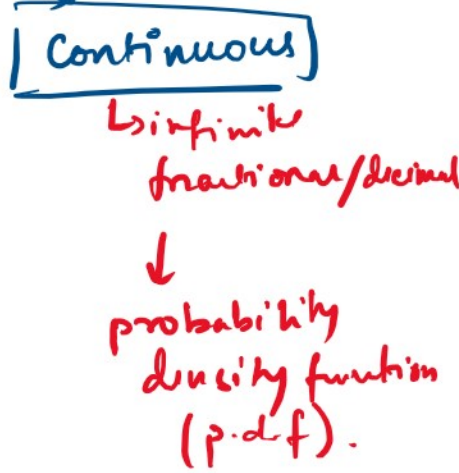
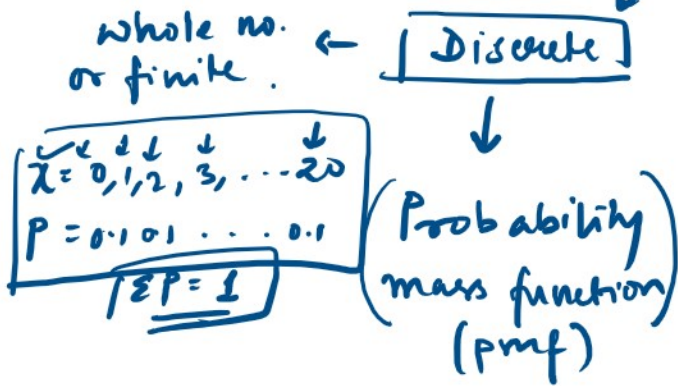


Random Variables → probability distribution



P.m.f

$x \rightarrow P(x=x)$
$\left. \begin{matrix} 0 \\ 1 \\ 2 \end{matrix} \right\} \begin{matrix} 0.3 \\ 0.3 \\ 0.4 \end{matrix} \right\}$
$\Sigma P = 1.0$

① *discrete*  $x$  is a r.v.? or  $x$  follow p.m.f.?

- Conditions: (a) for all values of  $x$ ,  $P(x=x)$  or  $f(x) \geq 0$
- (b)  $\sum_x f(x) = 1$ .

②  $x$  is a cont. r.v.? or  $x$  follow p.d.f.?

- Conditions: (a) for all values of  $x$ ,  $P(x=x)$  or  $f(x) \geq 0$
- (b)  $\int_{-\infty}^{\infty} f(x) dx = 1$ .

# **Mean** or **Expectation** of a r.v.  $x$ .

①  $x$  is a discrete r.v.

$E(x) = \sum_x x \cdot P$

$\bar{x} = \frac{1}{n} \sum x$

$= E(x) = \text{Mean}$

$$V(x) = E(x^2) - [E(x)]^2$$

$$E(x^2) = \sum x^2 \cdot p$$

$$= E(x) = \text{mean}$$

$$\text{Var}(x) = \frac{1}{n} \sum x^2 - \bar{x}^2$$

$$E(x^2) - [E(x)]^2$$

(2)  $x$  is a continuous r.v.

$$E(x) = \int_{-\infty}^{\infty} x \cdot f(x) dx$$

again  $E(x^2) = \int_{-\infty}^{\infty} x^2 \cdot f(x) dx$

$$V(x) = E(x^2) - \{E(x)\}^2$$

Q Calculate expected value of discrete r.v.  $x$ .

$x$	$P(x)$	$x \cdot p$
0	0.12	0
1	0.20	0.20
2	0.25	0.50
3	0.30	0.90
4	0.13	0.52

$$\sum x p = \text{ans.}$$

$$= 2.12$$

$$E(x) = \sum x \cdot P(x)$$

$$\therefore E(x) = 2.12$$

(ans.)

## Binomial Distribution

- (1)  $n$  no. of trials
- (2)  $x$  is no. of success out of  $n$  trials.

(2)  $x$  is no. of success out of  $n$  trials.

