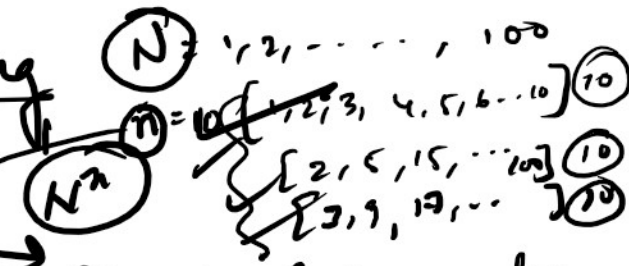
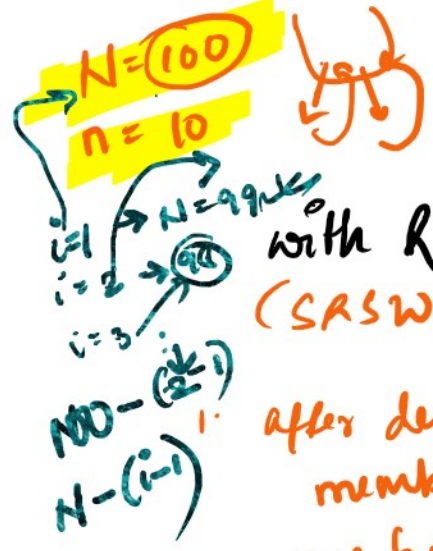


Random Sampling



with Replacement (SRSWR)

1. After drawing a member for sample one by one, the member is returned back to population.
2. So at each drawing any member of the population has same probability $\frac{1}{N}$ of being selected.
3. Any member of sample can appear more than once in the sample.
4. Total no. of samples is N^n and probab of selecting one sample is $\frac{1}{N^n}$

without Replacement

1. it is not returned back to population after a member is drawn for a sample

2. So at each drawing the probability of selecting a member is $\frac{1}{N - (i-1)}$.
3. No member in the sample is repeated.
4. Total no. sample $N C_n$ and probability of selecting the sample is $\frac{1}{N C_n}$.

$E(\bar{x})$ and $SE(\bar{x})$ Expectation and Standard Error of

(Expectation) and Standard error of Sample mean

$$\hookrightarrow \bar{x}$$

Suppose N members of variable $x \rightarrow x_1, x_2, \dots, x_N$
and we select a random sample of size ' n '
from this population and denote the i th member
of this sample as x_i ($i = 1, 2, \dots, n$)

If μ and σ^2 are population mean
and population variance
then we have,

$$\mu = \frac{1}{N} \sum_{r=1}^N X_r \quad \text{--- (1)}$$

$$\sigma^2 = \frac{1}{N} \sum_{r=1}^N (X_r - \mu)^2 \quad \text{--- (2)}$$

Again sample mean, $\bar{x} = \sum_{i=1}^n x_i / n$ (3)

$$\text{Now, } E(\bar{x}) = E\left(\frac{\sum x_i}{n}\right) = \frac{1}{n} \sum_{i=1}^n E(x_i) \quad \text{--- (4)}$$

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Further,

$$V(\bar{x}) = V\left(\frac{1}{n} \sum x_i\right) = \frac{1}{n^2} V(\sum x_i)$$

$$= \frac{1}{n^2} \left[\sum V(x_i) + \sum_{i \neq j} \text{Cov}(x_i, x_j) \right]$$

(5)

Case I SRWR

$$P(x_i = x_h) = \frac{1}{N}$$

$$E(x_i) = \sum x_h P(x_i = x_h) = \sum_{h=1}^N x_h \times \frac{1}{N} = \frac{1}{N} \sum x_h = \mu \quad \text{--- (6)}$$

$$\therefore E(\bar{x}) = \frac{1}{n} \sum E(x_i) = \frac{1}{n} \sum_{i=1}^n \mu = \frac{n}{n} \times \mu = \mu$$

$\therefore E(x) = E(\bar{x}) = \mu$ in case of SRWR.

$$\text{Var}(x_i) = E(x_i - \mu)^2 = \sum_{h=1}^N (x_h - \mu)^2 P(x_i = x_h)$$

$$= \frac{1}{N} \sum (x_h - \mu)^2$$

$$V(x_i) = \sigma^2 \quad \text{--- (7)}$$

$$\begin{aligned} \text{Cov}(x_i, x_j) &= E(x_i - \mu)(x_j - \mu) \\ &= \sum_{h=1}^N \sum_{s=1}^N (x_h - \mu)(x_s - \mu) P(x_i = x_h, x_j = x_s) \end{aligned}$$

$$\begin{aligned}
 &= \sum_{r=1}^N \sum_{s=1}^N (x_r - \mu)' (x_s - \mu) P(x_i = x_r, x_j = x_s) \\
 &= \frac{1}{N} \sum_{r=1}^N (x_r - \mu) \cdot \frac{1}{N} \sum_{s=1}^N (x_s - \mu) \\
 &= 0 \quad \checkmark \quad \text{--- (9)}
 \end{aligned}$$

$$\begin{aligned}
 \therefore v(\bar{x}) &= \frac{1}{n^2} \left[\sum v(x_i) + \sum \sum \text{cov}(x_i, x_j) \right] \\
 &= \frac{1}{n^2} \left[\sum_{i=1}^n \delta^2 + 0 \right] \\
 &= \frac{n \delta^2}{n^2}
 \end{aligned}$$

$$v(\bar{x}) = \frac{\delta^2}{n}$$

$$\therefore SE(\bar{x}) = \sqrt{v(\bar{x})} = \delta / \sqrt{n} \quad \Rightarrow \text{for SRSWR.}$$

