LINEAR ALGEBRA 28.3.2024

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Linear Algebra U...

1. Let W be the vector space of all real polynomials of degree at most 3. Define $T: W \to W$ by $(Tp)(x) = p'(x)$, where p' is the derivative of p. The matrix of T in the basis $\{1, x, x^2, x^3\}$, considered as column vectors, is given by

   1. $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$
   2. $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{pmatrix}$
   3. $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
   4. $\begin{pmatrix} 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

2. Let $S = \{A : A = [a_{ij}]_{5\times 5}, a_{ij} = 0 \text{ or } 1\ \forall i, j, \sum_j a_{ij} = 1\ \forall i \text{ and } \sum_i a_{ij} = 1\ \forall j\}$. Then the number of elements in S is

   1. 5^2
   2. 5^5
   3. $5!$
   4. 55

3. Let ξ be a primitive fifth root of unity. Define $A = \begin{pmatrix} \xi^{-2} & 0 & 0 & 0 & 0 \\ 0 & \xi^{-1} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \xi & 0 \\ 0 & 0 & 0 & 0 & \xi^2 \end{pmatrix}$

   For a vector $v = (v_1, v_2, v_3, v_4, v_5) \in \mathbb{R}^5$, define $|v|_A = \sqrt{|vAv^T|}$, where v^T is transpose of v. If $w = (1, -1, 1, 1, -1)$, then $|w|_A$ equals

   1. 0
   2. 1
   3. -1
   4. 2

4. The dimension of the vector space of all symmetric matrices of order $n \times n$ $(n \geq 2)$ with real entries and trace equal to zero is

   1. $\left(\frac{n^2 - n}{2}\right) - 1$
   2. $\left(\frac{n^2 + n}{2}\right) - 1$
   3. $\left(\frac{n^2 - 2n}{2}\right) - 1$
   4. $\left(\frac{n^2 + 2n}{2}\right) - 1$

5. Let D be a non-zero $n \times n$ real matrix with $n \geq 2$. Which of the following implications is valid?

   1. $\det(D) = 0$ implies rank $(D) = 0$
   2. $\det(D) = 1$ implies rank $(D) \neq 1$
   3. rank$(D) = 1$ implies $\det(D) \neq 0$
   4. rank$(D) = n$ implies $\det(D) \neq 1$

6. Suppose A, B are $n\times n$ positive definite matrices and I be the $n\times n$ identity matrix. Then which of the following are positive definite.

1. $A+B$
2. ABA
3. A^2+I
4. AB

7. Let N be a 3×3 non-zero matrix with the property $N^3=O$. Which of the following is/are true?

1. N is not similar to a diagonal matrix.
2. N is similar to a diagonal matrix.
3. N has one non-zero eigenvector.
4. N has three linearly independent eigenvectors.

8. Let $a_{ij}=a_ia_j$, $1\le i, j\le n$, where $a_1,\dots,a_n$ are real numbers. Let $A=((a_{ij}))$ be the $n\times n$ matrix (a_{ij}). Then

1. it is possible to choose $a_1,\dots,a_n$ so as to make the matrix A non-singular.
2. the matrix A is positive definite if $(a_1,\dots,a_n)$ is a non-zero vector.
3. the matrix A is positive semidefinite for all $(a_1,\dots,a_n)$.
4. for all $(a_1,\dots,a_n)$, zero is an eigenvalue of A.

9. Let T be a linear transformation on the real vector space $\mathbb{R}^n$ over $\mathbb{R}$ such that $T^2=\lambda T$ for some $\lambda\in\mathbb{R}$. Then

1. $\|Tx\|=|\lambda|\,\|x\|$ for $x\in\mathbb{R}^n$.
2. If $\|Tx\|=\|x\|$ for some non-zero vector $x\in\mathbb{R}^n$, then $\lambda=\pm 1$.
3. $T=\lambda I$, where I is the identity transformation on $\mathbb{R}^n$.
4. If $\|Tx\|>\|x\|$ for a non-zero vector $x\in\mathbb{R}^n$, then T is necessarily singular.

10. Let M be the vector space of all 3×3 real matrices and let $A=\begin{pmatrix}2&1&0\\0&2&0\\0&0&3\end{pmatrix}$. Which of the following are subspaces of M?

1. $\{X\in M : XA=AX\}$
2. $\{X\in M : X+A=A+X\}$
3. $\{X\in M : trace(AX)=0\}$
4. $\{X\in M : \det(AX)=0\}$

11. Let $W=\{p(B): p$ is a polynomial with real coefficients$\}$, where $B=\begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix}$. The dimension d of the vector space W satisfies

1. $4\le d\le 6$
2. $6\le d\le 9$
3. $3\le d\le 8$
4. $3\le d\le 4$

12. The determinant of the matrix $\begin{bmatrix}1&0&0&0&0&2\\0&1&0&0&2&0\\0&0&1&2&0&0\\0&0&2&1&0&0\\0&2&0&0&1&0\\2&0&0&0&0&1\end{bmatrix}$ is

1. 0
2. -9
3. -27
4. 1

13. For a positive integer n, let P_n denote the space of all polynomials $p(x)$ with coefficients in $\mathbb{R}$ such that $\deg p(x)\le n$, and let B_n denote the standard basis of P_n given by $B_n=\{1,x,x^2,\dots,x^n\}$. If $T:P_3\to P_4$ is the linear transformation defined by $T(p(x))=x^2p'(x)+\int_0^x p(t)dt$ and $A=(a_{ij})$ is the 5×4 matrix of T with respect to standard bases B_3 and B_4, then

1. $a_{32}=\frac{3}{2}$ and $a_{33}=\frac{7}{3}$
2. $a_{32}=\frac{3}{2}$ and $a_{33}=0$
3. $a_{32}=0$ and $a_{33}=\frac{7}{3}$
4. $a_{32}=0$ and $a_{33}=0$

14. Let A be a 5×4 matrix with real entries such that the space of all solutions of the linear system $AX^t=[1,2,3,4,5]^t$ is given by $\{[1+2s,2+3s,3+4s,4+5s]^t : s\in\mathbb{R}\}$. (Here, M^t denotes the transpose of a

14. Let A be a 5×4 matrix with real entries such that the space of all solutions of the linear system $AX^t = [1,2,3,4,5]^t$ is given by $\{[1+2s, 2+3s, 3+4s, 4+5s]^t : s \in \mathbb{R}\}$. (Here, M^t denotes the transpose of a matrix M). Then the rank of A is equal to

1. 4
2. 3
3. 2
4. 1

15. Let A be a 3×3 matrix with real entries such that $\det(A)=6$ and the trace of A is 0. If $\det(A+I)=0$, where I denotes the 3×3 identity matrix, then the eigenvalues of A are

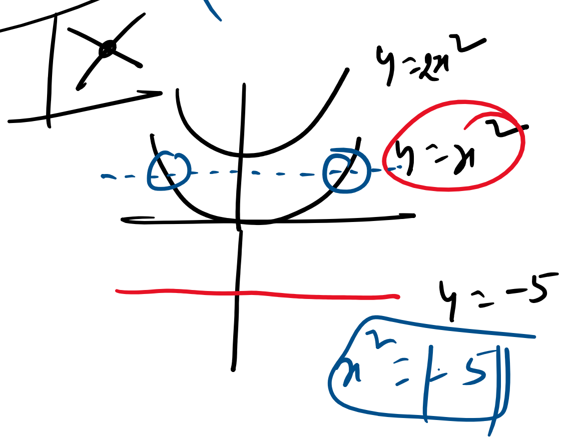
1. -1,2,3
2. -1,2,-3
3. 1,2,-3
4. -1,-2,3

16. Suppose the matrix $A = \begin{bmatrix} 40 & -29 & -11 \\ -18 & 30 & -12 \\ 26 & 24 & -50 \end{bmatrix}$ has a certain complex number $\lambda \neq 0$ as an eigenvalue. Which of the following numbers must also be an eigenvalue of A?

1. $\lambda + 20$
2. $\lambda - 20$
3. $20 - \lambda$
4. $-20 - \lambda$

17. Let $A = \begin{bmatrix} 0 & 0 & 0 & -4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \end{bmatrix}$. Then a Jordan canonical form of A is

1. $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$
2. $\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$
3. $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$
4. $\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$



18. Let n be a positive integer and let H_n be the space of all $n \times n$ matrices $A = (a_{ij})$ with entries in $\mathbb{R}$ satisfying $a_{ij} = a_{rs}$, whenever $i+j = r+s$ $(i,j,r,s = 1,\dots,n)$. Then the dimension of H_n, as a vector space over $\mathbb{R}$, is

1. n^2
2. n^2-n+1
3. $2n+1$
4. $2n-1$

PART - C

19. Consider a matrix $A = (a_{ij})_{n \times n}$ with integer entries such that $a_{ij} = 0$ for $i > j$ and $a_{ii} = 1$ for $i = 1,\dots,n$. Which of the following properties must be true?

1. A^{-1} exists and it has integer entries
2. A^{-1} exists and it has some entries that are not integers
3. A^{-1} is a polynomial function of A with integer coefficients
4. A^{-1} is not a power of A unless A is the identity matrix

20. Let J be the 3×3 matrix all of whose entries are 1. Then

1. 0 and 3 are the only eigenvalues of J
2. J is positive semidefinite, i.e., $\langle Jx, x \rangle \geq 0$ for all $x \in \mathbb{R}^3$
3. J is diagonalizable
4. J is positive definite, i.e., $\langle Jx, x \rangle > 0$ for all $x \in \mathbb{R}^3$ with $x \neq 0$.

21. Let A, B be complex $n \times n$ matrices. Which of the following statements are true?

1. If A, B and $A+B$ are invertible, then $A^{-1} + B^{-1}$ is invertible.
2. If A, B and $A+B$ are invertible, then $A^{-1} - B^{-1}$ is invertible.
3. If AB is nilpotent, then BA is nilpotent.
4. Characteristic polynomials of AB and BA are equal if A is invertible.

22. Let ω be a complex number such that $\omega^3 = 1$, but $\omega \neq 1$. If $A = \begin{bmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \dots \end{bmatrix}$, then which of the

22. Let ω be a complex number such that $\omega^3 = 1$, but $\omega \neq 1$. If $A = \begin{bmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & \omega & 1 \end{bmatrix}$, then which of the following statements are true?

1. A is invertible
2. rank(A) = 2
3. 0 is an eigenvalue of A
4. there exist linearly independent vectors $v, w \in \mathbb{C}^3$ such that $Av=Aw=0$

23. Let A be a 4×4 matrix with real entries such that -1,1,2,-2 are its eigenvalues. If $B = A^4 - 5A^2 + 5I$, where I denotes the 4×4 identity matrix, then which of the following statements are correct?

1. det(A+B)=0
2. det(B)=1
3. trace of A-B is 0
4. trace of A+B is 4

24. Let $M_2(\mathbb{R})$ denote the set of 2×2 real matrices. Let $A \in M_2(\mathbb{R})$ be of trace 2 and determinant -3. Identifying $M_2(\mathbb{R})$ with $\mathbb{R}^4$, consider the linear transformation $T: M_2(\mathbb{R}) \to M_2(\mathbb{R})$ defined by $T(B)=AB$. Then which of the following statements are true?

