DIFFERENTIAL EQ 2 14 12 2023

Thursday, December 14, 2023 7:37 PM

I order & degree question + answer

II 1st order 1st degree

III 1 or + 2nd ord

IV ~~clariris form~~

$$\left[\left(\frac{d^3y}{dx^3}\right)^{7/9}\right]^9 = \left[\left(\frac{d^4y}{dx^4}\right)^{5/3}\right]^9$$

906239523

$$\left(\frac{d^3y}{dx^3}\right)^7 = \left(\frac{d^4y}{dx^4}\right)^{15}$$

order 4

~~degree~~ 15

Q

$$(\text{Subtangent})^m = (\text{Subnormal})^n$$

$\boxed{0 < n < m}$

$$y/dy/dx \rightarrow y\left(\frac{dy}{dx}\right)^{-1}$$

$$\left[y\left(\frac{dy}{dx}\right)^{-1}\right]^m = \left(\frac{y\,dy}{dx}\right)^n$$

$$y^{m-n} = \left(\frac{dy}{dx}\right)^{n+m}$$

order → 1

degree n+m

Q

$y^2 = 2c(x+\sqrt{c})$

$2yy' = 2c$ — (1) (2) in (1)

$c = yy'$ — (2)

$y^2 = 2yy'\{x + (yy')^{1/2}\}$

$(y^2 - 2xyy')^2 = [2yy'(yy')^{1/2}]^2$ — (3)

$y^4 - 4xy^3y' + 4x^2y^2y'^2 = 4y^3y'^3$

order 1

Degree 3

Differential Equations

LD   LID

$x, 2x$ x, x^2

$c_1f_1 + c_2f_2 + \dots + c_nf_n = 0$ $[a, b)$

$c_i = 0$

$c_i \neq 0$

Wronskian

$f_1, f_2, \dots f_n$ over I

for which a derivative of order $(n-1)$ on I

interval

then determinant

$$\begin{vmatrix} f_1 & f_2 & \cdots & f_n \\ f_1' & f_2' & \cdots & f_n' \\ f_1'' & f_2'' & \cdots & f_n'' \\ \vdots & & & \\ f_1^{n+1} & f_2^{n+1} & \cdots & f_n^{n+1} \end{vmatrix} = W(f_1, f_2 \cdots f_n)$$

$$W[f_1(x), f_2(x) \cdots f_n(x)]$$

The Patriot 2001 → Mel Gibson

45 4 min 167 no Actors

123

20-27

1/4

30 y.

Beta Gamma Function

2D 3D

$\int\int$ $\int\int\int$

$D \to \frac{d}{dx}$ D^2 $\frac{d^2}{dx^2}$

$P_0 D^n + P_1 D^{n-1} + \cdots$

$P_0 \frac{d^n y}{dx^n} + \cdots$

For xy $(D^2 - 5D + 6)y = 0$

For y_1

$(D^2 - 5D + 6)y = 0$

$m^2 - 5m + 6 = 0 \quad m = 2, 3$

$y_1 = c_1 e^{2x} + c_2 e^{3x}$

Q.

$\frac{d^2y}{dx^2} + y = 0 \qquad y(0) = 0, \quad y'(0) = 1$

$m^2 + 1 = 0$

$y = A\sin x + B\cos x$

$B = 0$

$A = 1$

$y = \sin x$ — PS

CF + PI

$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{1x}$

$m^2 - 5m + 6 = 0 \qquad y = c_1 e^{2x} + c_2 e^{3x}$

$m = 2, 3$

e^{1x}

$1^2 - 5 \cdot 1 + 6 = 2$

$\frac{e^x}{2}$

If by putting $e^{\alpha x}$ in the LHS

the LHS $\neq 0$ $e^{\alpha x}$

the LHS $\neq 0$ $\dfrac{e^{\alpha x}}{f(\alpha)}$

the

5 Standard Cases

$\dfrac{1}{f(D)} e^{\alpha x}$ Put $D = \alpha$ $f(\alpha) \neq 0$

$f(\alpha) = 0$

method to evaluate $\dfrac{e^{\alpha x}}{f(D)}$

$$\frac{1}{f(D)} e^{\alpha x} = x \cdot \frac{e^{\alpha x}}{\frac{d}{dD}\left(f(D)\right)}$$