Case II : SRSWOR

$$P(x_i = x_r) = \frac{1}{N}$$

$$E(x_i) = \mu \quad v(x_i) = \delta^2$$

$$\text{cov}(x_i, x_j) = E \left[(x_i - \mu) (x_j - \mu) \right]$$

$$= \sum_{r=1}^N \sum_{s=1}^N \dots P(x_i = x_r, x_j = x_s)$$

Cor (x_i, x_j)

$$= \sum_{i=1}^N \sum_{j=1}^N (x_i - \mu)(x_j - \mu) P(x_i = x_j)$$

$$\begin{aligned} (x_1 + x_2)^2 &= x_1^2 + x_2^2 + 2x_1x_2 \\ 2x_1x_2 &= (x_1 + x_2)^2 - x_1^2 - x_2^2 \end{aligned}$$

cov → $(\sum x_i)^2 - (\sum x_i^2)$

$$= \sum_{i=1}^N \sum_{s=1}^N (x_i - \mu)(x_s - \mu) \frac{1}{N} \cdot \frac{1}{N}$$

$$= \frac{1}{N} \frac{1}{N} \sum_{i \neq s} (x_i - \mu)(x_s - \mu)$$

$$\left(\sum_{i=1}^N (x_i - \mu) \right)^2 - \sum_{i=1}^N (x_i - \mu)^2 = \frac{1}{N(N-1)} \left[\left(\sum_{i=1}^N (x_i - \mu) \right)^2 - \sum_{i=1}^N (x_i - \mu)^2 \right]$$

$$= \frac{1}{N(N-1)} [0 - N\sigma^2]$$

$$\text{Cor}(x_i, x_j) = \frac{1}{N(N-1)} (-N\sigma^2) = \frac{-\sigma^2}{(N-1)}$$

$$\therefore E(\bar{x}) = \frac{1}{n} \sum_{i=1}^n E(x_i) = \frac{1}{n} \sum_{i=1}^n \mu = \frac{n\mu}{n} = \mu$$

$$V(\bar{x}) = \frac{1}{n^2} \left[\sum_{i=1}^n V(x_i) + \sum_{i \neq j} \text{cov}(x_i, x_j) \right]$$

$$= \frac{1}{n^2} \left[n\sigma^2 + \sum_{i \neq j} \left(\frac{-\sigma^2}{N-1} \right) \right]$$

$$= \frac{1}{n^2} \left[n\sigma^2 + n(n-1) \left(\frac{-\sigma^2}{N-1} \right) \right]$$

$$= \frac{\sigma^2}{n} - \frac{(n-1)\sigma^2}{n(N-1)}$$

$$= \frac{1}{n} - \frac{1}{n(N-1)}$$

$$= \frac{\sigma^2}{n} \left[1 - \frac{n-1}{N-1} \right]$$

$$= \frac{\sigma^2}{n} \left[\frac{N-1-n+1}{N-1} \right]$$

$$V(\bar{x}) = \frac{\sigma^2}{n} \left[\frac{N-n}{N-1} \right]$$

$$\therefore SE(\bar{x}) = \sqrt{V(\bar{x})} = \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$$

Q

Marks of 5 students are

20, 24, 20, 30, 26.

If a simple random sample of size 2 is drawn without replacement

Find the sample mean of each possible sample

Also find the standard error of the sample mean.

$N=5$, $n=2$

\therefore In case of SRSWOR \Rightarrow total no. of samples possible
 $= {}^N C_n = {}^5 C_2 = 10$

probable distribution of \bar{x} possible samples are

value of \bar{x}	P of \bar{x}
20	1/10
22	2/10
23	2/10
25	3/10
27	1/10
28	1/10

Sample values	Sample mean
20, 24	22
20, 20	20
20, 30	25 ✓
20, 26	23 ✓
24, 20	22
24, 30	27 ✓
24, 26	25 ✓
20, 30	25
20, 26	23 ✓
30, 26	28 ✓

$$E(\bar{x}) = \mu = \frac{1}{N} \sum x_i = \frac{1}{5} (20 + 24 + 20 + 26 + 30)$$

$$= \text{ans } \textcircled{24}$$

$$V(\bar{x}) = \frac{\sigma^2}{\sqrt{n}} \sqrt{\frac{N-1}{n-1}}$$

$$= \frac{\sigma^2}{\sqrt{5}}$$

$$\sigma^2 = \frac{1}{N} \sum (x_i - \mu)^2$$

$$= \frac{1}{5} \left\{ \begin{aligned} &+(20-24)^2 \\ &+(24-24)^2 \\ &+(20-24)^2 \\ &+(26-24)^2 \\ &+(30-24)^2 \end{aligned} \right.$$