LIMITS 11-1-24

Thursday, January 11, 2024 5:35 PM

Limits Basic to advanced

9062395123

Standard Situation

Sample space

Finite

Time

Definition

Money

Agree

Asymptote

W

840cm

420

neighbourhood

~~Vamp~~

unable option

$x = \frac{1}{y}$

$y \to \infty$

$y = \frac{1}{x}$

$-y = -\frac{1}{x}$

$y = \frac{1}{\infty}$

$y = 0$

$y = \frac{1}{|x|}$

0

$Y \to 0$

$X \to \infty$

Somnath Jaga

Sonanati jayga

14th Feb Valentine's Day

Treat

Rice Pulao

Pea Mutti Carbu

$\lim_{x \to 0}$

Infinite

limit

Diffn

Integrability

Advanced Algebra

Intermediate forms

$\frac{0}{0}$ $\frac{7}{0}$ $\infty - \infty$ $= \infty$ ✓

$= 5$ ✓

$\infty + \infty$ $= 0$ ✓

$\infty + \infty = 2\infty - \infty$

(45 - - - - 7 - 63 - - - -)

1^{∞} 0^{0} $5^{0} = 1$

$\frac{0^{x}}{\cancel{0^{x}}} = 0^{0}$ $7^{0} = 1$

$0^{0} \neq 1$

$\cancel{0^x}$ $\qquad 0^0 \neq 1$

$\frac{0}{0}$, $\frac{\infty}{\infty}$, $\infty - \infty$, 1^∞, 0^0, ∞^0, $\infty \times 0$

Lengor me Single Space
↓
Better use font of Selhyce frula methods

<u>L' Hospital'S</u> → <u>french</u>

Some Special expansions

<u>9062395123</u>

5^x (i) $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$

(ii) $a^x = 1 + \frac{x \cdot \ln a}{1!} + \frac{(\ln a)^2 \cdot x^2}{2!} + \ldots$

$(1+x)^2 = 1 + 2x + x^2$

→ Coefficient a falling

(iii) $(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \ldots$

(iv) $\ln(1+x) = x - x^2 + x^3 - x^4 \ldots$

(iv) $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$

note: if know more functions ..

(v) $\frac{x^n - a^n}{x-a} = x^{n-1} + x^{n-2} \cdot a + x^{n-3} \cdot a^2 + \cdots$

(vi) $(1+x)^{1/x} = e\left(1 - \frac{x}{2} + \frac{11x^2}{24} - \cdots\right)$

(vii) $\sin^{-1} x = x + \frac{1^2}{3!} x^3 + \frac{1^2 \cdot 3^2 x^5}{5!} + \frac{1^2 \cdot 3^2 \cdot 5^2 x^7}{7!} + \cdots$

2 variable limit problems ..

$y = \min\{a, b\}$ find the non-diff. of y ?? when $a = b$

$\cancel{\frac{dy}{dx}} =$

Left Slope	Right Slope
5	7

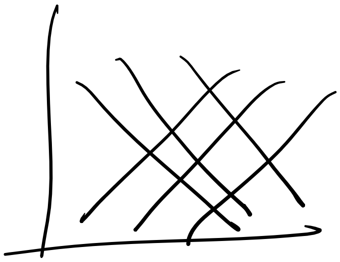
$y = \min\{5, 7\}$

$y = 5$

If $a = b$ then only y is max

∞ many max pts.

$\boxed{a = y}$ z



Shoo 1/2/3/4/5

Shoes in Shops
Shoes in homes

value of $\lim_{\substack{x \to 1 \\ y \to 0}} \left(\dfrac{y^3}{x^3 - y^2 - 1} \right)$

$(x, y) \to (1, 0)$ along the line $y = x - 1$ is

$x \to y + 1$

0 $\quad \lim_{y \to 0} \dfrac{(y)^3}{(y+1)^3 - y^2 - 1}$

$= \lim_{y \to 0} \dfrac{3y^2}{3(y+1)^2 - 2y}$ $\quad = \lim_{y \to 0} \dfrac{6y}{6(y+1) - 2}$

