STAT 4.4.24

04 April 2024 18:37

MCQ + Simulation

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Exponential distn

Y exponential distn mean = θ

Conditional distn .. of X given $Y = y$

N → 0 mean var y ($y > 0$)

Identify the CF of X [$\phi(t) = E(e^{itX})$]

Ans: $Y \sim$ exp with mean θ

$$f(y) = \frac{1}{\theta} e^{-y/\theta} \; ; \; y > 0$$

$$X / Y = y \sim N(0, y) \; ; \; y > 0$$

C/F of $X/Y = y$

$$\phi_{X|Y=y} = E\left[e^{itX} / Y = y\right] = E\left[e^{-\frac{1}{2}yt^2}\right]$$

Using Conditional expectation.

$$E\left[e^{itX}\right] = E\left[E\left(e^{itX} / Y = y\right)\right] = E\left[e^{-\frac{1}{2}yt^2}\right]$$

$$\int_0^\infty e^{-\frac{1}{2}yt^2} f(y)\,dy$$

1/2 (3/4

20-22

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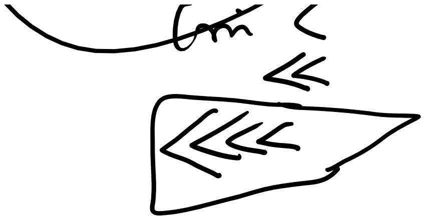
$$\int_0^\infty \left(-y/\theta + t^2/2 \right) \ldots$$

$$= \frac{1}{\theta}\int_0^\infty e^{-y\left(\theta + t^2/2\right)}\,dy$$

$$= \left(\frac{2}{\theta t^2 + 2}\right)$$

$$= \frac{1}{1 + \frac{1}{2}\theta t^2}$$



Distribution Theory

(JUNE 2021)

1. Suppose that Y has exponential distribution with mean θ and that the conditional distribution of X given $Y = y$ is Normal with mean 0 and variance y, for all $y > 0$. Identify the characteristic function of X (defined as $\phi(t) = E[e^{itX}]$) from the following.

(a) $e^{-\frac{\theta}{2}t^2}$ (b) $e^{-\frac{1}{2\theta}t^2}$

(c) $\frac{1}{1+\frac{1}{2}\theta t^2}$ (d) $\frac{\theta}{\theta+\frac{1}{2}t^2}$

2. Let $X_1, X_2, \ldots. X_n$ be random variable whose marginal distributions are $N(0,1)$. Suppose $E(X_i X_j) =$ for $i, j, i \neq j$. Let $Y = X_1 + X - 2 + \cdots + X_n$ and $V = X_1^2 + X_2^2 + .. + X_n^2$. which of the following statement follow from the give conditions?

(a) Y has normal distribution with mean zero and variance n

(b) V has Chi-square distributon with n degrees of freedom

(c) $E(X_i^3 X_j^3) = 0$ for all $i, j, i \neq j$

(d) $P(|Y| > t) \leq \frac{n}{t^2}$ for all $t > 0$

3. Suppose $X \sim$ Geometric (1/2) (taking values in {1,2,3 ...,}) and conditional on X, the variable Y has Poisson (X) distribution. Similarly suppose $U \sim$ Poisson (1) and conditional on U, the variable V has Geometric $(1/(U+1))$ distribution. Then,

(a) $E[Y] \geq E[V]$ (b) $E[Y] \leq E[V]$

(c) $Var\ [Y] \geq Var[V]$ (d) $Var\ [Y] \leq Var[V]$

(NOV 2020)

4. Let $X_1, X_2, \ldots$ be i.i.d random variables having a $\chi^2 - distribution$ with 5 degree of freedom. Let $a \in R$ be constant. Then the limiting distribution of $a\left(\frac{X_1 +.+ X_n - 5n}{\sqrt{n}}\right)$ is

(a) Gamma distribution for an appropriate value of a

(b) χ^2 –Distribution for an appropriate value of a

(c) Standard normal distribution for an appropriate value of a

(d) A degenerate distribution for an appropriate value of a

(NOV 2020)

5. Suppose that X has uniform distribution on the interval [0,100]. Let Y denote the greatest integer smaller than or equal to X. Which of the following is true?

(a) $P(Y \leq 25) = 1/4$ (b) $P(Y \leq 25) = 26/100$

(c) $E(Y) = 50$ (d) $E(Y) = 101/2$

(NOV 2020)

6. Suppose $X_1, X_2, \ldots, X_n$ are i.i.d. random variables with characteristic function $\phi(t;\theta) = E[e^{itX_i}]$ where $\underline{\theta} \epsilon R^k$ is the parameter of the distribution. Let $Z = X_1 + X_2 + \cdots + X_n$. Then for which of the following distributions of X_1 would the characteristic function of Z be of the form $\phi(t;\underline{\alpha})$ for some $\underline{\alpha} \epsilon R^k$?