1. T is diagonalizable
2. 2 is an eigenvalue of T
3. T is invertible
4. $T(B)=B$ for some $0 \neq B$ in $M_2(\mathbb{R})$

25. Let A be a 2×2 non-zero matrix with entries in $\mathbb{C}$ such that $A^2=O$. Which of the following statements must be true?

1. PAP^{-1} is diagonal for some invertible 2×2 matrix P with entries in $\mathbb{R}$
2. A has two distinct eigenvalues in $\mathbb{C}$
3. A has only one eigenvalue in $\mathbb{C}$ with multiplicity 2
4. $Av=v$ for some $v \in \mathbb{C}^2$, $v \neq 0$

26. Consider the linear transformation $T: \mathbb{R}^7 \to \mathbb{R}^7$ defined by $T(x_1, x_2, \ldots, x_6, x_7) = (x_7, x_6, \ldots, x_2, x_1)$. Which of the following statements are true?

1. The determinant of T is 1
2. There is a basis of $\mathbb{R}^7$ with respect to which T is a diagonal matrix
3. $T^7 = I$
4. The smallest n such that $T^n = I$, is even

27. Let λ, μ be distinct eigenvalues of a 2×2 matrix A. Then, which of the following statements must be true?

1. A^2 has distinct eigenvalues
2. $A^3 = \dfrac{\lambda^3 - \mu^3}{\lambda - \mu} A - \lambda\mu(\lambda+\mu) I$
3. trace of A^n is $\lambda^n + \mu^n$ for every positive integer n
4. A^n is not a scalar multiple of identity for any positive integer n

28. *Let A, B be $n \times n$ real matrices. Which of the following statements is correct?*

1. rank (A+B) = rank (A) + rank (B)
2. rank (A+B) ≤ rank (A) + rank (B)
3. rank (A+B) = min {rank (A), rank (B)}
4. rank (A+B) = max {rank (A), rank (B)}

29. *Let ξ be a primitive cube root of unity. Define* $A = \begin{bmatrix} \xi^{-1} & 0 \\ 0 & \xi \end{bmatrix}$. *For a vector* $\mathbf{v} = (v_1, v_2, v_3) \in \mathbb{R}^3$ *define* $|\mathbf{v}|_A = \sqrt{|\mathbf{v}A\mathbf{v}^T|}$, *where* $\mathbf{v}^T$ *is transpose of* $\mathbf{v}$. *If* $\mathbf{w} = (1,1,1)$ *then* $|\mathbf{w}|_A$ *equals*

1. 0
2. 1
3. − 1
4. 2

30. *The dimension of the vector space of all symmetric matrices* $A = (a_{ij})$ *of order* $n \times n (n \geq 2)$ *with real entries,* $a_{11} = 0$ *and trace zero is*

1. $(n^2 + n - 4)/2$
2. $(n^2 - n + 4)/2$
3. $(n^2 + n - 3)/2$
4. $(n^2 - n + 3)/2$

31. *Let N be the vector space of all real polynomials of degree at most 3. Define* $S : N \to N$ *by* $S(p(x)) = p(x+1)$, $p \in N$. *Then the matrix of S in the basis* $\{1, x, x^2, x^3\}$ *considered as column vectors is given by:*

1. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$
2. $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
3. $\begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & 1 & 2 & 3 \\ 2 & 2 & 2 & 3 \\ 3 & 3 & 3 & 3 \end{bmatrix}$
4. $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

PART - C

32. *Which of the following matrices are positive definite?*

1. $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$
2. $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$
3. $\begin{bmatrix} 4 & -1 \\ -1 & 4 \end{bmatrix}$
4. $\begin{bmatrix} 0 & 4 \\ 4 & 0 \end{bmatrix}$

33. *Let A be a non-zero linear transformation on a real vector space V of dimension n. Let the subspace* $V_0 \subset V$ *be the image of V under A. Let* $k = \dim V_0 < n$ *and suppose that for some* $\lambda \in \mathbb{R}$, $A^2 = \lambda A$. *Then*

1. $\lambda = 1$
2. $\det A = |\lambda|^n$
3. λ *is the only eigenvalue of* A
4. *there is a nontrivial subspace* $V_1 \subset V$ *such that* $Ax = 0$ *for all* $x \in V_1$

34. Let C be an $n\times n$ real matrix. Let W be the vector space spanned by $\{I, C, C^2, \ldots, C^{2n}\}$. The dimension of the vector space W is

1. $2n$
2. atmost n
3. n^2
4. atmost $2n$

35. Let V_1, V_2 be subspaces of a vector space V. Which of the following is/are necessarily a subspace of V?

1. $V_1 \cap V_2$
2. $V_1 \cup V_2$
3. $V_1 + V_2 = \{x + y : x \in V_1, y \in V_2\}$
4. $V_1/V_2 = \{x \in V_1 \text{ and } x \notin V_2\}$

36. Let N be a non-zero 3×3 matrix with the property $N^2 = O$. Which of the following is/are true?

1. N is not similar to a diagonal matrix.
2. N is similar to a diagonal matrix.
3. N has one non-zero eigenvector.
4. N has three linearly independent eigenvectors.

37. Let n be a positive integer and let $M_n(\mathbb{R})$ denote the space of all $n \times n$ real matrices. If $T: M_n(\mathbb{R}) \to M_n(\mathbb{R})$ is a linear transformation such that $T(A)=0$ whenever $A \in M_n(\mathbb{R})$ is symmetric or skew-symmetric, then the rank of T is

1. $\frac{n(n+1)}{2}$
2. $\frac{n(n-1)}{2}$
3. n
4. 0

38. Let $S: \mathbb{R}^3 \to \mathbb{R}^4$ and $T: \mathbb{R}^4 \to \mathbb{R}^3$ be linear transformations such that $T \circ S$ is the identity map of $\mathbb{R}^3$. Then

1. $S \circ T$ is the identity map of $\mathbb{R}^4$
2. $S \circ T$ is one–one, but not onto.
3. $S \circ T$ is onto, but not one-one.
4. $S \circ T$ is neither one–one nor onto.

39. Let V be a 3-dimensional vector space over the field $F_3 = \mathbb{Z}/3\mathbb{Z}$ of 3 elements. The number of distinct 1- dimensional subspaces of V is

1. 13
2. **26**
3. 9
4. 15

40. Let V be the inner product space consisting of linear polynomials, $p: [0,1] \to \mathbb{R}$ (i.e., V consists of polynomials p of the form $p(x) = ax + b$; $a, b \in \mathbb{R}$), with the inner product defined by

$\langle p, q \rangle = \int_0^1 p(x)q(x)\,dx$ for $p, q \in V$. An orthonormal basis of V is

1. $\{1, x\}$
2. $\{1, x\sqrt{3}\}$
3. $\{1, (2x-1)\sqrt{3}\}$
4. $\left\{1, x - \frac{1}{2}\right\}$

41. Let $f(x)$ be the minimal polynomial of the 4×4 matrix $A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$. Then the rank of the 4 × 4 matrix $f(A)$ is

1. 0 2. 1 3. 2 4. 4

42. Let a, b, c be positive real numbers such that $b^2 + c^2 < a < 1$. Consider the 3×3 matrix $A = \begin{bmatrix} 1 & b & c \\ b & a & 0 \\ c & 0 & 1 \end{bmatrix}$

1. All the eigenvalues of A are negative real numbers.
2. All the eigenvalues of A are positive real numbers.
3. A can have a positive as well as a negative eigenvalue.
4. Eigenvalues of A can be non-real complex numbers.

43. The system of equations $x + y + z = 1$, $2x + 3y - z = 5$, $x + 2y - kz = 4$, where $k \in \mathbb{R}$, has an infinite number of solutions for

1. $k = 0$ 2. $k = 1$ 3. $k = 2$ 4. $k = 3$

PART - C

44. Let n be an integer, $n \geq 3$ and let $u_1, u_2, \ldots, u_n$ be n linearly independent elements in a vector space over $\mathbb{R}$. Set $u_0 = 0$ and $u_{n+1} = u_1$. Define $v_i = u_i + u_{i+1}$ and $w_i = u_{i-1} + u_i$ for $i = 1, 2, \ldots, n$. Then

1. $v_1, v_2, \ldots, v_n$ are linearly independent, if $n = 2010$.
2. $v_1, v_2, \ldots, v_n$ are linearly independent, if $n = 2011$.
3. $w_1, w_2, \ldots, w_n$ are linearly independent, if $n = 2010$.
4. $w_1, w_2, \ldots, w_n$ are linearly independent, if $n = 2011$.

45. Let V and W be finite-dimensional vector spaces over $\mathbb{R}$ and let $T_1: V \to V$ and $T_2: W \to W$ be linear transformations whose minimal polynomials are given by $f_1(x) = x^3 + x^2 + x + 1$ and $f_2(x) = x^4 - x^2 - 2$. Let $T: V \oplus W \to V \oplus W$ be the linear transformation defined by $T((v, w)) = (T_1(v), T_2(w))$ for $(v, w) \in V \oplus W$ and let $f(x)$ be the minimal polynomial of T. Then

1. $\deg f(x) = 7$ 2. $\deg f(x) = 5$ 3. nullity $(T) = 1$ 4. nullity $(T) = 0$

46. Let $a, b, c, d \in \mathbb{R}$ and let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$ for $\begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2$. Let $S: \mathbb{C} \to \mathbb{C}$ be the corresponding map defined by $S(x + iy) = (ax + by) + i(cx + dy)$ for $x, y \in \mathbb{R}$. Then

1. S is always $\mathbb{C}$ – linear, that is $S(z_1 + z_2) = S(z_1) + S(z_2)$ for all $z_1, z_2 \in \mathbb{C}$ and $S(\alpha z) = \alpha S(z)$ for all $\alpha \in \mathbb{C}$ and $z \in \mathbb{C}$.
2. S is $\mathbb{C}$ – linear if $b = -c$ and $d = a$.
3. S is $\mathbb{C}$ – linear only if $b = -c$ and $d = a$.
4. S is $\mathbb{C}$ – linear if and only if T is the identity transformation.

47. Let $A = [a_{ij}]$ be an $n \times n$ complex matrix and let A^* denote the conjugate transpose of A. Which of the following statements are necessarily true?
    1. If A is invertible, then $tr(A^*A) \neq 0$, i.e., the trace of A^*A is non zero.
    2. If $tr(A^*A) \neq 0$, then A is invertible.
    3. If $|tr(A^*A)| < n^2$, then $|a_{ij}| < 1$ for some i,j.
    4. If $tr(A^*A) = 0$, then A is the zero matrix.

48. Let n be a positive integer and V be an $(n + 1)$-dimensional vector space over $\mathbb{R}$. If $\{e_1, e_2, \ldots, e_{n+1}\}$ is a basis of V and $T: V \to V$ is the linear transformation satisfying $T(e_i) = e_{i+1}$ for $i = 1, 2, \ldots, n$ and $T(e_{n+1}) = 0$. Then
    1. trace of T is non-zero.
    2. rank of T is n.
    3. nullity of T is 1
    4. $T^n = T \circ T \circ \ldots \circ T$ (n times) is the zero map.

