. it's average variable cost is ₹55. and output is 50 units. What is it's total fixed cost?

AC = 60 AVC = 55 Q = 50 TFC = ?

AC = AVC + AFC

AFC = AC − AVC

AFC = 60 − 55

AFC = 5

$\frac{TFC}{Q} = 5$ ⇒ TFC = 5 × Q

TFC = 5 × 50

TFC = 250

(2)

Q (Total Output)	Total cost (TC)	$MC = \frac{\Delta TC}{\Delta Q}$
0	10	—
1	70	$\frac{70-10}{1-0} = 60$
2	80	$\frac{80-70}{2-1} = 10$

(3)

Calculate values of TC, TVC, TFC, AFC and AVC

TC = TVC + TFC

When Q = 0, TC = TFC, 10 = TFC

TVC = TC − TFC

Q	TC	TVC = TC − TFC	TFC	AFC = TFC/Q	AVC = TVC/Q
0	10	0	10	—	—
9	70	70 − 10 = 60	10	10/9	60/9
10	80	80 − 70 = 70	10	10/10	70/9

(3)

Total cost function, $C = 5Q^2 + 20Q + 5$ at $Q = 5$

Calculate Marginal cost of production (AC).

$TC = 5Q^2 + 20Q + 5$

$AC = \frac{TC}{Q} = \frac{5Q^2 + 20Q + 5}{Q} = \frac{5Q^2}{Q} + \frac{20Q}{Q} + \frac{5}{Q}$

$AC = 5Q + 20 + \frac{5}{Q}$

at $Q = 5$ units $\Rightarrow AC = 5 \times 5 + 20 + \frac{5}{5}$

$AC = 25 + 20 + 1$

$AC = 46$ (ans)