(a) Negative Binomial (b) Geometric

(c) Hypergeometric (d) Discrete Uniform

(JUNE 2019)

7. Suppose a normal $Q - Q$ plot is drawn using a reasonably large sample $x_1, \ldots, x_n$ from an unknown probability distribution. For which of the following distribution would you expect the $Q - Q$ plot to be convex (J –shaped)?

(a) Beta (5, 1) (b) Exponential (1)

(c) Uniform (0,1) (d) Geometric (1/2)

(JUNE 2019)

8. Let $X_1, X_2, \ldots, X_{2n-1} (n > 5)$ be $i.i.d.$ with p.d.f. f_θ, which is symmetric about θ having bounded support. Let $X_{(1)} < X_{(2)} < \cdots < X_{(2n-1)}$ be the order statistics of the random variables $X_1, X_2, \ldots, X_{2n-1}$. Which of the following statements are correct?

(a) $X_{(1)} - \theta$ and $\theta - X_{(1)}$ have the same distributioin

(b) $X_{(1)} - \theta$ and $\theta - X_{(2n-1)}$ have the same distribution

(c) The distribution of $X_{(n)}$ is symmetric about θ

(d) $E[X_{(k)} + X_{(2n-k)}]$ is same for all $k = 1,2, \ldots, n$

(DEC 2019)

9. Let X and Y be independent Exponential random variables with means $\frac{1}{\lambda}$ and $\frac{1}{\mu}$ respectively with $\neq \mu$. Let $f_z(z)$ denote the density function of Z = X +Y. Then for z>0,

(a) $f_z(z) = (\lambda + \mu)e^{-(\lambda+\mu)z}$

(b) $f_z(z) = \frac{\lambda\mu}{\lambda+\mu} e^{\frac{-\lambda\mu}{e\lambda+\mu}z}$

(c) $f_z(z) = \frac{\lambda\mu}{\lambda-\mu}(e^{-\mu z} - e^{-\lambda z})$

(d) $f_z(z) = \begin{cases} \frac{\lambda\mu}{\lambda-\mu} e^{\frac{-\lambda}{\lambda-\mu}z} \; if\ \lambda > \mu \\ \frac{\lambda\mu}{\mu-\lambda} e^{\frac{-\lambda\mu}{\mu-\lambda}z} \; if\ \mu>\lambda \end{cases}$

(DEC 2019)

10. A random variable T has a symmetric distribution if T and −T have the same distribution. Let X and Y be independent random variables. Then which of the following statements are correct?
    (a) If X and Y have the same distribution then X – Y has a symmetric distribution
    (b) If X ~ N(3,1) and Y ~ N(2,2), then 2X – 3Y has a symmetric distribution
    (c) If X and Y have the same symmetric distribution, then X + Y has a symmetric distribution
    (d) If X has a symmetric distribution, then XY has a symmetric distribution

(JUNE 2018)

11. Let X and Y be i.i.d. uniform (0, 1) random variables, Let $Z = \max(X,Y)$ and $W = \min(X,Y)$.
    Then $P((Z - W) > 1/2)$ is
    (a) 1/2 (b) 3/4
    (c) 1/4 (d) 2/3

(JUNE 2018)

12. Let X_1, X_2, X_3 be i.i.d. standard normal variables. Which of the following is true?
    (a) $\frac{\sqrt{2}|X_1|}{\sqrt{X_2^2+X_3^2}} \sim t_2$ (b) $\frac{X_1-2X_2+X_3}{\sqrt{2}|X_1+X_2+X_3|} \sim t_1$
    (c) $\frac{(X_1-X_2)^2}{(X_1+X_2)^2} \sim F_{2,2}$ (d) $\frac{3X_1^2}{X_1^2+X_2^2+X_3^2} \sim F_{1,3}$

(JUNE 2018)

13. Let X and Y be i.i.d. exponential random variables with parameter 1. Define, $W = X + Y$ and $U = X/(X+Y)$. Which of the following are true?
    (a) $E(U) = 1/2$
    (b) U is unforn on (0, 1)
    (c) W, U are independent
    (d) W, U are uncorrelated, but dependent

(JUNE 2018)

14. Suppose that for $n \geq 3$, $X_1, X_2, \ldots, X_n$ are i.i.d. $\sim N(\mu_1, \sigma_1^2)$ and $Y_1, Y_2, \ldots, Y_n$ are i.i.d. $\sim N(\mu_2, \sigma_2^2)$. Assume further that the $X_i's$ and the $Y_j's$ are independent. Let r be the correlation coefficient computed from the bivariate data $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$. Then
    (a) $\frac{r^2(n-2)}{1-r^2}$ has $F_{1,n-2}$ distribution (F- distribution with 1 and $n-2$ d.f.) for all $n \geq 3$
    (b) $\frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$ has t_{n-2} distribution (t- distribution with $n-2$ d.f.) for all $n \geq 3$
    (c) $\frac{r^2}{1-r^2}$ has the distribution of the square of a Cauchy variable for $n = 3$
    (d) r^2 has a beta distribution for all $n \geq 3$

(DEC. 2018)

