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151 2073

$t \quad F \quad \chi^2 \quad Z \quad U$

Sampling & Statistical Inference

$\boxed{X_1, X_2, X_3, X_4, X_5}$

$X_1 \sim N(200, 8)$
$X_2 \sim N(104, 8)$
$X_3 \sim N(108, 15)$
$X_4 \sim N(120, 15)$
$X_5 \sim N(210, 15)$

$$U = \frac{X_1 + X_2}{2}$$

$$V = \frac{X_3 + X_4 + X_5}{3}$$

$$\boxed{P(U > V)} = ??$$

or,

$$W = U - V$$

$$P(U - V > 0)$$

$$P(W > 0)$$

$$W \sim N(6, 9)$$

$$P\left(\frac{W-6}{3} > \frac{0-6}{3}\right)$$

$$P\left(\frac{W-6}{3} > -2\right)$$

$$P(Z > -2)$$

$$= 1 - \Phi(+2)$$

$$= \Phi(2)$$

$$= 0.9772$$

$x_1 = -2,\ x_2 = 1,\ x_3 = -1$

$\boxed{n=3}$ from a discrete distribution

pmf: $f(x,\theta) = P(X=x) \begin{cases} \frac{1}{2\theta+1} & x \in \{-\theta, -\theta+1, \ldots 0, \theta\} \\ 0 & \text{otherwise} \end{cases}$

$\theta \in \Theta = \{1, 2 \ldots\}$ then method of moment estimator of θ?

0/1/2/3/4

Uniform distb. $P(X=x) = \frac{1}{b-a+1}$ $x = a, a+1, \ldots b-1, b$

$E(X) = \frac{a+b}{2}$, $V(X) = \frac{(b-a+1)^2-1}{12}$

$P(X=x) = \frac{1}{2\theta+1}$, $x = -\theta, -\theta+1, \ldots 0, \ldots \theta-1, \theta$

using Method of moments

$E(X) = \frac{-\theta+\theta}{2} = 0$ $\quad V(X) = \frac{(\theta-(-\theta)+1)^2-1}{12} = \theta$

$= \frac{(2\theta+1)^2-1}{12}$

$E(X) = \sum \frac{x_i}{n}$

$E(X^2) = \frac{\sum x_i^2}{n} = \frac{1}{3}(4+1+1) = 2$

$V(X) = \frac{(2\theta+1)^2-1}{12} = 2$

$(2\theta+1)^2 = 25$

$2\theta+1 = \pm 5$ $\quad \boxed{\theta > 0}$ taking +ve only

$2\theta = 4$

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$\theta = 2$

MLE Based

$$f_X(x) = (\theta+1)x^{\theta} \quad 0<x<1$$
$$= 0, \text{ otherwise}$$

$\boxed{\theta<0}$ find MLE of θ ?

$$L(\theta) = (\theta+1)^n \prod (x_i^{\theta}) = $$

$$\cancel{\frac{d}{d\theta} L(\theta) = \frac{n}{\theta+1} +}$$

$$\log L(\theta) = n \ln(\theta+1) + \theta \sum \ln x_i$$

$$\frac{d}{d\theta} \ln L(\theta) = \frac{n}{1+\theta} + \sum \ln x_i$$

$$\frac{n}{1+\theta} = -\sum \ln x_i$$

$$1+\theta = \frac{n}{-\sum \ln x_i}$$

$$\theta = -1 - \frac{n}{\sum \ln x_i} = \frac{n + \sum \ln x_i}{-\sum \ln x_i}$$

2nd order derivative < 0

What if you have to make the functions yourself??

let, $x_1 = 0$, $x_2 = 1$, $x_3 = 2$, $x_4 = 3$, $x_5 = 0$

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$\boxed{n=5}$ $P(X=x) = \theta/3$ $x=0$

$2\theta/3$ $x=1$

$\cancel{(1-\theta)/3}$ $x=2,3$

Then MLE of θ ?

Likelihood function formation

$$L(\theta) = (\theta/3)^2 \times \left(\frac{2\theta}{3}\right)^1 \left(\frac{1-\theta}{3}\right)^1 \left(\frac{1-\theta}{2}\right)^{\cancel{1}}$$

$$= (1-\theta)^2/4$$

$$\ln L(\theta) = 3\ln\theta + 2\ln(\cancel{4}-\theta)$$

$$\frac{d}{d\theta}\ln L(\theta) = 3/\theta - \frac{2}{1-\theta} = \frac{3(1-\theta) - 2\theta}{\theta(1-\theta)}$$

$$= \frac{3 - 3\theta - 2\theta}{\theta(1-\theta)}$$

$\underline{\text{Uniform}}$

$x_1 = 1.1$, $x_2 = 2.2$, $x_3 = 3.3$

$\boxed{n=3}$

$$f(x;\theta) = \frac{1}{\theta} e^{-x\theta} \quad x>0$$
$$= 0, \text{ otherwise.}$$

MLE of $\underline{\theta}$?

$$L(\theta) = \prod_{i=1}^{3} \frac{1}{\theta} e^{\cancel{x_i}/\theta}$$

Σx_i

$$L(\theta) = \prod_{i=1} \frac{1}{\theta} e^{\cdots}$$

$$\Rightarrow L(\theta) = \frac{1}{\theta^3} e^{-\frac{\sum x_i}{\theta}}$$

$$\ln L(\theta) = -3\ln\theta - \frac{1}{\theta}\sum_{i=1}^{3} x_i$$

$$\frac{d}{d\theta} \ln L(\theta) = \frac{1}{\theta}\left[-3 + \frac{1}{\theta}\sum x_i\right] = 0$$

$$\hat{\theta} = \frac{\sum x_i}{3} = \frac{1.1 + 2.2 + 3.3}{3} = 2.2$$

$L(\theta)\uparrow$ for $\theta \in (0, 2.2)$

$L(\theta)\downarrow$ for $\theta\ (2.2, \infty)$

$$\max_{\theta} L(\theta) = \max\{L(2), L(3)\}$$

$$= \max\{e^{-3.3}/8,\ e^{-2.2}/27\}$$