49. Let A and B be $n \times n$ real matrices such that $AB = BA = O$ and $A + B$ is invertible. Which of the following are always true?
    1. rank (A) = rank (B)
    2. rank (A) + rank (B) = n.
    3. nullity (A) + nullity (B) = n.
    4. $A - B$ is invertible.

50. Let n be an integer ≥ 2 and let $M_n(\mathbb{R})$ denote the vector space of $n \times n$ real matrices. Let $B \in M_n(\mathbb{R})$ be an orthogonal matrix and let B^t denote the transpose of B. Consider $W_B = \{B^t AB : A \in M_n(\mathbb{R})\}$. Which of the following are necessarily true?
    1. W_B is the subspace of $M_n(\mathbb{R})$ and dim $W_B \leq$ rank (B).
    2. W_B is the subspace of $M_n(\mathbb{R})$ and dim W_B = rank (B) rank (B^t).
    3. $W_B = M_n(\mathbb{R})$.
    4. W_B is not a subspace of $M_n(\mathbb{R})$.

51. Let A be a 5×5 skew-symmetric matrix with entries in $\mathbb{R}$ and B be the 5×5 symmetric matrix whose $(i,j)^{th}$ entry is the binomial coefficient $\binom{i}{j}$ for $1 \leq i \leq j \leq 5$. Consider the 10×10 matrix, given in block form by $C = \begin{pmatrix} A & A+B \\ 0 & B \end{pmatrix}$. Then
    1. det $C = 1$ or -1
    2. det $C = 0$
    3. trace of C is 0.
    4. trace of C is 5

52. *Suppose A is a* 3×3 *symmetric matrix such that* $[x, y, 1]A\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = xy - 1$. *Let p be the number of positive eigenvalues of A and let* $q = rank(A) - p$. *Then*

1. $p = 1$
2. $p = 2$
3. $q = 2$
4. $q = 1$

53. Let $A=\begin{bmatrix}1 & 3 & 5 & a & 13\\ 0 & 1 & 7 & 9 & b\\ 0 & 0 & 1 & 11 & 15\end{bmatrix}$, where $a,b\in\mathbb{R}$. Choose the correct statement.

1. There exist values of a and b for which the columns of A are linearly independent.
2. There exist values of a and b for which Ax=0 has x=0 as the only solution.
3. For all values of a and b, the rows of A span a 3-dimensional subspace of $\mathbb{R}^5$.
4. There exist values of a and b for which rank (A)=2.

54. Consider $\mathbb{R}^3$ with the standard inner product. Let W be the subspace of $\mathbb{R}^3$ spanned by (1,0,-1). Which of the following is a basis for the orthogonal complement of W?

1. {(1,0,1),(0,1,0)}
2. {(1,2,1),(0,1,1)}
3. {(2,1,2),(4,2,4)}
4. {(2,-1,2),(1,3,1),(-1,-1,-1)}

55. A linear transformation T rotates each vector in $\mathbb{R}^2$ clockwise through 90^0. The matrix T relative to the standard ordered basis $\left(\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}0\\1\end{bmatrix}\right)$ is

1. $\begin{bmatrix}0 & -1\\ -1 & 0\end{bmatrix}$
2. $\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$
3. $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$
4. $\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$

56. Let $T:\mathbb{R}^n\to\mathbb{R}^n$ be a linear transformation. Which of the following statements implies that T is bijective?

1. Nullity (T) =n
2. Rank(T) = Nullity (T) =n
3. Rank (T) + Nullity (T) =n
4. Rank(T) – Nullity (T) =n

PART - C

57. Let $A\in M_{10}(\mathbb{C})$, the vector space of 10×10 matrices with entries in $\mathbb{C}$. Let W_A be the subspace of $M_{10}(\mathbb{C})$ spanned by $\{A^n \mid n\geq 0\}$. Choose the correct statements.

1. For any A, $\dim(W_A)\leq 10$
2. For any A, $\dim(W_A) < 10$
3. For some A, $10 < \dim(W_A) < 100$
4. For some A, $\dim(W_A)=100$

58. *Let A be a complex 3×3 matrix with $A^4 = -I$. Which of the following statements are correct?*

1. *A has three distinct eigenvalues*
2. *A is diagonalizable over $\mathbb{C}$*
3. *A is triangularizable over $\mathbb{C}$*
4. *A is non-singular*

59. *Consider the quadratic forms q and p given by* $q(x, y, z, w) = x^2 + y^2 + z^2 + bw^2$ *and* $p(x, y, z, w) = x^2 + y^2 + czw$. *Which of the following statements are true?*

1. *p and q are equivalent over $\mathbb{C}$ if b and c are non-zero complex numbers.*
2. *p and q are equivalent over $\mathbb{R}$ if b and c are non-zero real numbers.*
3. *p and q are equivalent over $\mathbb{R}$ if b and c are non-zero real numbers with b negative.*
4. *p and q are NOT equivalent over $\mathbb{R}$ if $c=0$*

60. *A linear operator T on a complex vector space V has characteristic polynomial $x^3(x-5)^2$ and minimal polynomial $x^2(x-5)$. Choose all correct options.*

1. *The Jordan form of T is uniquely determined by the given information*
2. *There are exactly 2 Jordan blocks in the Jordan decomposition of T*
3. *The operator induced by T on the quotient space V/Ker(T-5I) is nilpotent, where I is the identity operator*
4. *The operator induced by T on the quotient space V/Ker(T) is a scalar multiple of the identity operator*

61. *Let S denote the set of all primes p such that the following matrix is invertible when considered as a matrix with entries in $\mathbb{Z}/p\mathbb{Z}$.*

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & -1 \\ -2 & 0 & 2 \end{pmatrix}$$

Which of the following statements are true?

1. *S contains all the prime numbers*
2. *S contains all the prime numbers greater than 10*
3. *S contains all the prime numbers other than 2 and 5*
4. *S contains all the odd prime numbers.*

62. *For a fixed positive integer* $n \geq 3$, *let A be the* $n \times n$ *matrix defined as* $A = I - \frac{1}{n}J$, ***where I is the*** *identity matrix and J is the* $n \times n$ *matrix with all entries equal to 1.* ***Which of the following statements is*** *NOT true?*

1. $A^k = A$ *for every positive integer k.*
2. *Trace* $(A) = n-1$
3. $Rank(A)+Rank(I - A)=n$.
4. *A is invertible.*

63. *Let A be a* 5×4 *matrix with real entries such that* $A\underline{x}=\underline{0}$ *if and only if* $\underline{x}=\underline{0}$, *where* $\underline{x}$ *is a* 4×1 *vector and* $\underline{0}$ *is a null vector. Then, the rank of A is*

1. 4
2. 5
3. 2
4. 1

64. *Consider the following row vectors*

$\alpha_1=(1,1,0,1,0,0)$, $\alpha_2=(1,1,0,0,1,0)$

$\alpha_3=(1,1,0,0,0,1)$, $\alpha_4=(1,0,1,1,0,0)$

$\alpha_5=(1,0,1,0,1,0)$, $\alpha_6=(1,0,1,0,0,1)$

The dimension of the vector space spanned by these row vectors is

1. 6
2. 5
3. 4
4. 3

65. *Let* $A_{n\times n}=((a_{ij}))$, $n\geq 3$, *where* $a_{ij}=\left(b_i^2-b_j^2\right)$, $i,j=1,2,\ldots,n$ *for some distinct real numbers* $b_1,b_2,\ldots,b_n$. *Then det(A) is*

1. $\prod_{i<j}(b_i-b_j)$
2. $\prod_{i<j}(b_i+b_j)$
3. 0
4. 1

66. *Let A be an* $n\times n$ *matrix with real entries. Which of the following is correct?*

1. *If* $A^2=O$, *then A is diagonalizable over complex numbers.*
2. *If* $A^2=I$, *then A is diagonalizable over real numbers*
3. *If* $A^2=A$, *then A is diagonalizable only over complex numbers.*
4. *The only matrix of size n satisfying the characteristic polynomial of A is A.*

67. *Let A be a* 4×4 *invertible real matrix. Which of the following is NOT necessarily true?*

1. *The rows of A form a basis of* $\mathbb{R}^4$.
2. *Null space of A contains only the 0 vector.*
3. *A has 4 distinct eigenvalues.*
4. *Image of the linear transformation* $x\mapsto Ax$ *on* $\mathbb{R}^4$ *is* $\mathbb{R}^4$.

PART - C

68. *Let* $\{v_1,\ldots,v_n\}$ *be a linearly independent subset of a vector space V, where* $n\geq 4$. *Set* $w_{ij}=v_i-v_j$. *Let W be the span of* $\{w_{ij}\mid 1\leq i,j\leq n\}$. *Then*

1. $\{w_{ij}\mid 1\leq i<j\leq n\}$ *spans W.*
2. $\{w_{ij}\mid 1\leq i<j\leq n\}$ *is a linearly independent subset of W.*
3. $\{w_{ij}\mid 1\leq i\leq n-1, j=i+1\}$ *spans W.*
4. *dim W = n*

69. For any real square matrix M, let $\lambda^+(M)$ be the number of positive eigenvalues of M counting multiplicities. Let A be an $n\times n$ real symmetric matrix and Q be an $n\times n$ real invertible matrix. Then

1. Rank A=Rank $Q^T AQ$
2. Rank A=Rank $Q^{-1}AQ$
3. $\lambda^+(A) = \lambda^+(Q^T AQ)$
4. $\lambda^+(A) = \lambda^+(Q^{-1} AQ)$

70. Let T_1, T_2 be two linear transformations from $\mathbb{R}^n$ to $\mathbb{R}^n$. Let $\{x_1, x_2, \ldots, x_n\}$ be a basis of $\mathbb{R}^n$. Suppose that $T_1(x_i) \neq 0$ for every $i=1,2,\ldots,n$ and that $x_i \perp \text{Ker } T_2$ for every $i = 1, 2, \ldots, n$. Which of the following is/are necessarily true?

1. T_1 is invertible
2. T_2 is invertible
3. Both T_1, T_2 are invertible
4. Neither T_1 nor T_2 is invertible

71. Let $S: \mathbb{R}^n \to \mathbb{R}^n$ be given by $v \mapsto \alpha v$ for a fixed $\alpha \in \mathbb{R}$, $\alpha \neq 0$. Let $T: \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation such that $B = \{v_1, \ldots, v_n\}$ is a set of linearly independent eigenvectors of T. Then

1. The matrix of T with respect to B is diagonal.
2. The matrix of $T - S$ with respect to B is diagonal.
3. The matrix of T with respect to B is not necessarily diagonal, but upper triangular.
4. The matrix of T with respect to B is diagonal but the matrix of $(T-S)$ with respect to B is not diagonal.

