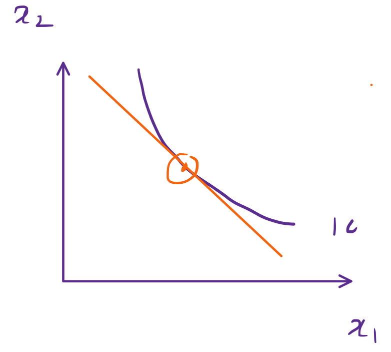
Q. $u(x_1, x_2) = \sqrt{x_1} + \sqrt{x_2}$, $P_1 = 5$, $P_2 = 1$, $M = 60$. Find opt x_1, x_2.

To find whether IC's are convex / concave:-

Convex: $\frac{\partial MRS}{\partial x_1} < 0 \Rightarrow$ Interior soln

Concave: $\frac{\partial MRS}{\partial x_1} > 0 \Rightarrow$ Corner soln.



Given: $u(x_1, x_2) = \sqrt{x_1} + \sqrt{x_2}$.

$$\therefore MRS = \frac{MU_1}{MU_2} = \frac{\partial u/\partial x_1}{\partial u/\partial x_2} = \sqrt{\frac{x_2}{x_1 \uparrow}}$$

$\therefore \frac{\partial MRS}{\partial x_1} < 0 \Rightarrow$ IC's are convex.

At opt: $MRS = \frac{P_1}{P_2} \Rightarrow \sqrt{\frac{x_2}{x_1}} = \frac{5}{1} \Rightarrow \left(\frac{x_2}{x_1}\right) = \frac{25}{1} \Rightarrow x_2 = 25x_1$.

$\therefore$ From B.L $M = P_1x_1 + P_2x_2$

$60 = 5x_1 + 1(25x_1)$

$60 = 30x_1 \Rightarrow x_1^* = 2$, $x_2^* = 25 \times 2 = 50$.

Q. $u(x_1, x_2) = \frac{x_1 x_2}{x_1 + x_2}$. Check IC's are concave/convex.

$$MRS = \frac{MU_1}{MU_2} = \frac{\partial u/\partial x_1}{\partial u/\partial x_2} = \left(\left(\frac{x_2}{x_1 \uparrow}\right)\right)^2 \downarrow \Rightarrow \frac{\partial MRS}{\partial x_1} = 2\left(\frac{x_2}{x_1}\right)\left(-\frac{1}{x_1^2}\right) < 0$$

...(convex)

$\frac{\partial u}{\partial x_1} = \frac{x_2^2}{(x_1 + x_2)^2}$ $\qquad$ $\frac{\partial u}{\partial x_2} = \frac{x_1^2}{(x_1 + x_2)^2}$

Utility fns with more than 2 goods:-

Recap: $u(x_1, x_2) = x_1^a x_2^b$; $x_1^* = \left(\frac{a}{a+b}\right)\left(\frac{M}{P_1}\right)$ $\qquad$ $x_2^* = \left(\frac{b}{a+b}\right)\left(\frac{M}{P_2}\right)$

Q. $u(x_1, x_2, x_3) = x_1^1 x_2^1 x_3^1$. Find the Marshallian demands for the 3 Goods.

Budget Line: $M = P_1 x_1 + P_2 x_2 + P_3 x_3$.

(*) For finding interior solutions use the method of Lagrange Multipliers.

Max $u = \underbrace{x_1 x_2 x_3}_{\text{obj fn}}$ subject to: $\underbrace{M - P_1 x_1 - P_2 x_2 - P_3 x_3}_{\text{const fn}} = 0$.

Define Lagrangian as:

$$\mathcal{L} = x_1 x_2 x_3 + \lambda [M - P_1 x_1 - P_2 x_2 - P_3 x_3]$$, λ = Lagrange Multiplier.

Here variables: x_1, x_2, x_3, λ.

