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Application of derivatives

Let f (x) be a polynomial function such that f (x) + f ' (x) + f "(x) = x^5 + 64. Then, the value of $\lim_{x\to 1}\frac{f(x)}{x-1}$ $= f'(1)$ and $f(1)=0$

(A) – 15 (B) – 60

(C) 60 (D) 15

Degree of $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots$

" " $f'(x) \to (n-1)$

" " $f''(x) \to (n-2)$

" " Sum $\to$ n first term $a_n x^n$

$\boxed{x-1=h}$

$x = 1+h$

$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

degree of $f(x) = 5$

$f^5(x) = a_n 5!$ $a_n = 1$

$\boxed{f^5(x) = 5!}$

$\lim_{h\to 0}\frac{f(1+h)-f(1)}{h} = f'(1)$ $f(1) \to 0$

$f(1) + f'(1) + f''(1) = 65$

$f(1) = 0$

$\boxed{f'(1) + f''(1) = 65 \quad (1)}$

$f'(1) + 80 = 65$

$\boxed{f'(1) = 65 - 80 = -15}$

$f(x) + f'(x) + f''(x) = x^5 + 64$

$f'(x) + f''(x) + f'''(x) = 5x^4$

$f''(x) + f'''(x) + f^4(x) = 20x^3$

$f'''(x) + f^4(x) + f^5(x) = 60x^2$

$f^5(x) - f''(x) = 60x^2 - 20x^3$

$5! - f''(x) = 60x^2 - 20x^3$

$f''(x) = 120 - 60x^2 + 20x^3$

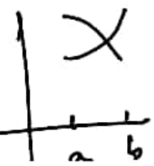
$\boxed{f''(1) = 120 - 60 + 20 = 80}$

Let f : R → R and g : R → R be two functions defined by f (x) = $\log_e(x^2 + 1) - e^{-x} + 1$ and

$1 - 2e^{2x}$

$f(a) > f(b)$

$f(x) < f(a)$ ... $f(x)$



Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be two functions defined by $f(x) = \log_e(x^2 + 1) - e^{-x} + 1$ and $g(x) = \frac{1-2e^{2x}}{e^x}$. Then, for which of the following range of α, the inequality

$$f\left(g\left(\frac{(\alpha-1)^2}{3}\right)\right) > f\left(g\left(\alpha - \frac{5}{3}\right)\right)$$ holds?

(A) (2, 3) (B) (–2, – 1)

(C) (1, 2) (D) (–1, 1)

$f(b) > f(a) \Rightarrow f(x)\uparrow$

$f(b) < f(a) \Rightarrow f(x)\downarrow$

$b > a$

$a \quad b$

$g(x) = e^{-x} - 2e^{x}$

$g'(x) = -e^{-x} - 2e^{x}$

$g'(x) = -(e^{-x} + 2e^{x})$

$g'(x) < 0$

$g(x)$ is decreasing

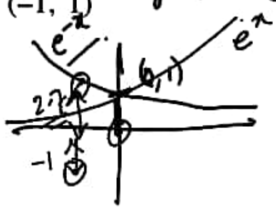
$f'(x) = \frac{2x}{x^2+1} - (-e^{-x}) = \frac{2x}{x^2+1} + e^{-x}$

$= \frac{2x}{x^2+1} + \frac{1}{e^x} > 0$

$f(x)$ is increasing.

$x < 0 \quad e^{-x} > 1$

$\frac{2x}{x^2+1}$



$g\left(\frac{(\alpha-1)^2}{3}\right) > g\left(\alpha - \frac{5}{3}\right)$

$\Downarrow$

$\frac{(\alpha-1)^2}{3} < \alpha - \frac{5}{3}$

$\frac{(\alpha-1)^2}{3} < \frac{3\alpha-5}{3}$

$(\alpha-1)^2 < 3\alpha - 5$

$\alpha^2 - 2\alpha + 1 - 3\alpha + 5 < 0$

$\alpha^2 - 5\alpha + 6 < 0$

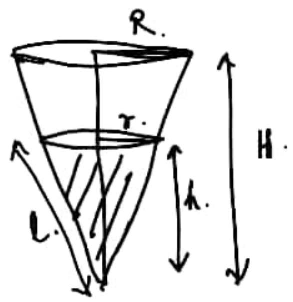
$(\alpha-2)(\alpha-3) < 0$

$2 < \alpha < 3$

Water is being filled at the rate of $1\ \text{cm}^3/\text{sec}$ in a right circular conical vessel (vertex downwards) of height 35 cm and diameter 14 cm. When the height of the water level is 10 cm, the rate (in cm^2/sec) at which the wet conical surface area of the vessel increases is

(A) 5 (B) $\frac{\sqrt{21}}{5}$

(C) $\frac{\sqrt{26}}{}$ (D) $\frac{\sqrt{26}}{}$



$\frac{r}{h} = \frac{R}{H} = \frac{7}{35} = \frac{1}{5}$

(A) 5 (B) $\frac{}{5}$

(C) $\frac{\sqrt{26}}{5}$ (D) $\frac{\sqrt{26}}{10}$

$\frac{r}{h} = \frac{R}{H} = \frac{7}{35} = \frac{1}{5}$

$\boxed{h = 5r}$

$5r = 10$

$r = 2$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 \cdot 5r = \frac{5}{3}\pi r^3$

$\frac{dV}{dt} = \frac{5}{3}\pi \cdot 3r^2 \frac{dr}{dt} = 5\pi r^2 \frac{dr}{dt} = 1$