(3)  $p$  is the probab of success

(4)  $q = \text{prob of failure} = 1-p$   
because  $p+q=1$

(5) The probability mass function (p.m.f) of a discrete r.v  $x$  that follows a binomial distribution with parameters  $n$  and  $p$  is written as

$$f(x) = {}^n C_x p^x q^{n-x} \quad ; x=0,1,\dots,n$$

$= 0$  ; otherwise

$${}^n C_x = \frac{n!}{x!(n-x)!}$$

$$n! = n(n-1)(n-2)\dots 2 \cdot 1.$$

$$5! = 5 \times 4 \times 3 \times 2 \times 1$$

$$10! = 10 \times 9 \times 8 \times \dots \times 3 \times 2 \times 1.$$

$${}^{10}C_5 = \frac{10!}{5!(10-5)!} = \frac{10!}{5!5!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times \cancel{5!}}{5! \cancel{5!}} = \frac{2 \times 3 \times 4 \times 2 \times 1}{\cancel{2} \times \cancel{3} \times \cancel{4} \times 2 \times 1} = 2 \times 3 \times 2 \times 1 = 6 \times 2 \times 2 = 24 \times 2 = 252 \text{ (ans)}$$

$${}^8C_3 = \frac{8!}{3!5!} = \frac{8 \times 7 \times \cancel{6} \times \cancel{5!}}{3 \times \cancel{2} \times 1 \times \cancel{5!}} = 56 \text{ (ans)}$$

Properties of BD:

(1) Mean or Expectation

$$E(x) = n \cdot p$$

(2) Variance  $v(x) = n \cdot p \cdot q$   
 $= np(1-p)$

$$\therefore SD(x) = \sqrt{npq}$$

Q A coin is thrown  $n=7$  times. Find the probability of getting at least  $r=5$  heads.

$$n=7 \quad \underline{f(x \geq 5)} \quad \text{ie } x=5, 6, 7$$

$$x=0, 1, 2, 3, \dots, 7$$

$p$  = prob of getting head

$$= 1/2$$

$$\therefore q = 1/2$$

$$f(x \geq 5) = \underbrace{f(x=5)} + f(x=6) + f(x=7)$$

$$= {}^7C_5 p^5 q^{7-5} + {}^7C_6 p^6 q^{7-6} + {}^7C_7 p^7 q^{7-7}$$

$$= \frac{7!}{5!2!} \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^2 + \frac{7!}{6!1!} \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^1 + 1 \left(\frac{1}{2}\right)^7$$

$$= \frac{7 \times 6 \times 5}{2 \times 1} \left(\frac{1}{2}\right)^7 + 7 \left(\frac{1}{2}\right)^7 + \left(\frac{1}{2}\right)^7$$

$$= \left(\frac{1}{2}\right)^7 [21 + 7 + 1]$$

$$= \left(\frac{1}{2}\right)^7 \times 29$$

$$= \frac{29}{128} \quad (\text{ans})$$

$$\boxed{0, 1, 2, \dots, 8} \quad 1$$

② mean = 4      variance = 2

$$f(x) = {}^8C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{8-x}$$

$$(2) \text{ mean} = 4$$

$$\text{B.D: } np = 4$$

$$\therefore n \cdot \frac{1}{2} = 4$$
$$\boxed{n = 8}$$

$$\text{variance} = 2$$

$$\underline{npq} = 2$$

$$4 \cdot q = 2$$

$$\boxed{q = \frac{2}{4} = \frac{1}{2}}$$

$$\boxed{p = 1 - q = \frac{1}{2}}$$

$$f(x) = {}^8 C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{8-x}$$

for  $x=0, 1, 2, \dots, 8$ .

(i) at least 2  
sum

$$P(x > 2) = 1 - P(x < 2)$$
$$= 1 - [P(x=0) + P(x=1)]$$

$$= 1 - \left[ {}^8 C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^8 + {}^8 C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^7 \right]$$

$$= 1 - \left[ 1 \left(\frac{1}{2}\right)^8 + \frac{8!}{7!} \left(\frac{1}{2}\right)^8 \right]$$

$$= 1 - \left[ \left(\frac{1}{2}\right)^8 + 8 \left(\frac{1}{2}\right)^8 \right]$$

$$= 1 - \left(\frac{1}{2}\right)^8 (9)$$

$$= 1 - \frac{9}{256}$$

(ans).