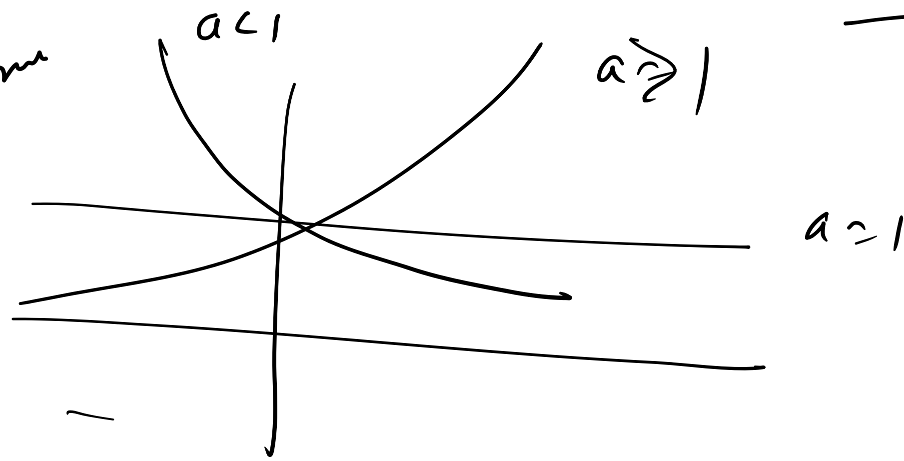
$= \dfrac{0}{6} = 0$

#

$\lim_{x \to \infty} a^x \to \infty$ .. $\quad a > 0$

3 dyn $\quad a < 1 \quad a \geq 1$

3 dym



$$\lim_{x\to\infty} a^x = \begin{cases} \infty & a>1 \\ 1 & a=1 \\ 0 & 0\le a<1 \end{cases}$$

$\lim_{x\to\infty} \dfrac{a_0 x^m + a_1 x^{m-1} + \dots + a_{m-1} x + a_m}{b_0 x^n + b_1 x^{n-1} + \dots + b_{n-1} x + b_n}$

$= 0 \quad m<n$

$= \dfrac{a_0}{b_0} \quad m=n$

$= \infty \quad m>n$ when $a_0 b_0 > 0$

$= -\infty \quad m>n$ when $a_0 b_0 < 0$

$\lim_{x\to\infty} \left(\dfrac{x^2+1}{x+1} - ax - b \right) = 2$

a, b

$b = -2, -3$

$a = 1$

$$\lim_{x\to\infty} \frac{x^2(1-a) - x(a+b) + (1-b)}{x+1} = 2$$

Limit is finite non-zero number

$\therefore$ Degree of num = degree of denominator

$1-a=0 \quad a=1$

$$\lim_{x\to\infty} \frac{-x(1+b)+(1+b)}{x+1} = 2$$

$-(1+b) = 2 \quad \boxed{b=-3}$

$S_n = (1+2+ \cdots +n)$

$$P_n = \frac{S_2}{S_2-1} \cdot \frac{S_3}{S_3-1} \cdot \frac{S_4}{S_4-1} \cdots \frac{S_n}{S_n-1}$$

where $n \in N \ (n \geq 2)$ find $\lim_{n\to\infty} P_n$.

$$S_n = \frac{n(n+1)}{2}$$

$$S_n - 1 = \frac{n(n+1)}{2} - 1 = \frac{n^2+n-2}{2}$$

$$= \frac{(n+2)(n-1)}{2}$$

$$\frac{S_n}{S_{n-1}} = \frac{n}{n-1} \times \frac{n+1}{n+2}$$

$$S_n \times \frac{1}{S_{n-1}} = \frac{n}{n-1} \times \frac{n+1}{n+2}$$

$$P_n = \left(\frac{2}{1} \cdot \frac{3}{2} \cdot \frac{4}{3} \cdots \frac{n}{n-1}\right)\left(\frac{3}{4} \cdot \frac{4}{5} \cdot \frac{5}{6} \cdots \frac{n+1}{n+2}\right)$$

$$= \frac{n}{1} \times \frac{3}{n+2}$$

$$\lim_{n \to \infty} \frac{3n}{n+2} = 3$$

Interval finding Problems

$$\frac{1}{\infty} \Rightarrow 0 + 1$$

$$\lim_{n \to \infty} \frac{1}{(\sin^{-1} x)^n + 1} = 1$$

find interval of X.

$$\sin^{-1} x \to 0$$

$$\frac{1}{0+1} \to 1$$

$x^2 - 3x - 4$

$x^2 - 3x - 4$

$$-1 < \sin^{-1} x < 1$$

$$x \in (-\sin 1, \sin 1)$$

Trigonometric limits

$$\lim_{x\to 0} \frac{\sin x}{x} = 1 = \lim_{x\to 0} \frac{\tan x}{x} = \frac{\sin^{-1} x}{x} = \frac{\tan^{-1} x}{x}$$

$$\lim_{x\to 0} \frac{\sin x^{0}}{x} = \frac{\pi}{180}$$

$$\lim_{x\to 0} \cos x = 1$$

$$\lim_{x\to a} \frac{\sin(x-a)}{(x-a)} = 1 = \lim_{x\to a} \frac{\tan(x-a)}{(x-a)}$$

$$-\frac{\sin x}{x} = -\frac{0}{1} \to \frac{0}{}$$

Q 15) 2016

$$\lim_{x\to\infty} \left[2^{-x} \cdot \sin 2^{x} \right]$$

$2^{-x} = \frac{1}{2^{x}}$ as $x \to \infty$ $2^{x} \to \infty$

Then the limit $= 0 \times$ [finite value b/w -1 to 1) $= 0$

$$\lim_{x\to\infty} \frac{\sin 2^{x}}{2^{x}} = 0 \; \#$$