72. For an $n\times n$ real matrix A, $\lambda \in \mathbb{R}$ and a nonzero vector $v \in \mathbb{R}^n$, suppose that $(A - \lambda I)^k v = 0$ for some positive integer k. Let I be the $n\times n$ identity matrix. Then which of the following is/are always true?

1. $(A - \lambda I)^{k+r} v = 0$ for all positive integers r.
2. $(A - \lambda I)^{k-1} v = 0$
3. $(A - \lambda I)$ is not injective
4. λ is an eigenvalue of A

73. Let y be a non-zero vector in an inner product space V. Then which of the following are subspaces of V?

1. $\{x \in V \mid \langle x, y \rangle = 0\}$.
2. $\{x \in V \mid \langle x, y \rangle = 1\}$.
3. $\{x \in V \mid \langle x, z \rangle = 0$ for all z such that $\langle z, y \rangle = 0\}$.
4. $\{x \in V \mid \langle x, z \rangle = 1$ for all z such that $\langle z, y \rangle = 1\}$.

74. *Let A be a* 5×5 *matrix with real entries such that the sum of the entries in each row of A is 1. Then the sum of all the entries in* A^3 *is*

1. 3 2. 15 3. 5 4. 125

75. *Let J denote a* 101×101 *matrix with all the entries equal to 1 and let I denote the identity matrix of order 101. Then the determinant of J-I is*

1. 101 2. 1 3. 0 4. 100

76. Let $M_{m\times n}(\mathbb{R})$ be the set of all $m\times n$ matrices with real entries. Which of the following statements is correct?

1. There exists $A\in M_{2\times 5}(\mathbb{R})$ such that the dimension of the null space of A is 2.
2. There exists $A\in M_{2\times 5}(\mathbb{R})$ such that the dimension of the null space of A is 0.
3. There exist $A\in M_{2\times 5}(\mathbb{R})$ and $B\in M_{5\times 2}(\mathbb{R})$ such that AB is the 2×2 identity matrix.
4. There exists $A\in M_{2\times 5}(\mathbb{R})$ whose null space is $\{(x_1, x_2, x_3, x_4, x_5)\in \mathbb{R}^5 : x_1=x_2, x_3=x_4=x_5\}$

77. For the matrix A as given below, which of them satisfy $A^6=I$?

1. $A=\begin{pmatrix} \cos\frac{\pi}{4} & \sin\frac{\pi}{4} & 0 \\ -\sin\frac{\pi}{4} & \cos\frac{\pi}{4} & 0 \\ 0 & 0 & 1 \end{pmatrix}$
2. $A=\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\frac{\pi}{3} & \sin\frac{\pi}{3} \\ 0 & -\sin\frac{\pi}{3} & \cos\frac{\pi}{3} \end{pmatrix}$
3. $A=\begin{pmatrix} \cos\frac{\pi}{6} & 0 & \sin\frac{\pi}{6} \\ 0 & 1 & 0 \\ -\sin\frac{\pi}{6} & 0 & \cos\frac{\pi}{6} \end{pmatrix}$
4. $A=\begin{pmatrix} \cos\frac{\pi}{2} & \sin\frac{\pi}{2} & 0 \\ -\sin\frac{\pi}{2} & \cos\frac{\pi}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

PART – C

78. Let V denote a vector space over a field F and with a basis $B=\{e_1, e_2, \ldots, e_n\}$. Let $x_1, x_2, \ldots, x_n \in F$. Let $C=\{x_1e_1, x_1e_1+x_2e_2, \ldots, x_1e_1+x_2e_2+\ldots+x_ne_n\}$. Then

1. C is linearly independent set implies that $x_i\neq 0$ for every $i=1,2,\ldots,n$.
2. $x_i\neq 0$ for every $i=1,2,\ldots,n$ implies that C is linearly independent set.
3. The linear span of C is V implies that $x_i\neq 0$ for every $i=1,2,\ldots,n$.
4. $x_i\neq 0$ for every $i=1,2,\ldots,n$ implies that the linear span of C is V.

79. Let V denote the vector space of all polynomials over $\mathbb{R}$ of degree less than or equal to n. Which of the following defines a norm on V?

1. $\|p\|^2=|p(1)|^2+\ldots+|p(n+1)|^2, p\in V$
2. $\|p\|=\sup_{t\in[0,1]}|p(t)|, p\in V$
3. $\|p\|=\int_0^1|p(t)|\,dt, p\in V$
4. $\|p\|=\sup_{t\in[0,1]}|p'(t)|, p\in V$

80. Let u, v, w be vectors in an inner- product space V, satisfying $\|u\|=\|v\|=\|w\|=2$ and $\langle u,v\rangle=0$, $\langle u,w\rangle = 1$, $\langle v,w\rangle = -1$. Then which of the following are true?

1. $\|w+v-u\| = 2\sqrt{2}$.
2. $\left\{\frac{1}{2}u, \frac{1}{2}v\right\}$ forms an orthonormal basis of a two dimensional subspace of V.
3. w and $4u-w$ are orthogonal to each other.
4. u, v, w are necessarily linearly independent.

81. Let A be a 4×4 matrix over $\mathbb{C}$ such that rank$(A)=2$ and $A^3 = A^2 \neq 0$. Suppose that A is not diagonalizable. Then

1. One of the Jordan blocks of the Jordan canonical form of A is $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.
2. $A^2 = A \neq 0$.
3. There exists a vector v such that $Av \neq 0$ but $A^2v = 0$.
4. The characteristic polynomial of A is $x^4 - x^3$.

82. Let $\varphi: \mathbb{R}^2 \to \mathbb{C}$ be the map defined by $\varphi(x, y) = z$, where $z=x+iy$. Let $f: \mathbb{C} \to \mathbb{C}$ be the function $f(z)=z^2$ and $F = \varphi^{-1} f \varphi$. Which of the following are correct?

1. The linear transformation $T(x, y) = 2\begin{pmatrix} x & -y \\ y & x \end{pmatrix}$ represents the derivative of F at (x,y).
2. The linear transformation $T(x, y) = 2\begin{pmatrix} x & y \\ y & x \end{pmatrix}$ represents the derivative of F at (x,y).
3. The linear transformation $T(z)=2z$ represents the derivative of f at $z \in \mathbb{C}$.
4. The linear transformation $T(z)=2z$ represents the derivative of f only at 0.

83. Let V be the vector space of polynomials over $\mathbb{R}$ of degree less than or equal to n. For $p(x) = a_0 + a_1 x + \ldots + a_n x^n$ in V, define a linear transformation $T: V \to V$ by $(Tp)(x) = a_0 - a_1 x + a_2 x^2 - \ldots + (-1)^n a_n x^n$. Then which of the following are correct?

1. T is one-to-one
2. T is onto
3. T is invertible
4. det $T=0$

84. Consider a homogeneous system of linear equations $Ax=0$, where A is an $m\times n$ real matrix and $n > m$. Then which of the following statements are always true?

1. $Ax=0$ has a solution.
2. $Ax=0$ has no non-zero solution.
3. $Ax=0$ has a non-zero solution.
4. Dimension of the space of all solutions is at least $n-m$.

85. Let A, B be $n\times n$ matrices such that $BA + B^2 = I - BA^2$, where I is the $n\times n$ identity matrix. Which of the following is always true?

1. A is nonsingular   2. B is nonsingular   3. A+B is nonsingular   4. AB is nonsingular

86. Which of the following matrices has the same row space as the matrix $\begin{pmatrix} 4 & 8 & 4 \\ 3 & 6 & 1 \\ 2 & 4 & 0 \end{pmatrix}$?

1. $\begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ 2. $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ 3. $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ 4. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

87. The determinant of the $n\times n$ permutation matrix $\begin{bmatrix} & & & & 1 \\ & & & 1 & \\ & & \cdot & & \\ & \cdot & & & \\ & 1 & & & \\ 1 & & & & \end{bmatrix}$

1. $(-1)^n$ 2. $(-1)^{\left[\frac{n}{2}\right]}$ 3. -1   4. 1

Here, [x] denotes the greatest integer not exceeding x.

88. The determinant $\begin{vmatrix} 1 & 1+x & 1+x+x^2 \\ 1 & 1+y & 1+y+y^2 \\ 1 & 1+z & 1+z+z^2 \end{vmatrix}$ is equal to

1. $(z-y)(z-x)(y-x)$ 2. $(x-y)(x-z)(y-z)$

3. $(x-y)^2(y-z)^2(z-x)^2$ 4. $(x^2-y^2)(y^2-z^2)(z^2-x^2)$

89. Which of the following matrices is not diagonalizable over $\mathbb{R}$?

1. $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 2. $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$ 3. $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ 4. $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

86. Let P be a 2×2 complex matrix such that P^*P is the identity matrix, where P^* is the conjugate transpose of P. Then the eigenvalues of P are

1. real
2. complex conjugates of each other
3. reciprocals of each other
4. of modulus 1

PART - C

91. Let A be a real $n\times n$ orthogonal matrix, that is, $A^tA=AA^t=I_n$, the $n\times n$ identity matrix. Which of the following statements are necessarily true?

1. $\langle Ax, Ay\rangle = \langle x, y\rangle\ \forall x, y \in \mathbb{R}^n$
2. All eigenvalues of A are either +1 or -1.
3. The rows of A form an orthonormal basis of $\mathbb{R}^n$.
4. A is diagonalizable over $\mathbb{R}$.

92. Which of the following matrices have Jordan canonical form equal to $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$?

1. $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
2. $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$
3. $\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
4. $\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

93. Let A be a 3×4 and b be a 3×1 matrix with integer entries. Suppose that the system $Ax=b$ has a complex solution. Then

1. $Ax=b$ has an integer solution
2. $Ax=b$ has a rational solution
3. The set of real solutions to $Ax=0$ has a basis consisting of rational solutions.
4. If $b\neq0$, then A has positive rank.

94. Let f be a non-zero symmetric bilinear form on $\mathbb{R}^3$. Suppose that there exist linear transformations $T_i : \mathbb{R}^3 \to \mathbb{R}$, $i = 1,2$ such that for all $\alpha, \beta \in \mathbb{R}^3$, $f(\alpha, \beta) = T_1(\alpha) T_2(\beta)$. Then

1. rank $f=1$
2. $\dim\{\beta \in \mathbb{R}^3 : f(\alpha, \beta) = 0 \text{ for all } \alpha \in \mathbb{R}^3\} = 2$
3. f is positive semi-definite or negative semi-definite.
4. $\{\alpha : f(\alpha,\alpha)=0\}$ is a linear subspace of dimension 2

95. The matrix $A = \begin{pmatrix} 5 & 9 & 8 \\ 1 & 8 & 2 \\ 9 & 1 & 0 \end{pmatrix}$ satisfies

1. A is invertible and the inverse has all integer entries.

2. det(A) is odd.
3. det(A) is divisible by 13.
4. det(A) has at least two prime divisors.

96. Let A be 5×5 matrix and let B be obtained by changing one element of A. Let r and s be the ranks of A and B respectively. Which of the following statements is/are correct?