15. Let X and Y be i.i.d. random variables uniformly distributed on (0,4). Then $P(X > Y | X < 2Y)$ is.
    (a) $\frac{1}{3}$ (b) $\frac{5}{6}$
    (c) $\frac{1}{4}$ (d) $\frac{2}{3}$

(DEC. 2018)

16. Suppose $X \sim$ Cauchy (0,1). Then the distribution of $\frac{1-X}{1+X}$ is
    (a) Uniform (0,1) (b) Normal (0,1)
    (c) Double exponential (0,1) (d) Cauchy (0,1)

(DEC. 2018)

17. Suppose $X_1, X_2 \ldots, X_3$ is a random sample for the uniform distribution on (0,2) and $M_n = \max\{X_1, X_2 \ldots, X_3\}$ for every positive integer n. Then which of the following statement are true?
    (a) $M_n \to 2$ almost surely
    (b) $M_n \to 2$ in probability
    (c) $M_n \to 2$ in distribution
    (d) $\frac{M_n-2}{\sqrt{n}}$ converges in distribution to normal distribution

(DEC. 2018)

18. Let $X_1, X_2, \ldots$ be i.i.d. N (0,1) random variables. Let $S_n = X_1^2 + X_2^2 + \cdots X_n^2, \forall n \geq 1$. Which of the following statements are correct?
    (a) $\frac{S_n-n}{\sqrt{2}} \sim N(0,1)$ for all $n \geq 1$
    (b) For all $\varepsilon > 0, P\left(\left|\frac{S_n}{n} - 2\right| > \epsilon\right) \to 0\ as\ n \to \infty$
    (c) $\frac{S_n}{n} \to 1$ with probability 1
    (d) $P(S_n \leq 1 + \sqrt{n}x) \to P(Y \leq x)\ \forall x \in R$ where $Y \sim N(0,2)$

(JUNE 2017)

19. $X_1, X_2, \ldots.$ are independent identically distributed random variables having common density f. Assume $f(x) = f(-x)$ for all $x \in \mathbb{R}$. Which of the following statements is correct?
    (a) $\frac{1}{n}(X_1 + \cdots + X_n) \to 0$ in probability
    (b) $\frac{1}{n}(X_1 + \cdots + X_n) \to 0$ almost surely
    (c) $P\left(\frac{1}{\sqrt{n}}(X_1 + \cdots + X_n) < 0\right) \to \frac{1}{2}$
    (d) $\sum_{i=1}^{n} X_i$ has the same distribution as $\sum_{i=1}^{n}(-1)^i X_i$

SJV

(JUNE 2017)

20. X and Y are independent random variables each having the density

$f(t) = \frac{1}{\pi}\frac{1}{1+t^2}$; $-\infty < t < \infty$.

Then the density function of $\frac{X+Y}{3}$ for $-\infty < t < \infty$ is given by

(a) $\frac{6}{\pi}\frac{1}{4+9t^2}$ (b) $\frac{6}{\pi}$ $\frac{1}{9+4t^2}$

(c) $\frac{3}{\pi}\frac{1}{1+9t^2}$ (d) $\frac{3}{\pi}$ $\frac{1}{9+t^2}$

(JUNE 2017)

21. Let $\{X_n, n \geq 1\}$ be i.i.d. uniform $(-1, 2)$ random variables. Which of the following statements are true?

(a) $\frac{1}{n}\sum_{i=1}^{n} X_i \to 0$ almost surely

(b) $\left\{\frac{1}{2n}\sum_{i=1}^{n} X_{2i} - \frac{1}{2n}\sum_{i=1}^{n} X_{2i-1}\right\} \to 0$ almost surely

(c) $\sup\{X_1, X_2, \ldots\} = 2$ almost surely

(d) $\inf\{X_1, X_2, \ldots\} = -1$ almost surely

(JUNE 2017)

22. Suppose X follows an exponential distribution with parameter $\lambda > 0$. $Fox\ a > 0$. Define the random variable Y by $Y = k$, if $ka \leq X < (k+1)a,\ k = 0,1,2, \ldots$

Which of the following statement are correct?

(a) $P(4 < Y < 5) = 0$

(b) Y follows an exponential distribution

(c) Y follows a geometric distribution

(d) Y follows a Poisson distribution.

(DEC. 2017)

23. X, Y are independent exponential random variables with means 4 and 5, respectively. Which of the following statements is true?