II $\dfrac{1}{f(D^2)} \sin ax$ or $\dfrac{1}{f(D^2)} \cos ax$

Put $D^2 = -a^2$ $f(-a^2) \neq 0$

If failure If $f(-a^2) = 0$

$$\frac{1}{f(D^2)} \cos ax = x \cdot \frac{\cos ax}{\frac{d}{dD}\left[f(D^2)\right]}$$

Same for $\sin ax$.

III $\frac{x^m}{f(D)}$

(i) For $f(D)$ take the lowest degree term outside. Then the remaining factor will be of the type

$[1 \pm \phi(D)]$

(ii) Take $[1 \pm \phi(D)]$ to the numerator & expand it by Binomial theorem $\left(\because D^{m+1}(x^m) \to 0\right)$

(iii) Operate each term on x^m

Remark

$(1-D)^{-1} = 1 + D + D^2 + \cdots \infty$

$(1+D)^{-1} = 1 - D + D^2 - D^3 + \cdots \infty$

$(1-D)^{-2} = 1 + 2D + 3D^2 + 4D^3 + \cdots \infty$

$(1-D)^{-3} = 1 + 3D + 6D^2 + \cdots \infty$

IV $\frac{e^{ax} \cdot X}{f(D)}$

$\Rightarrow e^{ax} \frac{X}{f(D+a)}$

The

$\Rightarrow$ ... $f(\ldots)$

Thm

(V) $\dfrac{x f(x)}{f(D)}$

$= x \dfrac{f(x)}{f(D)} + \dfrac{d}{dD}\left[\dfrac{1}{f(D)}\right] f(x)$

Example

16- $(D^4 + 2D^3 - 3D^2)y = 3e^{2x} + 4\sin x$

$m^4 + 2m^3 - 3m^2 = 0$

$m^2(m^2 + 2m - 3) = 0$

$m = 0, 0, 1, -3$

$Cf \rightarrow (c_1 + c_2 x) + c_3 e^{x} + c_4 e^{-3x}$

$PI = \dfrac{3e^{(2)x}}{f(2)} = \boxed{\dfrac{3e^{2x}}{20}}$

$D^2 = -a^2$

$D^2 = -(1)^2$

PI for $4\sin x \rightarrow \dfrac{4\sin x}{D^2(D^2 + 2D - 3)}$

$\Rightarrow \dfrac{4\sin x}{(-1)(-1 + 2D - 3)}$

$\Rightarrow \dfrac{4\sin x}{(-2)(D-2)} \Rightarrow -2\left(\dfrac{\sin x}{D-2}\right)$

$\dfrac{\cancel{D}\sin x}{\cancel{(D+2)}}$

$$\Rightarrow \frac{-2(D+2)\sin x}{D^2-4} \Rightarrow \frac{-2(D+2)\sin x}{-1-4}$$

$$\Rightarrow +\frac{2}{5}(D+2)\sin x$$

$$\Rightarrow \frac{2}{5}\cos x + \frac{4}{5}\sin x$$

$$CF + PI_1 + PI_2$$

Q

$$\left(\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y\right) = \left(x^2+x\right)$$

$$PI = \frac{x^2+x}{(D^2-3D+2)} = \frac{1}{2}\left(1+\frac{D^2-3D}{2}\right)^{-1}(x^2+x)$$

$$= \frac{x^2+x}{2\left(1+\frac{D^2-3D}{2}\right)}$$

$$\Rightarrow \frac{1}{2}\left[1-\left(\frac{D^2-3D}{2}\right)+\left(\frac{D^2-3D}{2}\right)^2-\left(\frac{D^2-3D}{2}\right)^3 \ldots\right](x^2+x)$$

$$\Rightarrow \frac{1}{2}\left[1-\frac{D^2}{2}+\frac{3D}{2}+\frac{D^4}{4}-\frac{3D^3}{2}+\frac{9D^2}{4} \cdot \text{ignoring the rest}\right)$$

$$2 \cdot \frac{(D^2)(3D)}{4/2}$$

$$\Rightarrow \frac{1}{2}\left[1+\frac{3D}{2}+\frac{9D^2}{4}-\frac{D^2}{2} \ldots\right)(x^2+x)$$

$$\Rightarrow \frac{1}{2}\left(\ldots \; 3D(x^2+x) + \frac{1}{2}\,\frac{9}{4}D^2(x^2+x)\right)$$