1. $s \le r+1$ 2. $r-1 \le s$ 3. $s = r-1$ 4. $s \ne r$

97. Let $M_n(K)$ denote the space of all $n\times n$ matrices with entries in a field K. Fix a non-singular matrix $A=(A_{ij})\in M_n(K)$, and consider the linear map $T: M_n(K)\to M_n(K)$ given by $T(X)=AX$. Then

1. trace $(T)=n\sum_{i=1}^{n} A_{ii}$
2. trace$(T)=\sum_{i=1}^{n}\sum_{j=1}^{n} A_{ij}$
3. rank of T is n^2
4. T is non–singular

98. For arbitrary subspaces U, V and W of a finite dimensional vector space, which of the following hold?

1. $U\cap(V+W)\subset U\cap V+U\cap W$
2. $U\cap(V+W)\supset U\cap V+U\cap W$
3. $(U\cap V)+W\subset(U+W)\cap(V+W)$
4. $(U\cap V)+W\supset(U+W)\cap(V+W)$

99. Let A be a 4×7 real matrix and B be a 7×4 real matrix such that $AB=I_4$, where I_4 is the 4×4 identity matrix. Which of the following is/are always true?

1. rank $(A)=4$
2. rank $(B)=7$
3. nullity $(B)=0$
4. $BA=I_7$, where I_7 is the 7×7 identity matrix

100. Let $\mathbb{R}[x]$ denote the vector space of all real polynomials. Let D: $\mathbb{R}[x]\to\mathbb{R}[x]$ denote the map $Df=\frac{df}{dx}, \forall f$. Then,

1. D is one-one
2. D is onto
3. There exists $E:\mathbb{R}[x]\to\mathbb{R}[x]$ so that $D(E(f))=f, \forall f$.
4. There exists $E:\mathbb{R}[x]\to\mathbb{R}[x]$ so that $E(D(f))=f, \forall f$.

101. Which of the following are eigenvalues of the matrix $\begin{pmatrix}0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\end{pmatrix}$?

1. $+1$ 2. -1 3. $+i$ 4. $-i$

102. Let $A = \begin{pmatrix} x & y \\ -y & x \end{pmatrix}$, *where* $x, y \in \mathbb{R}$ *such that* $x^2 + y^2 = 1$. *Then we must have*

1. *For any* $n \geq 1$, $A^n = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, *where* $x = \cos(\theta / n), y = \sin(\theta / n)$
2. $tr(A) \neq 0$
3. $A^t = A^{-1}$
4. *A is similar to a diagonal matrix over* $\mathbb{C}$

103. Let V be the space of twice differentiable functions on $\mathbb{R}$ satisfying $f''-2f'+f=0$.
Define $T: V \to \mathbb{R}^2$ by $T(f)=(f'(0), f(0))$. Then T is

1. one –to-one and onto
2. one-to-one but not onto
3. onto but not one-to-one
4. neither one-to-one nor onto

104. The row space of a 20×50 matrix A has dimension 13. What is the dimension of the space of solutions of $Ax = 0$?

1. 7
2. 13
3. 33
4. 37

105. Let A, B be $n\times n$ matrices. Which of the following equals trace(A^2B^2)?

1. $(\text{trace}(AB))^2$
2. $\text{trace}(AB^2A)$
3. $\text{trace}((AB)^2)$
4. $\text{trace}(BABA)$

106. Given a 4×4 real matrix A, let $T:\mathbb{R}^4\to\mathbb{R}^4$ be the linear transformation defined by $Tv = Av$, where we think of $\mathbb{R}^4$ as the set of real 4×1 matrices. For which choices of A given below, do Image(T) and Image(T^2) have respective dimensions 2 and 1?
(* denotes a non-zero entry)

1. $A=\begin{bmatrix} 0 & 0 & * & * \\ 0 & 0 & * & * \\ 0 & 0 & 0 & * \\ 0 & 0 & 0 & 0 \end{bmatrix}$
2. $A=\begin{bmatrix} 0 & 0 & * & 0 \\ 0 & 0 & * & 0 \\ 0 & 0 & 0 & * \\ 0 & 0 & 0 & * \end{bmatrix}$
3. $A=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & * \\ 0 & 0 & * & 0 \end{bmatrix}$
4. $A=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & * & * \\ 0 & 0 & * & * \end{bmatrix}$

107. Let T be a 4×4 real matrix such that $T^4 = 0$. Let $k_i = \dim \operatorname{Ker} T^i$ for $1 \le i \le 4$. Which of the following is NOT a possibility for the sequence $k_1 \le k_2 \le k_3 \le k_4$?

1. $3 \le 4 \le 4 \le 4$.
2. $1 \le 3 \le 4 \le 4$.
3. $2 \le 4 \le 4 \le 4$.
4. $2 \le 3 \le 4 \le 4$.

108. Which of the following is a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$?

(a) $f\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ x+y \end{pmatrix}$ (b) $g\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} xy \\ x+y \end{pmatrix}$ (c) $h\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z-x \\ x+y \end{pmatrix}$

1. Only f.
2. Only g.
3. Only h.
4. all the transformations f, g and h.

109. Let A be an $m\times n$ matrix of rank n with real entries. Choose the correct statement.

1. $Ax = b$ has a solution for any b.
2. $Ax = 0$ does not have a solution.
3. If $Ax = b$ has a solution, then it is unique.
4. $y'A = 0$ for some nonzero y, where y' denotes the transpose of the vector y.

PART - C

110. Let $F: \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ be the function $F(x,y) = \langle Ax, y\rangle$, where $\langle , \rangle$ is the standard inner product of $\mathbb{R}^n$ and A is a $n\times n$ real matrix. Here D denotes the total derivative. Which of the following statements are correct?

1. $(DF(x,y))(u,v) = \langle Au, y\rangle + \langle Ax, v\rangle$
2. $(DF(x,y))(0,0) = 0$.
3. $DF(x,y)$ may not exist for some $(x,y) \in \mathbb{R}^n \times \mathbb{R}^n$
4. $DF(x,y)$ does not exist at $(x,y) = (0,0)$.

111. Let $f: \mathbb{R}^n \to \mathbb{R}^n$ be a continuous function such that $\int_{R^n} |f(x)|\,dx < \infty$. Let A be a real $n\times n$ invertible matrix and for $x, y \in \mathbb{R}^n$, let $\langle x, y\rangle$ denote the standard inner product in R^n. Then $\int_{R^n} f(Ax) e^{i\langle y,x\rangle} dx =$

1. $\int_{R^n} f(x) e^{i\langle (A^{-1})^T y, x\rangle} \frac{dx}{|\det A|}$
2. $\int_{R^n} f(x) e^{i\langle A^T y, x\rangle} \frac{dx}{|\det A|}$
3. $\int_{R^n} f(x) e^{i\langle (A^T)^{-1} y, x\rangle} dx$
4. $\int_{R^n} f(x) e^{i\langle A^{-1} y, x\rangle} \frac{dx}{|\det A|}$

112. *Let S be the set of 3×3 real matrices A with* $A^T A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. *Then the set S contains*

1. a nilpotent matrix.
2. a matrix of rank one.
3. a matrix of rank two.
4. a non-zero skew-symmetric matrix.

113. *An $n \times n$ complex matrix A satisfies $A^k = I_n$, the $n \times n$ identity matrix, where k is a positive integer > 1. Suppose 1 is not an eigenvalue of A. Then which of the following statements are necessarily true?*

1. A is diagonalizable.
2. $A + A^2 + \ldots + A^{k-1} = O$, the $n \times n$ zero matrix
3. $tr(A) + tr(A^2) + \ldots + tr(A^{k-1}) = -n$
4. $A^{-1} + A^{-2} + \ldots + A^{-(k-1)} = -I_n$

114. *Let $S: \mathbb{R}^n \to \mathbb{R}^n$ be given by $S(v) = \alpha v$ for a fixed $\alpha \in \mathbb{R}$, $\alpha \neq 0$. Let $T: \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation such that $B = \{v_1, \ldots, v_n\}$ is a set of linearly independent eigenvectors of T. Then*

1. The matrix of T with respect to B is diagonal.
2. The matrix of (T - S) with respect to B is diagonal.
3. The matrix of T with respect to B is not necessarily diagonal, but is upper triangular.
4. The matrix of T with respect to B is diagonal but the matrix of (T - S) with respect to B is not diagonal.

115. *Let $p_n(x) = x^n$ for $x \in \mathbb{R}$ and let $\wp = \text{span}\{p_0, p_1, p_2, \ldots\}$. Then*

1. $\wp$ is the vector space of all real valued continuous functions on $\mathbb{R}$.
2. $\wp$ is a subspace of all real valued continuous functions on $\mathbb{R}$.
3. $\{p_0, p_1, p_2, \ldots\}$ is a linearly independent set in the vector space of all continuous functions on $\mathbb{R}$.
4. Trigonometric functions belong to $\wp$.

116. *Let* $A = \begin{bmatrix} a & b & c \\ 0 & a & d \\ 0 & 0 & a \end{bmatrix}$ *be a 3×3 matrix, where a,b,c,d are integers. Then, we must have:*

1. If $a \neq 0$, there is a polynomial $p \in \mathbb{Q}[x]$ such that $p(A)$ is the inverse of A.
2. For each polynomial $q \in \mathbb{Z}[x]$, the matrix $q(A) = \begin{bmatrix} q(a) & q(b) & q(c) \\ 0 & q(a) & q(d) \\ 0 & 0 & q(a) \end{bmatrix}$
3. If $A^n = O$ for some positive integer n, then $A^3 = O$.
4. A commutes with every matrix of the form $\begin{bmatrix} a' & 0 & c' \\ 0 & a' & 0 \\ 0 & 0 & a' \end{bmatrix}$.

117. *Which of the following are subspaces of the vector space* $\mathbb{R}^3$?

1. $\{(x, y, z) : x + y = 0\}$
2. $\{(x, y, z) : x - y = 0\}$
3. $\{(x, y, z) : x + y = 1\}$
4. $\{(x, y, z) : x - y = 1\}$

118. *Consider non-zero vector spaces* V_1, V_2, V_3, V_4 *and linear transformations* $\phi_1 : V_1 \to V_2$, $\phi_2 : V_2 \to V_3$, $\phi_3 : V_3 \to V_4$ *such that* $Ker(\phi_1) = \{0\}$, $Range(\phi_1) = Ker(\phi_2)$, $Range(\phi_2) = Ker(\phi_3)$, $Range(\phi_3) = V_4$. *Then*

1. $\sum_{i=1}^{4} (-1)^i \dim V_i = 0$
2. $\sum_{i=2}^{4} (-1)^i \dim V_i > 0$
3. $\sum_{i=1}^{4} (-1)^i \dim V_i < 0$
4. $\sum_{i=1}^{4} (-1)^i \dim V_i \neq 0$

119. *Let A be an invertible* 4×4 *real matrix. Which of the following are NOT true?*

1. *Rank* $A = 4$.
2. *For every vector* $b \in \mathbb{R}^4$, $Ax = b$ *has exactly one solution.*
3. *dim (nullspace A)* ≥ 1.
4. *0 is an eigenvalue of A.*

120. *Let* $\underline{u}$ *be a real* $n \times 1$ *vector satisfying* $\underline{u}'\underline{u} = 1$, *where* $\underline{u}'$ *is the transpose of* $\underline{u}$. *Define* $A = I - 2\underline{u}\,\underline{u}'$ *where I is the* n^{th} *order identity matrix. Which of the following statements are true?*

1. *A is singular*
2. $A^2 = A$
3. $Trace(A) = n-2$
4. $A^2 = I$

121. Let S denote the set of all the prime numbers p with the property that the matrix $\begin{bmatrix} 91 & 31 & 0 \\ 29 & 31 & 0 \\ 79 & 23 & 59 \end{bmatrix}$ has an inverse in the field $\mathbb{Z}/p\mathbb{Z}$. Then

1. $S = \{31\}$
2. $S = \{31, 59\}$
3. $S = \{7, 13, 59\}$
4. S is infinite

122. For a positive integer n, let P_n denote the vector space of polynomials in one variable x with real coefficients and with degree $\leq n$. Consider the map $T: P_2 \to P_4$ defined by $T(p(x)) = p(x^2)$. Then

1. T is a linear transformation and dim range $(T) = 5$.
2. T is a linear transformation and dim range $(T) = 3$.
3. T is a linear transformation and dim range $(T) = 2$.
4. T is not a linear transformation.