$\frac{\partial \mathcal{L}}{\partial x_1} = 0 \Rightarrow x_2 x_3 + \lambda(-P_1) = 0 \Rightarrow x_2 x_3 = \lambda P_1$ --- (i)

$\frac{\partial \mathcal{L}}{\partial x_2} = 0 \Rightarrow x_1 x_3 + \lambda(-P_2) = 0 \Rightarrow x_1 x_3 = \lambda P_2$ --- (ii)

$\frac{\partial \mathcal{L}}{\partial x_3} = 0 \Rightarrow x_1 x_2 + \lambda(-P_3) = 0 \Rightarrow x_1 x_2 = \lambda P_3$ --- (iii)

$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow M = P_1 x_1 + P_2 x_2 + P_3 x_3$. --- (iv)

(i) ÷ (ii): $\frac{x_2}{x_1} = \frac{P_1}{P_2} \Rightarrow \boxed{x_2 = \left(\frac{P_1}{P_2}\right) x_1} \Rightarrow \boxed{x_2^* = \frac{M}{3P_2}}$

(i) ÷ (iii): $\frac{x_3}{x_1} = \frac{P_1}{P_3} \Rightarrow \boxed{x_3 = \left(\frac{P_1}{P_3}\right) x_1} \Rightarrow \boxed{x_3^* = \frac{M}{3P_3}}$

B.L: $M = P_1 x_1 + P_2 x_2 + P_3 x_3$.

$M = P_1 x_1 + \cancel{P_2}\left(\frac{P_1}{\cancel{P_2}}\right) x_1 + \cancel{P_3}\left(\frac{P_1}{\cancel{P_3}}\right) x_1$

$M = P_1 x_1 + P_1 x_1 + P_1 x_1$

$\boxed{x_1^* = \frac{M}{3P_1}} \Rightarrow \boxed{\frac{P_1 x_1^*}{M}} = \frac{1}{3}$.

↳ Sl...

$3P_1$ M 3.

↳ Share of exp on Good 1.

For Lagrange Multiplier: (i): $x_2 x_3 = \lambda P_1$

$$\left(\frac{M}{3P_2}\right)\left(\frac{M}{3P_3}\right) = \lambda P_1$$

$$\lambda^* = \frac{M^2}{9P_1 P_2 P_3}$$

Q. $u(x_1, x_2, x_3) = x_1 + \min\{x_2, x_3\}$. Find opt x_1, x_2, x_3.

[x_2, x_3 are perfect complements, but x_1 is a perfect substitute of (x_2, x_3)]

Perfect complements: $u(x_1, x_2) = \min\{x_1, x_2\}$

Perfect substitutes: $u(x_1, x_2) = x_1 + x_2$.

Note: $\min\{x_2, x_3\}$ is not differentiable, P.S gives corner solns, so Lagrange Method cannot be used.

At optimal: $x_2^* = x_3^*$

$$u = x_1 + \min\{x_2, x_3\}$$

$u = x_1 + a$ $a = \min\{x_2, x_3\}$ ⇒ considered as a bundle.

↳ Perfect substitutes b/w x_1 & a.

$P_a = P_2 + P_3$.

(i) If $P_1 < P_a$ ⇒ consume only Good 1.

$$x_1^* = \frac{M}{P_1}, \quad a^* = 0 \Rightarrow x_2^* = x_3^* = 0.$$

(ii) If $P_1 > P_a$ ⇒ consume only Bundle 'a'.

$$x_1^* = 0, \quad a^* = x_2^* = x_3^* = \frac{M}{P_2 + P_3}.$$

Q. $u(x_1, x_2, x_3) = x_1 \cdot \min\{x_2, x_3\}$. Find opt x_1, x_2, x_3.

Cobb Douglas.

At opt: $x_2^* = x_3^*$

$\min\{x_2, x_3\} = a$, $P_a = P_2 + P_3$.

$u(x_1, a) = x_1^1 \cdot a^1$, Prices P_1, P_a.

$x_1^* = \frac{1}{2} \cdot \frac{M}{P_1}$ $\quad a^* = \frac{1}{2} \cdot \frac{M}{P_a} \Rightarrow x_2^* = x_3^* = \frac{1}{2} \cdot \frac{M}{P_2 + P_3}$.

$x_1^* = \frac{M}{2P_1}$, $x_2^* = x_3^* = \frac{1}{2} \cdot \frac{M}{P_2 + P_3}$.