18 February 2024 11:06 AM

Random Variables → probability distribution

p.m.f

x	$P(x=x)$
0	0.3
1	0.3
2	0.4
	$\Sigma P = 1.0$

Discrete → whole no. or finite.

$x = 0, 1, 2, 3, \dots 20$

$P = 0.1\ 0.1 \dots 0.1$

$\boxed{\Sigma P = 1}$

↓

(Probability mass function (pmf))

Continuous → infinite, fractional/decimal

↓

probability density function (p.d.f).

① x is a discrete r.v? or x follow p.m.f.?

Conditions : (a) for all values of x, $P(x=x)$ or $f(x) \geq 0$

(b) $\sum_x f(x) = 1$.

② x is a cont. r.v? or x follow p.d.f?

Conditions: (a) for all values of x, $P(x=x)$ or $f(x) \geq 0$

(b) $\int_{-\infty}^{\infty} f(x)\, dx = 1$.

Mean or Expectation of a r.v x.

① x is a discrete r.v.

$E(x) = \sum_x x \cdot P$

$\bar{x} = \frac{1}{n} \Sigma x$

$= E(x) = \text{mean}$

x

$$V(x) = E(x^2) - [E(x)]^2$$

$$E(x^2) = \sum x^2 \cdot p$$

$= E(x) = \text{mean}$

$Var(x) = \frac{1}{n}\sum x^2 - \bar{x}^2$

$E(x^2) - [E(x)]^2$

② x is a continuous r.v.

$$E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$$

again $$E(x^2) = \int_{-\infty}^{\infty} x^2 \cdot f(x)\, dx$$

$$V(x) = E(x^2) - \{E(x)\}^2$$

Q. Calculate expected value of discrete r.v. x.

x	$P(x)$	$x \cdot p$
0	0.12	0
1	0.20	0.20
2	0.25	0.50
3	0.30	0.90
4	0.13	0.52

$\sum xp =$ ans.

$= 2.12$

$$E(x) = \sum_x x \cdot P(x)$$

$\therefore E(x) = 2.12$ (ans).

Binomial Distribution

① n no. of trials

② x is no. of success out of n trials.

② x is no. of success out of n trials.

③ p is the probab of success

④ q = prob of failure = $1-p$

because $p+q=1$

⑤ The probability mass function (p.m.f) of a discrete r.v X that follows a binomial distribution with parameter n and p is written as

$$f(x) = {}^{n}C_{x}\, p^{x}\, q^{n-x} \quad ; x = 0, 1, \ldots, n$$

$$= 0 \quad ; \text{otherwise}$$

$${}^{n}C_{x} = \frac{n!}{x!\,(n-x)!}$$

$n! = n(n-1)(n-2)\cdots 2\cdot 1.$

$5! = 5\times4\times3\times2\times1$

$10! = 10\times9\times8\times\cdots3\times2\times1.$

$${}^{10}C_{5} = \frac{10!}{5!\,(10-5)!} = \frac{10!}{5!\,5!} = \frac{10\times9\times8\times7\times6\times\cancel{5!}}{5!\,\cancel{5!}} = \frac{\cancel{10}\times\cancel{9}\times\cancel{8}\times7\times6}{\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times1}$$

$= 2\times3\times7\times6$

$= 6\times6\times7$

$= 6\times42$

$= 252$ (ans)

$${}^{8}C_{3} = \frac{8!}{3!\,5!} = \frac{8\times7\times\cancel{6}\times\cancel{5!}}{3\times\cancel{2}\times1\times\cancel{5!}} = 56 \text{ (ans)}$$

Properties of BD:

① Mean or Expectation

$E(X) = n\cdot p$

② Variance $V(X) = n\cdot p\cdot q$

$= np(1-p)$

$\therefore SD(x) = \sqrt{npq}$.

Q A coin is thrown 7 times. Find the probability of getting at least 5 heads.

$n = 7 \quad \underline{p(x \geq 5)}$ ie $x = 5, 6, 7$

$x = 0, 1, 2, 3, \ldots, 7$

p = prob of getting head

$= 1/2$

$\therefore q = 1/2$

$$p(x \geq 5) = \underbrace{p(x=5)}_{} + p(x=6) + p(x=7)$$

$$= {}^7C_5\, p^5 q^{7-5} + {}^7C_6\, p^6 q^{7-6} + {}^7C_7\, p^7 q^0$$

$$= \frac{7!}{5!2!}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^2 + \frac{7!}{6!1!}\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^1 + 1\left(\frac{1}{2}\right)^7$$

$$= \frac{7\times\cancel{6}^3}{\cancel{2}\times 1}\left(\frac{1}{2}\right)^7 + 7\left(\frac{1}{2}\right)^7 + \left(\frac{1}{2}\right)^7$$

$$= \left(\frac{1}{2}\right)^7\left[21 + 7 + 1\right]$$

$$= \left(\frac{1}{2}\right)^7 \times 29$$

$$= \frac{29}{128} \text{ (ans)}$$

$\boxed{0, 1, 2, \ldots 8}$ 1

(2) mean = 4 variance = 2

$f(x) = {}^8C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{8-x}$

(2) mean = 4 variance = 2

B.D: $np = 4$ $\underbrace{np}q = 2$

$4 \cdot q = 2$

$\therefore n\frac{1}{2} = 4$

$\boxed{n = 8}$

$\boxed{q = \frac{2}{4} = \frac{1}{2}}$

$\boxed{p = 1 - q = \frac{1}{2}}$

$f(x) = {}^{8}C_{x}\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{8-x}$ for $x = 0, 1, 2, \ldots, 8$.

(i) at least 2 sum

$P(x \geq 2) = 1 - P(x < 2)$

$= 1 - [P(x=0) + P(x=1)]$

$= 1 - \left[{}^{8}C_{0}\frac{1}{2}^{0}\frac{1}{2}^{8} + {}^{8}C_{1}\frac{1}{2}^{1}\frac{1}{2}^{7}\right]$

$= 1 - \left[1\left(\frac{1}{2}\right)^{8} + \frac{8!}{7!}\left(\frac{1}{2}\right)^{8}\right]$

$= 1 - \left[\left(\frac{1}{2}\right)^{8} + 8\left(\frac{1}{2}\right)^{8}\right]$

$= 1 - \left(\frac{1}{2}\right)^{8}(9)$

$= 1 - \frac{9}{256}$

(ans).