123. Let A be a real 3×4 matrix of rank 2. Then the rank of A^tA, where A^t denotes the transpose of A, is

1. exactly 2
2. exactly 3
3. exactly 4
4. at most 2 but not necessarily 2

124. Consider the quadratic form $Q(v) = v^tAv$, where $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $v = (x, y, z, w)$. Then

1. Q has rank 3.
2. $xy + z^2 = Q(Pv)$ for some invertible 4×4 real matrix P
3. $xy + y^2 + z^2 = Q(Pv)$ for some invertible 4×4 real matrix P
4. $x^2 + y^2 - zw = Q(Pv)$ for some invertible 4×4 real matrix P.

125. If A is a 5×5 real matrix with trace 15 and if 2 and 3 are eigenvalues of A, each with algebraic multiplicity 2, then the determinant of A is equal to

1. 0
2. 24
3. 120
4. 180

126. Let $A \neq I_n$ be an $n \times n$ matrix such that $A^2 = A$, where I_n is the identity matrix of order n. Which of the following statements is false?

1. $(I_n - A)^2 = I_n - A$.
2. Trace (A) = Rank (A).
3. Rank (A) + Rank $(I_n - A) = n$.
4. The eigenvalues of A are each equal to 1.

PART - C

127. Let A and B be $n \times n$ matrices over $\mathbb{C}$. Then,

1. AB and BA always have the same set of eigenvalues.
2. If AB and BA have the same set of eigenvalues then $AB = BA$.
3. If A^{-1} exists then AB and BA are similar.
4. The rank of AB is always the same as the rank of BA.

128. Let A be an $m \times n$ real matrix and $b \in \mathbb{R}^m$ with $b \neq 0$.

1. The set of all real solutions of $Ax = b$ is a vector space.
2. If u and v are two solutions of $Ax = b$, then $\lambda u + (1 - \lambda)v$ is also a solution of $Ax = b$, for any $\lambda \in \mathbb{R}$.
3. For any two solutions u and v of $Ax = b$, the linear combination $\lambda u + (1 - \lambda) v$ is also a solution of $Ax = b$ only when $0 \leq \lambda \leq 1$.
4. If rank of A is n, then $Ax = b$ has at most one solution.

129. *Let A be an n × n matrix over ℂ such that every nonzero vector of $\mathbb{C}^n$ is an eigenvector of A. Then*

1. *All eigenvalues of A are equal.*
2. *All eigenvalues of A are distinct.*
3. $A = \lambda I$ *for some* $\lambda \in \mathbb{C}$, *where I is the n × n identity matrix.*
4. *If* χ_A *and* m_A *denote the characteristic polynomial and the minimal polynomial respectively, then* $\chi_A = m_A$.

130. *Consider the matrices* $A = \begin{bmatrix} 2 & 2 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}$ *and* $B = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. *Then*

1. *A and B are similar over the field of rational numbers* ℚ.
2. *A is diagonalizable over the field of rational numbers* ℚ.
3. *B is the Jordan canonical form of A.*
4. *The minimal polynomial and the characteristic polynomial of A are the same*

131. *Let V be a finite dimensional vector space over* ℝ. *Let* $T : V \to V$ *be a linear transformation such that rank* (T^2) = *rank (T). Then,*

1. *Kernel* (T^2) = *Kernel (T).*
2. *Range* (T^2) = *Range (T).*
3. *Kernel (T)* ∩ *Range (T) = {0}.*
4. *Kernel* (T^2) ∩ *Range* (T^2) = *{0}.*

132. *Let V be the vector space of polynomials over* ℝ *of degree less than or equal to n. For* $p(x) = a_0 + a_1x + \dots + a_nx^n$ *in V, define a linear transformation* $T: V \to V$ *by* $(Tp)(x) = a_n + a_{n-1}x + \dots + a_0x^n$. *Then*

1. *T is one to one.* 2. *T is onto.* 3. *T is invertible.* 4. $\det T = \pm 1$.

133. *Given a $n \times n$ matrix B define e^B by $e^B = \sum_{j=0}^{\infty} \frac{B^j}{j!}$. Let p be the characteristic polynomial of B. Then the matrix $e^{p(B)}$ is*

1. $I_{n\times n}$ 2. $0_{n\times n}$ 3. $eI_{n\times n}$ 4. $\pi I_{n\times n}$

134. *Let A be a $n \times m$ matrix and b be a $n \times 1$ vector (with real entries). Suppose the equation Ax=b, $x \in R^m$ admits a unique solution. Then we can conclude that*

1. $m \geq n$ 2. $n \geq m$ 3. $n = m$ 4. $n > m$

135. Let V be the vector space of all real polynomials of degree ≤ 10. Let $T(p(x)) = p'(x)$ for $p \in V$ be a linear transformation from V to V. Consider the basis $\{1, x, x^2, \ldots, x^{10}\}$ of V. Let A be the matrix of T with respect to this basis. Then

1. Trace $A = 1$
2. $\det A = 0$
3. there is no $m \in \mathbb{N}$ such that $A^m = 0$
4. A has a non zero eigenvalue

136. Let $x = (x_1, x_2, x_3), y = (y_1, y_2, y_3) \in \mathbb{R}^3$ be linearly independent. Let $\delta_1 = x_2 y_3 - y_2 x_3$, $\delta_2 = x_1 y_3 - y_1 x_3$, $\delta_3 = x_1 y_2 - y_1 x_2$. If V is the span of x, y, then

1. $V = \{(u, v, w) : \delta_1 u - \delta_2 v + \delta_3 w = 0\}$
2. $V = \{(u, v, w) : -\delta_1 u + \delta_2 v + \delta_3 w = 0\}$
3. $V = \{(u, v, w) : \delta_1 u + \delta_2 v - \delta_3 w = 0\}$
4. $V = \{(u, v, w) : \delta_1 u + \delta_2 v + \delta_3 w = 0\}$

137. Let A be a $n \times n$ real symmetric non-singular matrix. Suppose there exists $x \in \mathbb{R}^n$ such that $x' Ax < 0$. Then we can conclude that

1. $\det(A) < 0$
2. $B = -A$ is positive definite
3. $\exists y \in \mathbb{R}^n; y' A^{-1} y < 0$
4. $\forall y \in \mathbb{R}^n: y' A^{-1} y < 0$

138. Let $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. Let $f : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(v, w) = w^T Av$.

Pick the correct statement from below

1. There exists an eigenvector v of A such that Av is perpendicular to v
2. The set $\{v \in \mathbb{R}^2 \mid f(v, v) = 0\}$ is a nonzero subspace of $\mathbb{R}^2$
3. If $v, w \in \mathbb{R}^2$ are non-zero vectors such that $f(v, v) = 0 = f(w, w)$, then v is a scalar multiple of w.
4. For every $v \in \mathbb{R}^2$, there exists a non zero $w \in \mathbb{R}^2$ such that $f(v, w) = 0$.

PART – C

139. Let V be the vector space of all complex polynomials p with deg $p \leq n$. Let $T: V \to V$ be the map $(Tp)(x) = p'(1)$, $x \in \mathbb{C}$. Which of the following are correct?

1. dim Ker $T = n$.
2. dim range $T = 1$.
3. dim Ker $T = 1$.
4. dim range $T = n+1$.

140. Let A be an $n \times n$ real matrix. Pick the correct answer(s) from the following

1. A has at least one real eigenvalue.
2. For all nonzero vectors $v, w \in \mathbb{R}^n$, $(Aw)^T(Av) > 0$.
3. Every eigenvalue of $A^T A$ is a non negative real number.
4. $I + A^T A$ is invertible.

141. Let T be a $n \times n$ matrix with the property $T^n = 0$. Which of the following is/are true?

1. T has n distinct eigenvalues
2. T has one eigenvalue of multiplicity n
3. 0 is an eigenvalue of T.
4. T is similar to a diagonal matrix.

142. Let $V = \{f:[0,1]\to\mathbb{R} \mid f$ is a polynomial of degree less than or equal to $n\}$. Let $f_j(x) = x^j$ for $0 \le j \le n$ and let A be the $(n+1) \times (n+1)$ matrix given by $a_{ij} = \int_0^1 f_i(x) f_j(x)dx$. Then which of the following is/are true?

1. $\dim V = n$.
2. $\dim V > n$.
3. A is nonnegative definite, i.e., for all $v \in \mathbb{R}^n$, $\langle Av, v\rangle \ge 0$.
4. $\det A > 0$.

143. Consider the real vector space V of polynomials of degree less than or equal to d. For $p \in V$ define $||p||_k = \max\{|p(0)|, |p^{(1)}(0)|, \ldots, |p^{(k)}(0)|\}$, where $p^{(i)}(0)$ is the i^{th} derivative of p evaluated at 0. Then $||p||_k$ defines a norm on V if and only if

1. $k \ge d - 1$ 2. $k < d$ 3. $k \ge d$ 4. $k < d - 1$

144. Let A, B be $n \times n$ real matrices such that $\det A > 0$ and $\det B < 0$. For $0 \le t \le 1$, consider $C(t) = tA + (1-t)B$. Then

1. $C(t)$ is invertible for each $t \in [0,1]$.
2. There is a $t_0 \in (0,1)$ such that $C(t_0)$ is not invertible.
3. $C(t)$ is not invertible for each $t \in [0,1]$.
4. $C(t)$ is invertible for only finitely many $t \in [0,1]$.

145. Let $\{a_1, \ldots, a_n\}$ and $\{b_1, \ldots, b_n\}$ be two bases of $\mathbb{R}^n$. Let P be $n\times n$ matrix with real entries such that $Pa_i = b_i$; $i=1,2,\ldots,n$. Suppose that every eigenvalue of P is either -1 or 1. Let $Q=I+2P$. Then which of the following statements are true?