$SA = \pi r l$

$A = \pi r\sqrt{R^2+r^2} = \pi r\sqrt{26}\cdot r = \sqrt{26}\pi r^2$

$\boxed{\frac{dr}{dt} = \frac{1}{5\pi r^2}}$

$\frac{dA}{dt} = \sqrt{26}\pi \cdot 2r\frac{dr}{dt} = 2\sqrt{26}\cancel{\pi} r \cdot \frac{1}{5\cancel{\pi} r^2} = \frac{2\sqrt{26}}{5r} = \frac{\cancel{2}\sqrt{26}}{5\times\cancel{2}}$

If the angle made by the tangent at the point (x_0, y_0) on the curve $x = 12(t + \sin t\cos t)$, $\boxed{y = 12(1+\sin t)^2}$, $0 < t < \frac{\pi}{2}$, with the positive x-axis is $\frac{\pi}{3}$, then y_0 is equal to

(A) $6(3+2\sqrt{2})$ (B) $3(7+4\sqrt{3})$

(C) 27 (D) 48

Parametric → Implicit derivative

Slope of the tangent $= \left(\frac{dy}{dx}\right)_p = \tan\frac{\pi}{3}$

$y_0 = 12\left(1+\frac{1}{2}\right)^2$

$= \cancel{12}^{3} \times \frac{9}{\cancel{4}} = 27$

$\frac{dy/dt}{dx/dt} = \sqrt{3}.$

$\frac{\cancel{12}\cdot 2(1+\sin t)\cos t}{\cancel{12}(1+\cos^2 t - \sin^2 t)} = \sqrt{3}.$

$\frac{\cancel{2}\cancel{\cos t}(1+\sin t)}{\cancel{2}\cos^{\cancel{2}} t} = \sqrt{3}$

$1 + \sin t = \sqrt{3}\cos t$

$a\cos\theta + b\sin\theta$

$\frac{\sqrt{3}}{2}\cos t - \frac{1}{2}\sin t = \frac{1}{2}$

$\sin 60^\circ \cos t - \cos 60^\circ \sin t = \frac{1}{2}$

$\sin(60^\circ - t) = \sin 30^\circ$

$60^\circ - t = 30^\circ$

$\boxed{t = 30^\circ}$

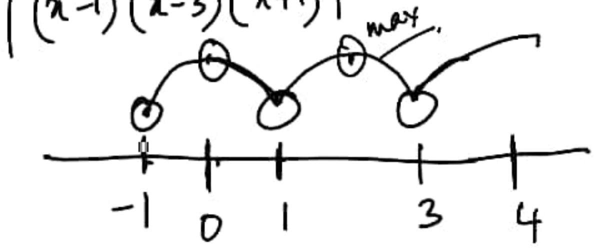
Let $f(x) = |(x-1)(x^2-2x-3)| + x - 3,\ x \in R$. If m and M are respectively the number of points of local minimum and local maximum of f in the interval (0, 4), then m + M is equal to 8.

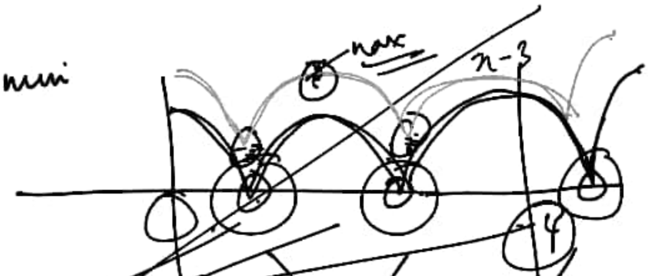
no of local mini

no of local max

g(x) g'(x)

$|(x-1)(x-3)(x+1)|$





mini values of function & function is not differentiable at these points.

Critical points

The sum of the absolute minimum and the absolute maximum values of the function $f(x) = |3x - x^2 + 2| - x$ in the interval $[-1, 2]$ is :

(A) $\dfrac{\sqrt{17}+3}{2}$ (B) $\dfrac{\sqrt{17}+5}{2}$

(C) 5 (D) $\dfrac{9-\sqrt{17}}{2}$

Let S be the set of all the natural numbers, for which the line $\frac{x}{a}+\frac{y}{b}=2$ is a tangent to the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ at the point (a, b), ab $\neq$ 0. Then:

(A) $S=\phi$ (B) $n(S)=1$

(C) $S=\{2k : k \in N\}$ (D) $S=N$

Let $f(x) = 2\cos^{-1}x + 4\cot^{-1}x - 3x^2 - 2x + 10$, $x \in [-1, 1]$. If $[a, b]$ is the range of the function then $4a - b$ is equal to:

(A) 11 (B) $11 - \pi$ (C) $11 + \pi$ (D) $15 - \pi$

Consider a cuboid of sides 2x, 4x and 5x and a closed hemisphere of radius r. If the sum of their surface areas is a constant k, then the ratio x : r, for which the sum of their volumes is maximum, is :

(A) 2 : 5 (B) 19:45 (C) 3 : 8 (D) 19 : 15

If y = y(x) is the solution of the differential equation $x\frac{dy}{dx}+2y=xe^x, y(1)=0$, then the local maximum value of the function $z(x) = x^2y(x)-e^x$, $x \in R$ is :

(A) $1-e$ (B) 0 (C) $\frac{1}{2}$ (D) $\frac{4}{e}-e$