21 December 2023 07:41 PM

Theory of Dispersion

- absolute Measure
  - 1. Range
  - 2. Mean deviation
  - 3. Standard deviation
- Relative Measure
  - 1. Coefficient of variation

Why Relative measure of Dispersion is better than absolute measure of dispersion

Q>

(1) Range = (Max - Min) value.

Q 3, 5, (-1), 8, 7

min = -1, max = 8

Range = 8 - (-1) = 9 (ans).

Q if $y = a + bx$

then Range of $(y) = |b|$ Range (x)

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Solution: Let x_{max} and x_{min} denote max and min value of given values of x.

Then $y_{max} = a + b x_{max}$

and $y_{min} = a + b x_{min}$

$\therefore$ Range $(y) = y_{max} - y_{min}$

$= a + b x_{max} - (a + b x_{min})$

$= b (x_{max} - x_{min})$

$= b$ Range (x)

(Proved)

Q If two variables x and y are related as $3y + 4x = 9$ and range of x is 3, then find find range of y.

Soln:

$3y + 4x = 9$

$3y = 9 - 4x$

$y = 3 - \frac{4}{3} x$

$$\text{Ran}(y) = (b)\ \text{Ran}\ x$$
$$= \tfrac{4}{3} \times 3 = 4\ (\text{cm})$$

(2) Mean Deviation (M.D)

$$M.D_{\bar{x}} = \frac{1}{n}\sum_{i=1}^{n} |x_i - \bar{x}|$$

$$M.D_{\bar{x}} = \frac{1}{\sum f} \sum |x_i - \bar{x}| f_i$$

Again, M.D about median $= \frac{1}{\sum f} \sum |x_i - Me| f_i$

Q Frequency Distribution of age (in years) of 80 persons

Age (in years)	Frequency
30 - 34	12
35 - 39	15
40 - 44	18
45 - 49	11
50 - 54	10
55 - 59	9
Total	80

Calculate mean deviation about mean and median.

mean about mean as well median.

Soln

CB Age	f	x_i	$x_i f$	cf	$\lvert x_i - \bar{x} \rvert f$	$\lvert x_i - \text{Me} \rvert f_i$
—	—	32		12		
—	—	37		27 cf		
Med class 39.5–49.5	18 f	42		45		
—	—	47		61		
—	—	52		71		
—	—	57		80		
	$\Sigma f = 80$		$\Sigma x_i f_i$ =		$\Sigma \lvert x_i - \bar{x} \rvert f_i$	$\Sigma \lvert x_i - \text{Me} \rvert f_i$

M.D(x̄) ↓ M.D(Me) ↓

Median → ans (6.5 years)

median value 43.11

(3) Standard Deviation

Variance (σ^2)

$$V(x) = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$= \frac{1}{n} \Sigma (x_i^2 - 2 x_i \bar{x} + \bar{x}^2)$$

$$= \frac{1}{n} \Sigma x_i^2 - 2\bar{x} \frac{1}{n} \Sigma x_i + \frac{n}{n} \bar{x}^2$$

$$= \frac{1}{n}\Sigma x_i^2 - 2\bar{x}\frac{1}{n}\Sigma x_i + \frac{n}{n}\bar{x}$$

$$V(x) = \frac{1}{n}\Sigma x_i^2 - \bar{x}^2$$

Standard deviation (s.d) or $\sigma = \sqrt{Var(x)}$

$$= \sqrt{\frac{1}{n}\Sigma(x_i - \bar{x})^2}$$

$$= \sqrt{\frac{1}{n}\Sigma x_i^2 - \bar{x}^2}$$

Combined s.d (in case of two groups)

$$s^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2} + \frac{n_1 n_2 (\bar{x}_1 - \bar{x}_2)}{(n_1 + n_2)^2}$$

$$s = \sqrt{s^2} = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2} + \frac{n_1 n_2 (\bar{x}_1 - \bar{x}_2)}{(n_1 + n_2)^2}}$$

(4) Quartile Deviation (Q.D) $= \frac{Q_3 - Q_1}{2}$

(semi-interquartile range)

Q

Income	No. of persons
150 - 300	232
300 - 450	128
450 - 600	60
600 - 750	40
750 - 900	28
900 - 1100	12
1100 - 1500	6

Calculate Standard deviation

Formula

$\sigma^2 = \frac{1}{\Sigma f} \Sigma x^2 f - \bar{x}^2$

CB	f	x_i	$x_i f_i$	$x_i^2 f$
	Σf_i	$\Sigma x_i f_i$	$\Sigma x_i^2 f$	

$\bar{x}$

Ans: 24 = sd

For two values say 'a' and 'b', $a < b$ of a variable x, the mean and standard deviation are respectively 25 and 4. Find 'a' and 'b'.