$\Rightarrow \frac{1}{2}\left(1+\ldots\right.$

$\Rightarrow \frac{1}{2}\cdot 1\cdot(x^2+x) + \frac{1}{2}\cdot\frac{3D}{2}(x^2+x) + \frac{1}{2}\frac{9}{4}D^2(x^2+x)$

$D^2(x^2) = 2$

$-\frac{1}{2}\frac{D^2}{2}(x^2+x)$

$\Rightarrow \frac{x^2+x}{2} + \frac{3}{4}(2x+1) + \frac{9}{8}(2)$

$-\frac{1}{4}(2)$

$\Rightarrow \frac{x^2}{2} + \frac{x}{2} + 5/2 \rightarrow$ PI

Q $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2e^{3x}$

$PI = \frac{x^2e^{3x}}{(D-1)^2}$

$= e^{3x}\left\{\frac{1}{(D+3-1)^2}\right\}x^2$

$\Rightarrow e^{3x}\frac{x^2}{(D+2)^2}$

$(D+2)^{-2}\,x^2 \quad \ldots\}x^2$

$\Rightarrow e^{3x} \cdot (D+2)^{-2}\, x^2$

$\Rightarrow \frac{e^{3x}}{4}\left\{1 - 2\left(\frac{D}{2}\right) + 3\left(\frac{D^2}{4}\right) - \cdots\right\} x^2$

$\Rightarrow \frac{e^{3x}}{4}\left(x^2 - 2x + \frac{3}{4} \cdot \frac{2}{2}\right)$

$\Rightarrow \frac{e^{3x}}{8}\left(2x^2 - 4x + 3\right) \to PI$

VARIABLE Coefficient Problem

$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \frac{1}{x}$

Putting $x = e^z$

Rule $P_0 \frac{d^ny}{dx^n} + P_1 \frac{d^{n-1}y}{dx^{n-1}} + \cdots + P_n y = Q(x)$

Put $x = e^z$ $\quad z = \log x \quad x > 0$

Put $\frac{d}{dz} = \theta$, $xD = \theta$

$x^2 D^2 = \theta(\theta - 1)$

$x^n D^n = \theta(\theta-1)(\theta-2) \cdots (\theta - n + 1)$

The eq will become

$$\left[P_0\theta(\theta-1)\cdots(\theta-n+1)+P_1\theta(\theta-1)\cdots(\theta-n+2)+\cdots+P_n\right]y = \phi(e^z)$$

Letting $z=\log x$ & solve

Here, $\left(\theta(\theta-1)(\theta-2)+2\theta(\theta-1)-\theta+1\right)y = e^{-z}$

$$PI = \frac{e^{-z}}{(\theta-1)^2(\theta+1)}$$

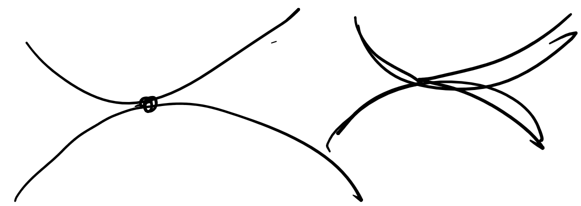
$$= \frac{1}{(\theta+1)}\left(\left(\frac{1}{\theta-1}\right)^2 e^{-z}\right)$$

