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Samuelson's Multiplier Accelarator Interaction Model.

$$Y_t = C_t + I_t + G_0 \quad \text{--- } ①$$

$$C_t = \gamma Y_{t-1} \quad \text{--- } ② \qquad (0 < \gamma = mpc < 1)$$

$$I_t = \alpha (C_t - C_{t-1}) \quad \text{--- } ③ \qquad (\alpha > 0)$$

Substituting ② in ③,

$$I_t = \alpha (\gamma Y_{t-1} - \gamma Y_{t-2})$$

$$I_t = \alpha\gamma (Y_{t-1} - Y_{t-2}) \quad \text{--- } ③'$$

The model can be written in single eqn as,

$$Y_t = \gamma Y_{t-1} + \alpha\gamma (Y_{t-1} - Y_{t-2}) + G_0$$

or, $$Y_t - \gamma Y_{t-1} - \alpha\gamma Y_{t-1} + \alpha\gamma Y_{t-2} = G_0$$

or, $$\boxed{Y_t - \gamma (1+\alpha) Y_{t-1} + \alpha\gamma Y_{t-2} = G_0} \quad \text{--- } ④$$

eq ④ is the 2nd order linear non-homogeneous difference eqn on Y.

P.I

$$Y_t = Y_{t-1} = Y_{t-2} = \bar{Y} \quad (\text{const}).$$

$$\bar{Y} - \gamma(1+\alpha)\bar{Y} + \alpha\gamma\bar{Y} = G_0$$

$$\bar{Y}\left(1 - \gamma - \cancel{\alpha\gamma} + \cancel{\alpha\gamma}\right) = G_0$$

$$\bar{Y} = \frac{G_0}{(1-\gamma)}$$ (intertemporal equilibrium income).

CF Considering the homogeneous part

$$y_t - \gamma(1+\alpha)y_{t-1} + \alpha\gamma y_{t-2} = 0$$

Let the trial soln be $y_t = \lambda x^t$

then $y_{t-1} = \lambda x^{t-1}$

$y_{t-2} = \lambda x^{t-2}$

So, $\lambda x^t - \gamma(1+\alpha)\lambda x^{t-1} + \alpha\gamma\lambda x^{t-2} = 0$

or, $\lambda x^{t-2}\left[x^2 - \gamma(1+\alpha)x + \alpha\gamma\right] = 0$

$$\therefore x^2 - \gamma(1+\alpha)x + \alpha\gamma = 0 \quad \text{— (5)}$$

$$x_1, x_2 = \frac{\gamma(1+\alpha) \pm \sqrt{\gamma^2(1+\alpha)^2 - 4\alpha\gamma}}{2}$$

Case1. Roots are real and distinct.

$$\gamma^2(1+\alpha)^2 - 4\alpha\gamma > 0$$

$$\gamma > \frac{4\alpha}{(1+\alpha)^2}$$

Case 2: equal and real roots : $\gamma = \dfrac{4\alpha}{(1+\alpha)^2}$

Case 3: roots are imaginary if $\gamma < \dfrac{4\alpha}{(1+\alpha)^2}$

Contd

inner eu

$$* \quad 1 - \quad - \left(\overset{a}{\frac{\gamma + \alpha}{}} + \overset{b}{\cancel{\frac{\sqrt{\quad + \quad 4\alpha\gamma}}{2}}} \right) + \left(\overset{a}{\frac{\gamma + \alpha\gamma}{}} - \overset{b}{\cancel{\frac{\sqrt{\quad}}{2}}} \right)$$

$x_1 + x_2$ -

$$* \quad x_1 x_2 = \left(\frac{\gamma + \alpha\gamma}{2}\right)^2 - \left\{ \frac{\gamma^2(1+\alpha)^2 - 4\alpha\gamma}{4} \right\}$$

$$= \frac{\cancel{\gamma^2(1+\alpha)^2} - \cancel{\gamma^2(1+\alpha)^2} + 4\alpha\gamma}{4}$$

$$\underline{x_1 x_2} = \underline{\alpha\gamma > 0}$$

Case 1

$$\underbrace{(1-x_1)}\ \underbrace{(1-x_2)} = 1 - (x_1 + x_2) + x_1 x_2$$

$$= 1 - \gamma(1+\alpha) + \alpha\gamma$$

$$= 1 - \gamma$$

$$0 < \gamma < 1 \Rightarrow 0 < (1-\gamma) < 1$$

same sign.

$0 < \gamma < 1 \Rightarrow 0 < (1- ...$

Since $\alpha\gamma > 0 \Rightarrow x_1 x_2$ posses same sign.

and since $x_1 + x_2 = \gamma(1+\alpha) > 0$

$\therefore x_1$ and x_2 +ve

$\Rightarrow$ time path of y_t is a step fn.

five combinations of $(x_1$ and $x_2)$

convergent (i) $0 < \boxed{x_1 < x_2} < 1 \Rightarrow 0 < \gamma < 1$; $\alpha\gamma < 1$ (mpc)

(ii) $0 < x_2 < x_1 = 1 \Rightarrow \gamma = 1$ ×

(iii) $0 < x_2 < 1 < x_1 \Rightarrow \gamma > 1$ ×

(iv) $1 = x_2 < x_1 \Rightarrow \gamma = 1$ ×

divergent (v) $1 < \boxed{x_1 < x_2} \Rightarrow 0 < \gamma < 1$; $\alpha\gamma < 1$

Case II for equal roots : $x = \dfrac{\gamma(1+\alpha)}{2} > 0$

$x^2 = \alpha\gamma > 0$

$x = \sqrt{\alpha\gamma}$

$\Rightarrow$ Time path will be a step fn.

(i) $0 < x < 1 \Rightarrow \gamma < 1$ ✓ ; $\alpha\gamma < 1$

(ii) $x = 1 \Rightarrow \gamma = 1$ ×

(iii) $x > 1 \Rightarrow \gamma < 1$; $\alpha\gamma > 1$

Case III : Let $x_1 = R(\cos\theta + i\sin\theta)$

$x_2 = R(\cos\theta - i\sin\theta)$

$R = \sqrt{h^2 + v^2}$

$h = -b/2a$

$v = \sqrt{b^2 - 4ac}$

$$h = -b/2a$$
$$v = \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x_1 x_2 = R^2 = \alpha\gamma \Rightarrow R = \sqrt{\alpha\gamma}$$

< 1

$R < 1 \Rightarrow \alpha\gamma < 1$ ✓

$R = 1 \Rightarrow \alpha\gamma = 1$ ✗

$R > 1 \Rightarrow \alpha\gamma > 1$ ✗