(a) $X + Y$ is exponential with mean 9

(b) XY is exponential with mean 20

(c) $\max(X, Y)$ is exponential

(d) $\min(X, Y)$ is exponential

(DEC. 2017)

24. Let X and Y be independent exponential random variables. If $E[X] = 1$ and $E[Y] = \frac{1}{2}$

then $P(X > 2Y | X > Y)$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

(DEC. 2017)

25. Which of the following are correct?

(a) if X and Y are N (0,1) then $\frac{X+Y}{\sqrt{2}}$ is $N(0,1)$

(b) if X and Y are independent $N(0,1)$ then $\frac{X}{Y}$ has t-distribution

(c) if X and Y are independent Uniform (0,1) then $\frac{X+Y}{2}$ is Uniform (0,1)

(d) if X is Binomial (n, p) then $n - X$ is Binomial $(n, 1-p)$

(DEC. 2017)

26. For $n \geq 1$, let X_n be a Poisson random variable with mean n^2. Which of the following are equal to $\frac{1}{\sqrt{2}}\int_2^{\infty} e^{-x^2/2} dx$

(a) $\lim_{n \to \infty} P\{X_n > (n+1)^2\}$

(b) $\lim_{n \to \infty} P\{X_n \leq (n+1)^2\}$

(c) $\lim_{n \to \infty} P\{X_n < (n-1)^2\}$

(d) $\lim_{n \to \infty} P\{X_n < (n-2)^2\}]$

(DEC. 2016)

27. Suppose $X_1, X_2, \ldots, X_n$ is a random sample from a distribution with probability density function

$f(x) = 3x^2 I_{(0,1)}(x)$, where $I_{(0,1)}(Z) = \begin{cases} 1 & if\ z \in (0,1) \\ 0 & otherwise \end{cases}$

What is the probability density function $g(y)$ of $Y = \min\{X_1, X_2, \ldots, X_n\}$?

(a) $g(y) = 3ny^{3n-1} I_{(0,1)}(y)$.

(b) $g(y) = 1 - (1-y^3)^n I_{(0,1)}(y)$.

(c) $g(y) = (1-y^3)^n I_{(0,1)}(y)$.

(d) $g(y) = 3ny^2(1-y^3)^{n-1} I_{(0,1)}(y)$.

(DEC. 2016)

28. Let $\{X_i; i \geq 1\}$ be a sequence of independent random variables each having a normal distribution with mean 2 and variance 5. Then which of the followings are true

(a) $\frac{1}{n}\sum_{i=1}^{n} X_i$ converges in probability to 2

(b) $\frac{1}{n}\sum_{i=1}^{n} X_i^2$ converges in probability to 9

(c) $\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right)^2$ converges in probability to 4

(d) $\sum_{i=1}^{n}\left(\frac{X_i}{n}\right)^2$ converges in probability to 0

(DEC. 2016)

29. Let X be a random variable with a certain non-degenerate distribution. Then identify the correct statements

(a) If X has an exponential distribution then median $(X) < E(X)$

(b) If X has a uniform distribution on an interval $[a, b]$, then $E(X) < median(X)$

(c) If X has a Binomial distribution then $V(X) < E(X)$

(d) If X has a normal distribution, then $E(X) < V(X)$

(JUNE 2016)

30. Let $X_1 \sim N(0,1)$ and let $X_2 = \begin{cases} -X_1, & -2 \leq X_1 \leq 2 \\ X_1, & otherwise \end{cases}$

Then identify the correct statement.

(a) $corr\ (X_1, X_2) = 1$

(b) X_2 does not have $N(0,1)$ distribution.

(c) (X_1, X_2) has a bivariate normal distribution

(d) (X_1, X_2) does not have a bivariate normal distribution

(JUNE 2016)

31. Suppose $X$and Y are independent and indentically distributed random variables and let $= X + Y$. Then the distribution of Z is in the same family as that of$X$ and$Y$ if$X$ is

(a) Normal (b) Exponential

(c) Uniform (d Binomial

(DEC. 2015)

32. Let X_i' s be independent random variables such that X_i' are symmetric about 0 and
$Var(X_i) = 2i - 1$, for $i \geq 1$.
Then, $\lim_{n\to\infty} P(X_1 + X_2 + \cdots + X_n > n \log n)$

(a) does not exist (b) equals ½

(c) equals 1 (d) equals 0

(DEC. 2015)

33. Let $X_1, X_2, \ldots$ be independent and identically distributed, each having a uniform distribution on $(0,1)$. Let $S_n = \sum_{i=1}^{n} X_i$ for $n \geq 1$. Then which of the following statements are true?

(a) $\frac{S_n}{n \log n} \to 0$ as $n \to \infty$ with probability 1

(b) $P\left\{\left\{S_n > \frac{2n}{3}\right\} \text{occurs for infinitely many } n\right\} = 1$

(c) $\frac{S_n}{\log n} \to 0$ as $n \to \infty$ with probability 1

(d) $P\left\{\left\{S_n > \frac{n}{3}\right\} \text{occurs for infinitely many } n\right\} = 1$

(JUNE 2015)

34. Assume that $X \sim$ Binomial (n, p) for some $n \geq 1$ and $0 < p < 1$ and $Y \sim$ Poisson (λ) for some $\lambda > 0$. Suppose $E[X] = E[Y]$. Then

(a) $Var(X) = Var\ (Y)$

(b) $Var(X) < Var\ (Y)$

(c) $Var(X) > Var\ (Y)$

(d) $Var(X)$ may be larger or smaller than $Var(Y)$ depending on the values of n, p and λ.

(JUNE 2015)

35. Suppose $X_i|\theta_i \sim N(\theta_i \sigma^2), i = 1,2$ are independently distributed. Under the prior distribution, θ_1 and θ_2 are i.i.d $N(\mu, \tau^2)$, where σ^2, μ and τ^2 are known. Then which of the following is true about the marginal distributions of X_1 and X_2?

