07 January 2024 09:59

$\frac{x-2}{x+3} > 1$ find x.

$x-2 > x+3 \Rightarrow -2 > 3$ ✗

$\boxed{\frac{x-2}{x+3} - 1 > 0}$

$\frac{x-2-x-3}{x+3} > 0$.

$\frac{a}{b} > 0$

$a>0$ & $b>0$ $\quad$ $a<0$ & $b<0$

$\frac{-5}{x+3} > 0$

$x+3 < 0 \Rightarrow \boxed{x < -3}$

$y = \sqrt{x - \sqrt{1-x^2}}$ find x.

$1-x^2 \geq 0$ and $x - \sqrt{1-x^2} \geq 0$.

$x^2 \leq 1$ and $x \geq \sqrt{1-x^2}$

$x^2 \geq a^2$

$x \geq a$ or $x \leq -a$

$x^2 \geq 1 - x^2$.

$\boxed{-1 \leq x \leq 1}$ $\boxed{\text{and.}}$ $2x^2 \geq 1$

$-\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} = -0.7$

$x^2 \geq \frac{1}{2}$

$\cancel{\left[-1, -\frac{1}{\sqrt{2}}\right]}$ $\cancel{\cup}$ $\left[\frac{1}{\sqrt{2}}, 1\right]$ $\quad$ $\boxed{x \geq \frac{1}{\sqrt{2}} \text{ or } \cancel{x \leq -\frac{1}{\sqrt{2}}}}$

Imp. $f(x+y) = f(x) \cdot f(y)$ find $f(x)$ if $f(1) = 2$.

$x=1 \quad y=1$.

$f(2) = f(1)^2$.

$f(3) = f(2) \cdot f(1) = [f(1)]^3$

$f(4) = f(2) \cdot f(2) = [f(1)]^4$

$\vdots$

$f(n) = [f(1)]^n$. $\longrightarrow$ $f(x) = [f(1)]^x = 2^x$.

$f(x+y) = f(x) + f(y)$ find $f(x)$ if $\cancel{f(0)=2}$

$f(1) = 4$

$x = y = 1$

$f(2) = 2f(1)$

$f(3) = 3f(1)$

$\vdots$

$f(n) = nf(1)$

$f(x) = x \cdot f(1)$

$f(x+y) = f(x) \cdot f(y) \to f(x) = [f(1)]^x$

$\sum_{k=1}^{n} f(k) = ?$

$f(1) + f(2) + \dots + f(n) = f(1) + [f(1)]^2 + \dots [f(1)]^n$

GINT function

Greatest Integer

$y = [x]$ where $[x] \to$ greatest integer less than or equal to x.

discontinuous for integer values of x.

$-5 < x < 5$

pts of discontinuity

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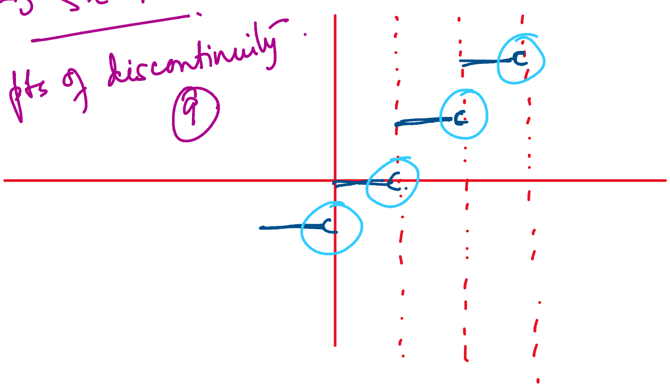

x	y
0	0
$0 < x < 1$	0
$x = 1$	1
$1 < x < 2$	1

$p_{min} = 0 - 4$ $p = x^2 - 4$

≥ 0

$y = [x^2 - 4]$

p

$y = [p] \to$ discont for int values of p.

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$-3 < x < 3$

find the no of pts of discontinuity

num of $x^2 = 0$

$x^2 \geq 0$

$-4 \leq p < 5$

$-3 \to 0, \to 3$

$5 \to -4, \to 5$

$4 \to -4 \to 4$

8 8

-3 to 4

8

$-3, -2, -1, 0, 1, 2, 3, 4$

x	$p = x^2 - 4$
-3	5
3	5
0	-4

x $-3 \to 0 \to 3$

p $5 \to -4 \to 5$

$\boxed{4 \to -4 \to 4}$

$y = [x] \to$ discont for int values of x.

$p \quad 5 \to -4 \to 5$

int values of x.

$\boxed{4 \to -4 \to 4}$

$8 \quad 1 \quad 8$

$y = [\sin x - 2]$

p.

$\boxed{p = \sin x - 2}$

$y = [p]$

min $= -3$ ✓

max $= -1$ ✓

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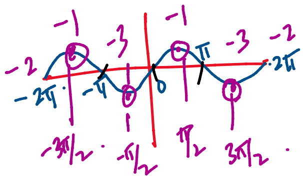
$-2\pi < x < 2\pi$

$-3 \le p \le -1$

-3 to -1

$\cancel{3}$

x	p
-2π	-2
2π	-2



x	p
$-3\pi/2$	-1
$-\pi$	-2 ✓
$-\pi/2$	-3
0	-2 ✓
$\pi/2$	-1
π	-2 ✓
$3\pi/2$	-3

$y = \sqrt{x - \sqrt{1-x^2}}$

$\frac{1}{\sqrt{2}} \le x \le 1$

x	y
$\frac{1}{\sqrt{2}}$	0
1	1

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x-\sqrt{1-x^2}}} \cdot \left[1 - \frac{1}{\cancel{2}\sqrt{1-x^2}}(-\cancel{2}x)\right]$$

$$= \frac{1}{2\sqrt{x-\sqrt{1-x^2}}} \left[\frac{\sqrt{1-x^2}+x}{\sqrt{1-x^2}}\right]$$

$\sqrt{1+x^2} + x = 0 \quad \times$

$\sqrt{1+x^2} = -x. \quad 1+x^2 = x^2. \times$