1. $\{a_i + 2b_i \mid i=1,2,\ldots,n\}$ is also a basis of V.
2. Q is invertible.
3. Every eigenvalue of Q is either 3 or -1.
4. $\det Q > 0$ if $\det P > 0$.

146. Let A be an $n \times n$ matrix with real entries. Define $\langle x, y\rangle_A = \langle Ax, Ay\rangle, x, y \in \mathbb{R}^n$. Then $\langle x, y\rangle_A$ defines an inner-product if and only if

1. $\ker A = \{0\}$.
2. $\operatorname{rank} A = n$.
3. All eigenvalues of A are positive.
4. All eigenvalues of A are non-negative.

147. Suppose $\{v_1, \ldots, v_n\}$ are unit vectors in $\mathbb{R}^n$ such that $\| v\|^2 = \sum_{i=1}^{n} |\langle v_i, v\rangle|^2, \forall\ v \in \mathbb{R}^n$

Then decide the correct statements in the following

1. $v_1, \ldots, v_n$ are mutually orthogonal.
2. $\{v_1, \ldots, v_n\}$ is a basis for $\mathbb{R}^n$.
3. $v_1, \ldots, v_n$ are not mutually orthogonal.
4. Atmost $n - 1$ of the elements in the set $\{v_1, \ldots, v_n\}$ can be orthogonal.

148. The matrix $\begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix}$ is

1. positive definite.
2. non-negative definite but not positive definite.
3. negative definite.
4. neither negative definite nor positive definite.

149. Which of the following subsets of $\mathbb{R}^4$ is a basis of $\mathbb{R}^4$?

$B_1 = \{(1,0,0,0), (1,1,0,0), (1,1,1,0), (1,1,1,1)\}$, $B_2 = \{(1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4)\}$

$B_3 = \{(1,2,0,0), (0,0,1,1), (2,1,0,0), (-5,5,0,0)\}$

1. B_1 and B_2 but not B_3
2. B_1, B_2 and B_3
3. B_1 and B_3 but not B_2
4. Only B_1

150. Let $D_1 = \det\begin{pmatrix} a & b & c \\ x & y & z \\ p & q & r \end{pmatrix}$ and $D_2 = \det\begin{pmatrix} -x & a & -p \\ y & -b & q \\ z & -c & r \end{pmatrix}$. Then

1. $D_1 = D_2$
2. $D_1 = 2D_2$
3. $D_1 = -D_2$
4. $2D_1 = D_2$

151. Consider the matrix $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, where $\theta = \frac{2\pi}{31}$. Then A^{2015} equals

1. A
2. I
3. $\begin{pmatrix} \cos 13\theta & \sin 13\theta \\ -\sin 13\theta & \cos 13\theta \end{pmatrix}$
4. $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

152. Let J denote the matrix of order $n \times n$ with all entries 1 and let B be a $(3n) \times (3n)$ matrix given by

$B = \begin{pmatrix} 0 & 0 & J \\ 0 & J & 0 \\ J & 0 & 0 \end{pmatrix}$. Then the rank of B is

1. $2n$
2. $3n - 1$
3. 2
4. 3

153. Which of the following sets of functions from $\mathbb{R}$ to $\mathbb{R}$ is a vector space over $\mathbb{R}$?

$S_1 = \{f \mid \lim_{x \to 3} f(x) = 0\}$

$S_2 = \left\{g \,\middle|\, \lim_{x \to 3} g(x) = 1\right\}$

$S_3 = \left\{ h \middle| \lim_{x\to 3} h(x) \text{ exists} \right\}$

1. Only S_1
2. Only S_2
3. S_1 and S_3 but not S_2
4. All the three are vector spaces

154. Let A be an $n \times m$ matrix with each entry equal to +1, -1 or 0 such that every column has exactly one +1 and exactly one -1. We can conclude that

1. Rank $A \le n - 1$
2. Rank $A = m$
3. $n \le m$
4. $n - 1 \le m$

155. What is the number of non-singular 3×3 matrices over F_2, the finite field with two elements?

1. 168
2. 384
3. 2^3
4. 3^2

PART - C

156. Let $A=[a_{ij}]$ be an $n \times n$ matrix such that a_{ij} is an integer for all i, j. Let $AB = I$ with $B= [b_{ij}]$ (where I is the identity matrix). For a square matrix C, det C denotes its determinant. Which of the following statements is true?

1. If det $A= 1$ then det $B=1$.
2. A sufficient condition for each b_{ij} to be an integer is that det A is an integer.
3. B is always an integer matrix.
4. A necessary condition for each b_{ij} to be an integer is det $A \in \{-1, +1\}$.

157. Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ and let α_n and β_n denote the two eigenvalues of A^n such that $|\alpha_n| \ge |\beta_n|$. Then

1. $\alpha_n \to \infty$ as $n \to \infty$
2. $\beta_n \to 0$ as $n \to \infty$
3. β_n is positive if n is even.
4. β_n is negative if n is odd.

158. Let M_n denote the vector space of all $n \times n$ real matrices. Among the following subsets of M_n, decide which are linear subspaces.

1. $V_1 = \{A \in M_n : A \text{ is nonsingular}\}$
2. $V_2 = \{A \in M_n : \det(A) = 0\}$
3. $V_3 = \{A \in M_n : \text{trace}(A) = 0\}$
4. $V_4 = \{BA : A \in M_n\}$, where B is some fixed matrix in M_n

159. If P and Q are invertible matrices such that $PQ = -QP$, then we can conclude that

1. $Tr(P) = Tr(Q) = 0$
2. $Tr(P) = Tr(Q) = 1$
3. $Tr(P) = -Tr(Q)$
4. $Tr(P) \ne Tr(Q)$

160. Let n be an odd number ≥ 7. Let $A=[a_{ij}]$ be an $n \times n$ matrix with $a_{i,i+1}=1$ for all $i=1,2,\ldots,n-1$ and $a_{n,1}=1$. Let $a_{ij}=0$ for all the other pairs (i,j). Then we can conclude that

1. A has 1 as an eigenvalue.
2. A has -1 as an eigenvalue.
3. A has at least one eigenvalue with multiplicity ≥ 2.
4. A has no real eigenvalues.

161. Let W_1, W_2, W_3 be three distinct subspaces of $\mathbb{R}^{10}$ such that each W_i has dimension 9. Let $W=W_1 \cap W_2 \cap W_3$. Then we can conclude that

1. W may not be a subspace of $\mathbb{R}^{10}$
2. $\dim W \leq 8$
3. $\dim W \geq 7$
4. $\dim W \leq 3$

162. Let A be a real symmetric matrix. Then we can conclude that

1. A does not have 0 as an eigenvalue
2. All eigenvalues of A are real
3. If A^{-1} exists, then A^{-1} is real and symmetric
4. A has at least one positive eigenvalue

163. Let A be a 4×4 matrix. Suppose that the null space $N(A)$ of A is $\{(x,y,z,w) \in \mathbb{R}^4 : x+y+z=0,\ x+y+w=0\}$. Then

1. dim (column space (A)) = 1
2. dim (column space (A)) = 2
3. rank (A) = 1
4. $S = \{(1, 1, 1, 0), (1, 1, 0, 1)\}$ is a basis of $N(A)$.

164. Let A and B be real invertible matrices such that $AB = -BA$. Then

1. Trace (A) = Trace (B) = 0
2. Trace (A) = Trace (B) = 1
3. Trace (A) = 0, Trace (B) = 1
4. Trace (A) = 1, Trace (B) = 0

165. Let A be an $n \times n$ self-adjoint matrix with eigenvalues $\lambda_1, \ldots, \lambda_n$

Let $\| X \|_2 = \sqrt{|x_1|^2 + \ldots + |x_n|^2}$ for $X = (x_1, \ldots, x_n) \in \mathbb{C}^n$. If $p(A) = a_0 I + a_1 A + \ldots + a_n A^n$ then $\sup_{\|X\|_2 = 1} \| p(A) X \|_2$ is equal to

1. $\max\{a_0 + a_1\lambda_j + \ldots + a_n\lambda_j^n : 1 \le j \le n\}$
2. $\max\{|a_0 + a_1\lambda_j + \ldots + a_n\lambda_j^n| : 1 \le j \le n\}$
3. $\min\{a_0 + a_1\lambda_j + \ldots + a_n\lambda_j^n : 1 \le j \le n\}$
4. $\min\{|a_0 + a_1\lambda_j + \ldots + a_n\lambda_j^n| : 1 \le j \le n\}$

166. Let $p(x) = \alpha x^2 + \beta x + \gamma$ be a polynomial, where $\alpha, \beta, \gamma \in \mathbb{R}$. Fix $x_0 \in \mathbb{R}$.

Let $S = \{(a,b,c) \in \mathbb{R}^3 : p(x) = a(x - x_0)^2 + b(x - x_0) + c$ for all $x \in \mathbb{R}\}$. Then the number of elements in S is

1. 0

2. 1

3. strictly greater than 1 but finite

4. infinite

167. Let $A=\begin{bmatrix}1 & 0 & 2\\ 1 & -2 & 0\\ 0 & 0 & -3\end{bmatrix}$ and I be the 3×3 identity matrix. If $6A^{-1}=aA^2+bA+cI$ for $a,b,c\in\mathbb{R}$ then (a,b,c) equals

1. (1, 2, 1)
2. (1, -1, 2)
3. (4, 1, 1)
4. (1, 4, 1)

168. Let $A=\begin{bmatrix}1 & 1 & 2\\ 1 & -2 & 5\\ 2 & 5 & -3\end{bmatrix}$. Then the eigenvalues of A are

1. -4, 3, -3
2. 4, 3, 1
3. $4, -4\pm\sqrt{13}$
4. $4, -2\pm 2\sqrt{7}$

PART - C

169. Consider the vector space V of real polynomials of degree less than or equal to n. Fix distinct real numbers $a_0, a_1, \ldots, a_k$. For $p\in V$, $\max\{|p(a_j)|: 0\le j\le k\}$ defines a norm on V

1. only if $k < n$
2. only if $k \ge n$
3. if $k+1\le n$
4. if $k\ge n+1$

170. Let V be the vector space of polynomials of degree at most 3 in a variable x with coefficients in $\mathbb{R}$. Let $T = d/dx$ be the linear transformation on V to itself given by differentiation. Which of the following are correct ?

1. T is invertible
2. 0 is an eigenvalue of T
3. There is a basis with respect to which the matrix of T is nilpotent.
4. The matrix of T with respect to the basis $\{1, 1+x, 1+x+x^2, 1+x+x^2+x^3\}$ is diagonal.