(a) X_1 and X_2 are i.i.d $N(\mu, \tau^2 + \sigma^2)$.

(b) X_1 and X_2 are not normally distributed.

(c) X_1 and X_2 are $N(\mu, \tau^2 + \sigma^2)$ but they are not independent.

(d) X_1 and X_2 are normally distributed but are not identically distributed.

(JUNE 2015)

36. Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed random variables having an exponential distribution with mean $\frac{1}{\lambda}$. Let $S_n = X_1 + X_2 + \ldots + X_n$ and $N = \inf\{n \geq 1 : S_n > 1\}$. Then $Var(N)$ equals

(a) 1 (b) λ

(c) λ^2 (d) ∞

(JUNE 2015)

37. Suppose X has density
$f(x|\theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, x > 0$ where $\theta > 0$ is unknown.
Define Y as follows:
$Y = k \text{ if } k \leq X < k + 1, k = 0, 1, 2 \ldots$
Then the distribution of Y is

(a) Normal (b) Binomial

(c) Poisson (d) Geometric

(JUNE 2015)

38. Let (X, Y) have the joint discrete distribution such that $X \mid Y = y \sim$ Binomials (y, 0.5) and $Y \sim$ poisson $(\lambda), \lambda > 0$ where λ is an unknown parameter. Let $T = T(X, Y)$ be any unbiased estimator of λ Then

(a) $Var(T) \leq Var(Y) for\ all\ \lambda$

(b) $Var(T) \geq Var(Y) for\ all\ \lambda$

(c) $Var(T) \geq \lambda\ for\ all\ \lambda$

(d) $Var(T) = Var(Y) for\ all\ \lambda$

(JUNE 2015)

39. Let X and Y be independent normal random variables with mean 0 and variance 1. Let the characteristics function of XY be denoted by φ Then

(a) $\varphi(2) = 1/2$

(b) φ is an even function

(c) $\varphi(t)\varphi\left(\frac{1}{t}\right) = |t|$ for all $t \neq 0$

(d) $\varphi(t) = E(e^{-t^2 y^2/2})$

(JUNE 2015)

40. Let X_1 and X_2 be independent and identically distributed normal random variables with mean 0 and variance 1. Let U_1 and U_2 be independent and identically distributed $U(0,1)$ random variables independent of X_1, X_2. Define $Z = \frac{X_1 U_1 + X_2 U_2}{\sqrt{U_1^2 + U_2^2}}$ then

(a) $E(Z) = 0$ (b) $Var(Z) = 1$

(c) Z is standard Cauchy (d) $Z \sim N(0,1)$

(JUNE 2015)

41. Suppose $X_1, X_2 \ldots$ are independent random variables. Assume that $X_1, X_3, \ldots$ are identically distributed with mean μ_1 and variance σ_1^2 and $X_2, X_4, \ldots$ are identically distributed with mean μ_2 and variance σ_2^2.

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Let $S_n = X_1 + X_2 + \ldots + X_n$.

Then $\frac{S_n - a_n}{b_n}$ coverges is distribution to N (0, 1) if

(a) $a_n = \frac{n(\mu_1+\mu_2)}{2}$ and $b_n = \sqrt{n}\sqrt{\frac{\sigma_1^2+\sigma_2^2}{2}}$

(b) $a_n = \frac{n(\mu_1+\mu_2)}{2}$ and $b_n = \frac{n(\sigma_1+\sigma_2)}{2}$

(c) $a_n = n(\mu_1 + \mu_2)$ and $b_n = \sqrt{n}\frac{(\sigma_1+\sigma_2)}{2}$

(d) $a_n = n(\mu_1 + \mu_2)$ and $b_n = \sqrt{n}\sqrt{\frac{\sigma_1^2+\sigma_2^2}{2}}$

(DEC. 2014)

42. Let $X_1, X_2, \ldots$ be independent and identically distributed random variables with $E(X_1) = 0$ and Variables with $E(X_i) = 0$ and $\text{Var}(X_i) = 1$ for all i. Let $S_n = X_1 + \cdots + X_n$. Let $\Phi(x)$ denote the cumulative distribution function of a standard normal random variable.