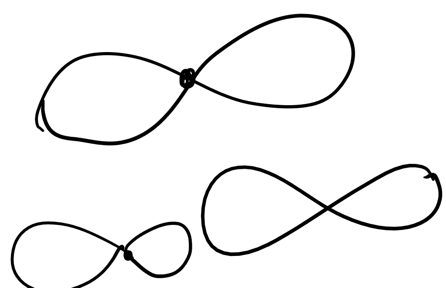
$$= \frac{1}{\theta+1}\left(\frac{1}{4}e^{-z}\right)$$

$$= \frac{1}{4}e^{-z}\frac{1}{(\theta-1+2)}$$

$$(z)e^{-z} = \frac{\log x}{4x}$$

$= \frac{1}{4}(2)e^{-2} = \frac{\ldots}{4\pi}$





Critical Point Analysis

$$\frac{dx}{dt} = 6x - 65y$$

$$\frac{dy}{dt} = 2x - 17y$$

then $\rightarrow (0,0)$

$$\begin{bmatrix} \frac{dx}{dt} \\ dy/dt \end{bmatrix} = \begin{bmatrix} 6 & -65 \\ 2 & -17 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Eigen value

$$\begin{vmatrix} 6-\lambda & -65 \\ 2 & -17-\lambda \end{vmatrix} = 0$$

$$\lambda^2 - (-11\lambda) + 28 = 0$$

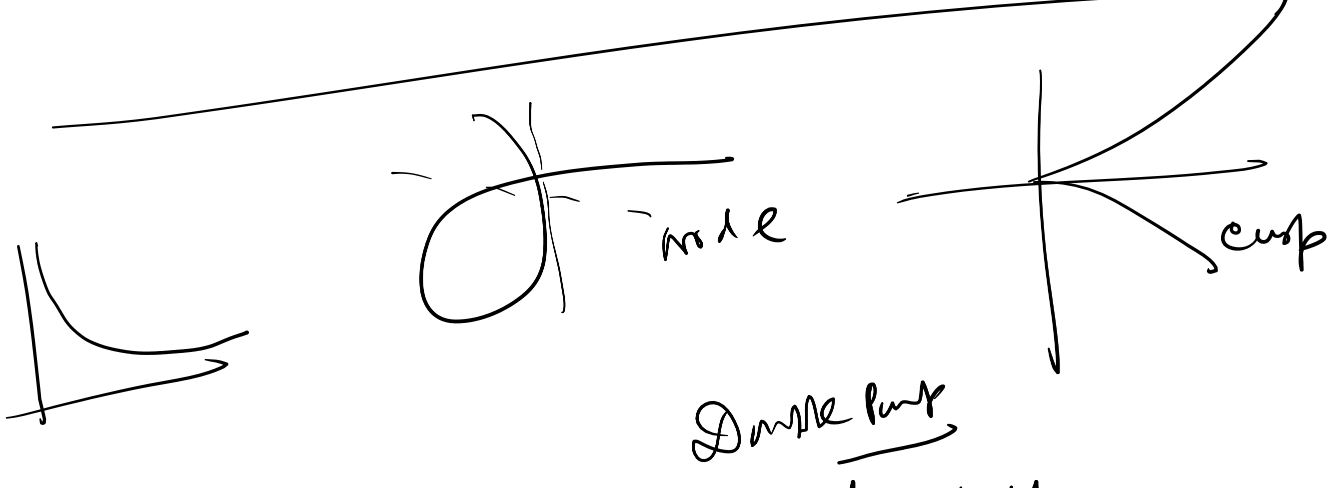
$$\lambda = -4, -7$$

Both EV < 0 $\rightarrow$ Asymptotically stable node

Link to Stability

$\lambda^2 - (a+d)\lambda + (ad-bc) = 0$

Root type	Critical point	Stability
Real / unequal / same sign	node	Asymptotically stable if -ve, Unstable if +ve
Real / unequal / opposite in sign	Saddle point	Unstable
Real / equal	node	Stable if -ve, unstable if +ve
Pure imaginary	Centre	Stable but not asymptotically stable
Not pure imaginary	Spiral point	Unstable



Double point

Stable vs Asymptotically stable

$t \to \infty$ guaranteed