171. Let m,n,r be natural numbers. Let A be $m\times n$ matrix with real entries such that $(AA^t)^r = I$, where I is the $m\times m$ identity matrix and A^t is the transpose of the matrix A. We can conclude that

1. $m=n$
2. AA^t is invertible
3. A^tA is invertible
4. if $m=n$, then A is invertible

172. Let A be an $n\times n$ real matrix with $A^2=A$. Then

1. the eigenvalues of A are either 0 or 1
2. A is a diagonal matrix with diagonal entries 0 or 1
3. $\mathrm{rank}(A) = \mathrm{trace}(A)$
4. $\mathrm{rank}(I-A) = \mathrm{trace}(I-A)$

173. For any $n \times n$ matrix B, let $N(B) = \{X \in \mathbb{R}^n : BX = 0\}$ be the null space of B. Let A be a 4×4 matrix with $\dim(N(A-2I)) = 2$, $\dim(N(A-4I)) = 1$ and rank $(A) = 3$. Then

1. 0,2 and 4 are eigenvalues of A
2. determinant $(A) = 0$
3. A is not diagonalizable
4. trace $(A) = 8$

174. Which of the following 3×3 matrices are diagonalizable over $\mathbb{R}$?

1. $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$
2. $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
3. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 1 \end{bmatrix}$
4. $\begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

175. Let H be a real Hilbert space and $M \subseteq H$ be a closed linear subspace. Let $x_0 \in H \setminus M$. Let $y_0 \in M$ be such that $\|x_0 - y_0\| = \inf\{\|x_0 - y\| : y \in M\}$. Then

1. such a y_0 is unique
2. $x_0 \perp M$
3. $y_0 \perp M$
4. $x_0 - y_0 \perp M$

176. Let $A = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ and $Q(X) = X^t AX$ for $X \in \mathbb{R}^3$. Then

1. A has exactly two positive eigenvalues
2. all the eigenvalues of A are positive
3. $Q(X) \geq 0$ for all $X \in \mathbb{R}^3$
4. $Q(X) < 0$ for some $X \in \mathbb{R}^3$

177. Consider the matrix $A(x) = \begin{pmatrix} 1+x^2 & 7 & 11 \\ 3x & 2x & 4 \\ 8x & 17 & 13 \end{pmatrix}$; $x \in \mathbb{R}$. Then

1. $A(x)$ has eigenvalue 0 for some $x \in \mathbb{R}$
2. 0 is not an eigenvalue of $A(x)$ for any $x \in \mathbb{R}$
3. $A(x)$ has eigenvalue 0 for all $x \in \mathbb{R}$
4. $A(x)$ is invertible for every $x \in \mathbb{R}$

178. *Let A be a real symmetric matrix and $B = I + iA$, where $i^2 = -1$. Then*

1. B is invertible if and only if A is invertible

2. all eigenvalues of B are necessarily real

3. $B - I$ is necessarily invertible

4. B is necessarily invertible

179. Let $A = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$. Then the smallest positive integer n such that $A^n = I$ is

1. 1
2. 2
3. 4
4. 6

180. Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 2 & 3 & \alpha \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 3 \\ \beta \end{bmatrix}$. Then the system $AX = b$ over the real numbers has

1. no solution whenever $\beta \neq 7$.
2. an infinite number of solutions whenever $\alpha \neq 2$.
3. an infinite number of solutions if $\alpha = 2$ and $\beta \neq 7$
4. a unique solution if $\alpha \neq 2$

181. Let $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \in M_2(\mathbb{R})$ and $\phi : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ be the bilinear map defined by $\phi(v, w) = v^T A w$.

Choose the correct statement from below:

1. $\phi(v, w) = \phi(w, v)$ for all $v, w \in \mathbb{R}^2$
2. there exists nonzero $v \in \mathbb{R}^2$ such that $\phi(v, w) = 0$ for all $w \in \mathbb{R}^2$
3. there exists a 2×2 symmetric matrix B such that $\phi(v, v) = v^T B v$ for all $v \in \mathbb{R}^2$
4. the map $\psi : \mathbb{R}^4 \to \mathbb{R}$ defined by $\psi\left(\begin{bmatrix} v_1 \\ v_2 \\ w_1 \\ w_2 \end{bmatrix}\right) = \phi\left(\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}\right)$ is linear

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182. Let $M = \{A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} | a, b, c, d \in \mathbb{Z}$ and the eigenvalues of A are in $\mathbb{Q}\}$. Then

1. M is empty
2. $M = \{\begin{pmatrix} a & b \\ c & d \end{pmatrix} | a, b, c, d \in \mathbb{Z}\}$
3. If $A \in M$, then the eigenvalues of A are in $\mathbb{Z}$
4. If $A, B \in M$ are such that $AB = I$ then $\det A \in (+1, -1)$

183. Let A be a 3×3 matrix with real entries. Identify the correct statements.

1. A is necessarily diagonalizable over $\mathbb{R}$.
2. If A has distinct real eigenvalues then it is diagonalizable over $\mathbb{R}$.

3. If A has distinct eigenvalues then it is diagonalizable over $\mathbb{C}$.
4. If all eigenvalues of A are non-zero then it is diagonalizable over $\mathbb{C}$.

184. Let V be the vector space over $\mathbb{C}$ of all polynomials in a variable X of degree at most 3. Let $D: V \longrightarrow V$ be the linear operator given by differentiation with respect to X. Let A be the matrix of D with respect to some basis for V. Which of the following are true ?

1. A is a nilpotent matrix
2. A is a diagonalizable matrix
3. the rank of A is 2
4. The Jordan canonical form of A is $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

185. For every 4×4 real symmetric non-singular matrix A, there exists a positive integer p such that

1. $pI + A$ is positive definite
2. A^p is positive definite
3. A^{-p} is positive definite
4. $\exp(pA) - I$ is positive definite

186. Let A be an $m \times n$ matrix of rank m with $n > m$. If for some non-zero real number 'α', we have $x^t AA^t x = \alpha x^t x$, for all $x \in \mathbb{R}^m$ then $A^t A$ has

1. exactly two distinct eigenvalues
2. 0 as an eigenvalue with multiplicity $n-m$
3. α as a non – zero eigenvalue
4. exactly two non-zero distinct eigenvalues

187. Let $\mathbb{R}^n$, $n \geq 2$, be equipped with standard inner product. Let $\{v_1, v_2, \ldots, v_n\}$ be n column vectors forming an orthonormal basis of $\mathbb{R}^n$. Let A be the $n \times n$ matrix formed by the column vectors $v_1, \ldots, v_n$. Then

1. $A = A^{-1}$
2. $A = A^T$
3. $A^{-1} = A^T$
4. $Det(A) = 1$

188. Let A be a $(m \times n)$ matrix and B be a $(n \times m)$ matrix over real numbers with $m < n$. Then

1. AB is always nonsingular
2. AB is always singular
3. BA is always nonsingular
4. BA is always singular

189. If A is a (2×2) matrix over $\mathbb{R}$ with $Det(A+I) = 1 + Det(A)$, then we can conclude that

1. $Det(A) = 0$
2. $A = 0$
3. $Tr(A) = 0$
4. A is nonsingular

190. *The system of equations:*

$1 \cdot x + 2 \cdot x^2 + 3 \cdot xy + 0 \cdot y = 6$

$2 \cdot x + 1 \cdot x^2 + 3 \cdot xy + 1 \cdot y = 5$

$1 \cdot x - 1 \cdot x^2 + 0 \cdot xy + 1 \cdot y = 7$

1. has solutions in rational numbers
2. has solutions in real numbers
3. has solutions in complex numbers
4. has no solution

191. *The trace of the matrix* $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}^{20}$ *is*

1. 7^{20}
2. $2^{20} + 3^{20}$
3. $2 \cdot 2^{20} + 3^{20}$
4. $2^{20} + 3^{20} + 1$

PART - C

192. *Let* $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 1 \end{pmatrix}$ *and define for* $x, y, z \in \mathbb{R}$, $Q(x, y, z) = (x \; y \; z) A \begin{pmatrix} x \\ y \\ z \end{pmatrix}$. *Which of the following statements are true?*

1. The matrix of second order partial derivatives of the quadratic form Q is $2A$.
2. The rank of the quadratic form Q is 2
3. The signature of the quadratic form Q is $(+ + 0)$
4. The quadratic form Q takes the value 0 for some non-zero vector (x, y, z)

193. *Let* $M_n(\mathbb{R})$ *denote the space of all* $n \times n$ *real matrices identified with the Euclidean space* $\mathbb{R}^{n^2}$. *Fix a column vector* $x \neq 0$ *in* $\mathbb{R}^n$. *Define* $f : M_n(\mathbb{R}) \to \mathbb{R}$ *by* $f(A) = \langle A^2x, x \rangle$. *Then*

1. f is linear
2. f is differentiable
3. f is continuous but not differentiable
4. f is unbounded

194. *Let* V *denote the vector space of all sequences* $a = (a_1, a_2, \ldots)$ *of real numbers such that* $\sum 2^n |a_n|$ *converges. Define* $\|.\| : V \to \mathbb{R}$ *by* $\|a\| = \sum 2^n |a_n|$. *Which of the following are true?*

1. V contains only the sequence $(0, 0, \ldots)$
2. V is finite dimensional
3. V has a countable linear basis
4. V is a complete normed space

195. *Let* V *be a vector space over* $\mathbb{C}$ *with dimension* n. *Let* $T : V \to V$ *be a linear transformation with only 1 as eigenvalue. Then which of the following must be true?*

1. $T - I = 0$
2. $(T - I)^{n-1} = 0$
3. $(T - I)^n = 0$
4. $(T - I)^{2n} = 0$

196. If A is a (5×5) matrix and the dimension of the solution space of $Ax = 0$ is at least two, then

1. $Rank(A^2) \leq 3$ 2. $Rank(A^2) \geq 3$ 3. $Rank(A^2) = 3$ 4. $Det(A^2) = 0$

197. Let $A \in M_3(\mathbb{R})$ be such that $A^8 = I_{3\times 3}$. Then

1. minimal polynomial of A can only be of degree 2
2. minimal polynomial of A can only be of degree 3
3. either $A = I_{3\times 3}$ or $A = -I_{3\times 3}$
4. there are uncountably many A satisfying the above

198. Let A be an $n \times n$ matrix (with $n > 1$) satisfying $A^2 - 7A + 12I_{n\times n} = O_{n\times n}$ where $I_{n\times n}$ and $O_{n\times n}$ denote the identity matrix and zero matrix of order n respectively. Then which of the following statements are true?

1. A is invertible
2. $t^2 - 7t + 12n = 0$ where $t = Tr(A)$
3. $d^2 - 7d + 12 = 0$ where $d = Det(A)$
4. $\lambda^2 - 7\lambda + 12 = 0$ where λ is an eigenvalue of A

199. Let A be a (6×6) matrix over $\mathbb{R}$ with characteristic polynomial $= (x-3)^2 (x-2)^4$ and minimal polynomial $= (x-3)(x-2)^2$. Then Jordan canonical form of A can be

1. $\begin{pmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$

2. $\begin{pmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$

3. $\begin{pmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$

4. $\begin{pmatrix} 3 & 1 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$

200. Let V be an inner product space and S be a subset of V. Let $\overline{S}$ denote the closure of S in V with respect to the topology induced by the metric given by the inner product. Which of the following statements are true?

1. $S = (S^{\perp})^{\perp}$
2. $\overline{S} = (S^{\perp})^{\perp}$
3. $\overline{span(S)} = (S^{\perp})^{\perp}$
4. $S^{\perp} = ((S^{\perp})^{\perp})^{\perp}$