Then, for any $x > 0$, $\lim_{n\to\infty} P(-nx < S_n < nx)$ equals

(a) $2\Phi(x) - 1$ (b) $\Phi(x)$

(c) 1 (d) $1 - \Phi(2x)$

(DEC. 2014)

43. Suppose $X_1, X_2, \ldots$ are random variables on a common probability space with $X_n \sim N(\mu_n, \sigma_n^2)$. Then, X_n converges in probability to 2 if and only if

(a) $\mu_n \to 0$ and $\sigma_n^2 \to 2$

(b) $\mu_n \to 2$ and $\sigma_n^2 \to 0$

(c) $\mu_n \to 0$ and σ_n^2 converges

(d) $\sigma_n^2 \to 0$ and μ_n converges

(DEC. 2014)

44. Suppose that X_1 , X_2 and X_3 are independent and identically distributed random variables, each having a Bernoulli distribution with parameter 1/2. Consider the 2 x 2 matrix $A = \begin{pmatrix} X_1 & 0 \\ X_2 & X_3 \end{pmatrix}$. Then, $P(A$ is invertible) equals

(a) 0 (b) 1

(c) 1/4 (d) 3/4

(DEC. 2014)

45. $N, A_1, A_2 \ldots$ are independent real valued random variables such that

$P(N = k) = (1-p)p^k, k = 0,1,2 \ldots$ where $0 < p < 1$, and $\{A_i : i = 1,2, \ldots\}$ is a sequence of independent and identically distributed bounded random variables. Let

$$X(w) = \begin{cases} 0 & \text{If } N(w) = 0 \\ \sum_{j=1}^{k} A_j & \text{If } N(w) = k, k = 1,2, \ldots \end{cases}$$

Which of the following are necessarily correct?

(a) X is a bounded random variable

(b) Moment m_A is the moment function m_X of X is $m_X(t) = \frac{(1-p)}{1-p\, m_A(t)}, t \in \mathbb{R}$, where m_A is the moment generating function of A_1.

(c) Characteristic function $\varphi_X(t) = \frac{(1-p)}{1-p\, \varphi_A(t)}, t \in \mathbb{R}$, where φ_A is the Characteristic function of A_1.

(d) X is symmetric about 0.

(JUNE 2014)

46. Suppose $X_1, X_2, \ldots X_n$ are independent random variables each having a Bin $\left(8, \frac{1}{2}\right)$ distribution. Then $\frac{1}{\sqrt{n}} \sum_{k=1}^{n} (-1)^k X_k$ converges in distribution to

(a) $N(0,1)$ (b) $N(0,2)$

(c) $N(4,2)$ (d) $N(4,1)$

(JUNE 2014)

47. Let X and Y be independent and identically distributed random variables having a normal distribution with mean 0 and variance 1. Define Z and W as follow:

$$\begin{pmatrix} Z \\ W \end{pmatrix} = \begin{cases} \begin{pmatrix} X \\ Y \end{pmatrix} & \text{if } XY > 0 \\ \begin{pmatrix} -X \\ Y \end{pmatrix} & \text{if } X < 0 \text{ and } Y > 0 \\ \begin{pmatrix} X \\ -Y \end{pmatrix} & \text{if } X > 0 \text{ and } Y < 0 \end{cases}$$. Then

(a) Z and W are independent

(b) Z has $N(0,1)$ distribution

(c) W has $N(0,1)$ distribution

(d) $Cov(Z, W) > 0$

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48. Let X_n be distributed as a Poisson random variable with parameter n. Then which of the following statements are correct?

(a) $\lim_{n\to\infty} P\left(X_n > n + \sqrt{n}\right) = 0$

(b) $\lim_{n\to\infty} P\left(X_n \le n + \sqrt{n}\right) = 0$

(c) $\lim_{n\to\infty} P(X_n \le n) = \frac{1}{2}$

(d) $\lim_{n\to\infty} P(X_n \le n) = 1$

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49. Let X and Y be two independent $N(0,1)$ random variables. Define $U = \frac{X}{Y}$ and $V = \frac{X}{|Y|}$ then

(a) U and V have the same distribution

(b) V has t distribution

(c) $E\left(\frac{V}{U}\right) = 0$

(d) U and V are independent

(DEC. 2013)

50. Suppose $D \sim N(0,1)$ and $U = \begin{cases} 1 & \text{if } D \ge 0 \\ 0 & \text{if } D < 0 \end{cases}$. Then the correlation coefficient between $|D|$ and U is equal to

(a) 0.5 (b) 0.25

(c) 1 (d) 0

(DEC. 2013)

51. Suppose $X_1, X_2, \ldots, X_n$ are independent and identically distributed random variables each having an exponential distribution with parameter $\lambda > 0$. Let $X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(n)}$ be the corresponding order statistics. Then the probability distribution of $\frac{X_{(n)} - X_{(n-1)}}{nX_{(1)}}$ is

(a) Chi-square with 1 degree of freedom.

(b) Beta with parameters 2 and 1.

(c) F with parameters 2 and 2.

(d) F with parameters 2 and 1.

(DEC. 2013)

52. Let $X_1, X_2, \ldots \ldots$ be independent and identically distributed random variables each having a uniform distribution on $[-1,1]$. For $n \geq 1$, let $S_n = \sum_{i=1}^{n} X_i$ and let $Z_n = S_n/n^p$ for some $p > 0$. Then, as $n \to \infty$,

(a) $Z_n \to 0$ almost surely for $p \geq 1$

(b) $Z_n \to 0$ in probability for $\frac{1}{2} < p < 1$

(c) Z_n converges in distribution to a non-degenerate random variable if $p = \frac{1}{2}$

(d) $Z_n \to \infty$ almost surely for $p < \frac{1}{2}$

(DEC. 2013)

53. Let $X_1, X_2, \ldots \ldots$ be independent and identically distributed standard normal random variables. Which of the following is true?