$\bar{x} = 25$

$\frac{a+b}{2} = 25$

$a+b = 50 \quad -(1)$

$\sigma = 4$

$\sigma^2 = 16$

$\frac{1}{2}(a^2+b^2) - 25^2 = 16$

$(a-b)^2 = a^2+b^2-2ab$

$= 1282 - 1218$

$(a-b)^2 = 64$

$\frac{1}{2}(a^2+b^2) = 16+25^2$

$a^2+b^2 = 2(16+25^2)$

$a^2+b^2 = 1282$

$ab = 8$ ——— (11)

$(a+b)^2 = a^2+b^2+2ab$

$50^2 - (a^2+b^2) = 2ab$

$2500 - 1282 = 2ab$

$2ab = 1218$

Adding (1) and (2)

$a+b = 50$

$a-b = 8$

(+) ———

$2a = 58$

$\Rightarrow a = 58/2 = 29$ (ans)

$\therefore b = 50 - a$

$= 50 - 29$

$= 21$ (ans).

For a batch of 10 boys, the mean and s.d of weight are found to be 50 kg and 5kg resp. On further verification it is detected that the weight of two boys have been wrongly included as 45kg, 55kg instead of [illegible] and 48 kg

actual values 42kg and 48 kg

Calculate the corrected mean and corrected S.d.

$n = 10$

$\bar{x} = 50$

$\frac{1}{n}\Sigma x = 50$

$\Sigma x = 50 \times 10 = 500$

$\therefore$ corrected sum of obs, $\Sigma x' = 500 - 45 - 55 + 42 + 48$

$= 490$

$\therefore$ corrected mean, $\bar{x}' = \frac{490}{10} = 49$ (ans).

$\sigma = S.d = 5$

$\sigma^2 = 25$

$\frac{1}{n}\Sigma x^2 - \bar{x}^2 = 25$

$\frac{1}{10}(\Sigma x^2) - 50^2 = 25$

$\Sigma x^2 = (25 + 50^2) \times 10$

$\Sigma x^2 = 25250$

Corrected sum of square $\Sigma x^{2'} = 25250 - 45^2 - 55^2$

Corrected sum of square $\Sigma x^2 = 25260 - 45^2 - 55^2 + 42^2 + 48^2$

$= 24268$

$\therefore$ corrected $\sigma^2 = \frac{1}{10} \times 24268 - 49^2$

$= 2426.8 - 2401$

$= 25.8$

$\sigma = \sqrt{25.8} = 5.08$ (ans)

Q The mean and s.d of 20 obs are found to be 10 and 2 resp At the time of checking it was found that one observation 8 was incorrect. Calculate the mean & s.d if the wrong obs (i) is omitted (ii) replaced by 12.

Ans (i) 2.02
(ii) 1.99

given monthly expenditure for 430 families

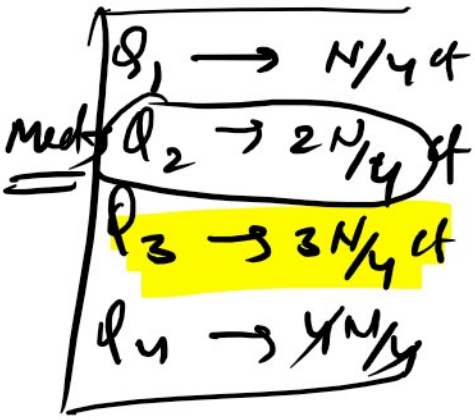
430 families

Monthly expenditure	Frequency	cf
less than 1000	30	30
1000 - 1250	45	75
1250 - 1500	70	145
1500 - 1750	82	227
1750 - 2000	66	293
2000 - 2250	57	350
2250 - 2500	28	378
2500 - 2750	22	400
2750 - 3000	18	418
more than 3000	12	430

Calculate the interquartile range

Monthly ex	cf
less than 1000	0
1000	30
1250 (Q_1)	75 ← 107.5
1500	145
1750	229
2000 (Q_3)	293 ← 322.5
2250	350
	378

43	2250	→	1350
2500		→	378
2750		→	400
3000		→	418
more than 3000		→	430



semi inter quartile range $= \frac{Q_3 - Q_1}{2}$

$N = 430$ $\quad N/4 = \frac{430}{4} = 107.5$

$\therefore Q_1$ corresponds to 107.5

and $\quad 3.\frac{N}{4} = 3 \times 107.5$

$= 322.5$

$Q_1 = 1366.07$

$Q_3 = 2129.38$

Ans : (381.65) ₹