(a) $\frac{\sqrt{n}X_1}{X_1^2 + \cdots + X_n^2}$ has a t −distribution with $n-1$ degrees of freedom

(b) $\frac{\sqrt{n}X_1}{X_1^2 + \cdots + X_n^2}$ has a t −distribution with n degrees of freedom

(c) $\frac{\sqrt{n}X_2}{X_2^2 + \cdots + X_{n+1}^2}$ has a t −distribution with n-1 degrees of freedom

(d) $\frac{\sqrt{n}X_2}{X_2^2 + \cdots + X_{n+1}^2}$ has a t −distribution with n degrees of freedom

(DEC. 2013)

54. Let X be a geometric random variable with probability mass function given by $P(X = k) = (1-p)^k p$ for $k \geq 0$ and $0 < p < 1$. For all $m, n \geq 1$ we have

(a) $P(X > m + n | X > m) = P(X \geq n)$

(b) $P(X > m + n | X > m) = P(X > n)$

(c) $P(X < m + n | X < m) = P(X < n)$

(d) $P(X < m + n | X < m) = P(X \leq n)$

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55. Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed random variables each having a uniform distribution on $(0, 1)$. Consider the histogram of these values with k equally spaced class intervals given by $\{(a_i, b_i], i = 1, 2, \ldots, k\}$ where $a_i = \frac{i-1}{k}$ and $b_i = a_i + \frac{1}{k}$. Let N_i be the number of values in the interval $(a_i, b_i]$. Then the covariance of N_1 and N_k is

(a) 0 (b) $-n/k^2$

(c) n/k^2 (d) $1/2$

(JUNE 2013)

56. Let $U_1, U_2, \ldots$ be independent and identically distributed random variables each having a uniform distribution on $(0, 1)$. Then $\lim_{n \to \infty} \left(U_1 + \cdots + U_n \leq \frac{3}{4}n\right)$

(a) does not exist (b) exists and equals 0

(c) exists and equals 1 (d) exists and equals $\frac{3}{4}$

(JUNE 2013)

57. Let X and Y independent random variables each following a uniform distribution on (0, 1). Let $W = X\, I_{[Y \leq X^2]}$, where I_A denotes the indicator function of the set A. Then which of the following statements are true?

(a) The cumulative distribution function of W is given by $F_W(t) = t^2\, I_{[0 \leq t \leq 1]} + I_{[t > 1]}$

(b) $P[W > 0] = \frac{1}{3}$

(c) The cumulative distribution function of W is continuous

(d) The cumulative distribution function of W is given by $F_W(t) = \left(\frac{2+t^3}{3}\right) I_{[0 \leq t \leq 1]} + I_{[t > 1]}$

(JUNE 2013)

58. Let $X_1, X_2, \ldots$ be independent and identically distributed random variables each following a uniform distribution on (0, 1). Denote $T_n = \max\{X_1, X_2, \ldots, X_n\}$. Then, which of the following statements are true?

(a) T_n converges to 1 in probability

(b) $n(1 - T_n)$ converges in distribution

(c) $n^2(1 - T_n)$ converges in distribution

(d) $\sqrt{n}(1 - T_n)$ converges to 0 in probability

(JUNE 2013)

59. Let $X_1, X_2, \ldots$ be independent random variables each following a normal distribution with unknown mean μ and unknown variance $\sigma^2 > 0$.

Define $\bar{X}_{n-2} = \frac{1}{n-2}\sum_{i=1}^{n-2} X_i$, $T_1 = \frac{\sum_{i=1}^{n-2}(X_i - \bar{X}_{n-2})^2}{n-3}$ and $T_2 = \frac{(X_{n-1} - X_n)}{\sqrt{2}}$; $n > 3$. Then which of the following statements are correct?

(a) T_1 is unbiased for σ^2

(b) $\frac{T_2}{\sqrt{T_1}}$ follows a t distribution with $(n-3)$ degrees of freedom

(c) $\frac{T_2^2}{T_1}$ follows a F distribution with 1 and $(n-3)$ degrees of freedom

(d) $\bar{X}_{n-2}$ is consistent for estimating μ

(JUNE 2013)

60. Let X_1, X_2, X_3, X_4, X_5 be independent and identically distributed random variables each following a uniform distribution on (0, 1), and let M denote their median. Then which of the following statements are true?

(a) $P\left(M < \frac{1}{3}\right) = P\left(M > \frac{2}{3}\right)$

(b) M is uniformly distributed on (0, 1)

(c) $E(M) = E(X_1)$

(d) $V(M) = V(X_1)$

(JUNE 2013)

61. Let $X_1, X_2, \dots$ be independent random variables each following exponential distribution with mean 1. Then which of the following statements are correct?

(a) $P(X_n > \log n \text{ for infinitely many } n \geq 1) = 1$

(b) $P(X_n > 2\log n \text{ for infinitely many } n \geq 1) = 1$

(c) $P(X_n > \frac{1}{2}\log n \text{ for infinitely many } n \geq 1) = 0$

(d) $P(X_n > \log n, X_{n+1} > \log(n+1) \text{ for infinitely many } n \geq 1) = 0$

(DEC. 2012)

62. Let $X_1, X_2, \dots$ be i.i.d. standard normal random variables and let $T_n = \frac{X_1^2 + \cdots + X_n^2}{n}$. Then

(a) The limiting distribution of $T_n - 1$ is χ^2 with 1 degree of freedom

(b) The limiting distribution of $\frac{T_n - 1}{\sqrt{n}}$ is normal with mean 0 and variance 2

(c) The limiting distribution of $\sqrt{n}(T_n - 1)$ is χ^2 with 1 degree of freedom

(d) The limiting distribution of $\sqrt{n}(T_n - 1)$ is normal with mean 0 and variance 2

(DEC. 2012)

63. Let X be a binomial random variable with parameters $\left(11, \frac{1}{3}\right)$. At which value(s) of k is $P(X = k)$ maximized?

(a) $k = 2$ (b) $k = 3$

(c) $k = 4$ (d) $k = 5$

(DEC. 2012)

64. X, Y, Z are independent random variables with N(0,1) (standard normal) distribution. Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 1$, if $x \geq 0$ and $f(x) = -1$, if $x < 0$. Let U, V, W be defined by $U = |X| \cdot f(Y), V = |Y| \cdot f(X), W = |Z| \cdot f(X)$. Then

(a) U and V are independent each having a N(0,1) distribution

(b) U and W are independent each having a N(0,1) distribution

(c) V and W are independent each having a N(0,1) distribution

(d) U, V and W are independent random variables.

(DEC. 2012)

65. Let X_1 and X_2 be two independent random variables with $X_1 \sim$ binomial $\left(m, \frac{1}{2}\right)$ and $X_2 \sim$ binomial $\left(n, \frac{1}{2}\right), m \neq n$. Which of the following are always true?

(a) $2X_1 + 3X_2 \sim$ binomial $\left(2m + 3n, \frac{1}{2}\right)$

(b) $X_2 - X_1 + m \sim$ binomial $\left(m + n, \frac{1}{2}\right)$

(c) Conditional distribution of X_2 given $(X_1 + X_2)$ is hypergeometric

(d) Distribution of $X_1 - X_2$ is symmetric about 0

(DEC. 2012)

66. Let X_1, X_2 and X_3 be independent with $X_1 \sim N(1,1), X_2 \sim N(-1,1)$ and $X_3 \sim N(0,1)$. Let

$q_1 = \frac{X_1^2 + X_2^2 + 2X_3^2 + 2X_1X_2}{2}$

$q_2 = \frac{X_1^2 + X_2^2 - 2X_1X_2}{2}$

Then which of the following statements are always true?

(a) q_1 has a central chi-square distribution

(b) q_2 has a central chi-square distribution

(c) $q_1 + q_2$ has a central chi-square distribution

(d) q_1 and q_2 are independent

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67. Let $X_1, X_2 \dots\dots$ as i.i.d.N(1, 1) random variables. Let $S_n = X_1^2 + X_2^2 + \ldots + X_n^2$ for $n \geq 1$. Then $\lim_{n \to \infty} \frac{Var(S_n)}{n}$ is

(a) 4 (b) 6

(c) 1 (d) 0

(JUNE 2012)

68. Let $X_1, X_2, \dots\dots$ be independent random variable with X_n being uniformly distributed between −n and 3n, n = 1, 2,...... Let $S_N = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} \frac{X_n}{n}$ for N = 1, 2, ... and let F_N be the distribution function of S_N. Also let Φ denote the distribution function of a standard normal random variable. Which of the following is/are true?

(a) $\lim_{N \to \infty} F_N(0) \leq \Phi(0)$ (b) $\lim_{N \to \infty} F_N(0) \geq \Phi(0)$

(c) $\lim_{N \to \infty} F_N(1) \leq \Phi(1)$ (d) $\lim_{N \to \infty} F_N(1) \geq \Phi(1)$

(JUNE 2012)

69. Suppose $\varepsilon_1, \varepsilon_2, \dots\dots, \varepsilon_n$ are i.i.d. $N(0, \sigma^2)$. Consider $Y_1, Y_2, \dots\dots, Y_n$ defined $Y_1 = \mu + \varepsilon_1$, $Y_{i+1} - \mu = \rho(Y_i - \mu) + \sqrt{1-\rho^2}\, \varepsilon_{i+1}, i = 1,2, \dots\dots n-1$.

Let $T = \frac{1}{n}\sum_{i=1}^{n} Y_i$. Suppose $0 < \rho < 1$ and $\sigma^2 > 0$. Then for $n \geq 2$

(a) T has a normal distribution

(b) T has mean μ and variance σ^2/n

(c) $E(T) = \mu, var\ (T) > \sigma^2/n$

(d) T follows N (μ, δ^2) where $\delta^2 > \sigma^2/n$