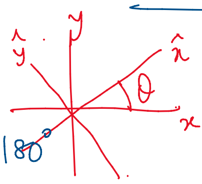
02 February 2024 15:08

$x = \hat{x}\cos\theta - \hat{y}\sin\theta$ and $y = \hat{x}\sin\theta + \hat{y}\cos\theta.$



Sketch the graph of $\overset{A}{4}x^2 - \overset{B}{4}xy + \overset{C}{7}y^2 - 24 = 0.$

Step 1 to find θ $\cot 2\theta = \frac{A-C}{B} = \frac{-3}{-4}$ $2\theta = 53^\circ + 180^\circ$

$2\theta = 233^\circ$

$\boxed{\theta = 116.5^\circ}$ $\boxed{\begin{aligned} \sin\theta &= 0.89 \\ \cos\theta &= -0.44 \end{aligned}}$

$\boxed{\begin{aligned} x &= -0.44\hat{x} - 0.89\hat{y} \\ y &= 0.89\hat{x} - 0.44\hat{y} \end{aligned}}$ $\rightarrow \frac{\hat{x}^2}{a^2} \pm \frac{\hat{y}^2}{b^2} = 1$

Step 2 $4(-0.44\hat{x} - 0.89\hat{y})^2 - 4(-0.44\hat{x} - 0.89\hat{y})(0.89\hat{x} - 0.44\hat{y}) + 7(0.89\hat{x} - 0.44\hat{y})^2$
$-24 = 0$

$3.1328, +2.394, -5.4824.$

$\cancel{0.7832}$ $+ 2.3816 - 5.4824.$

$3.1684 - 1.5664$
$+ 1.3552$

$0.7744 + 1.5664 + 5.5447$

$7.8855\hat{x}^2 + 2.9572\hat{y}^2 - 24 = 0.$

$\boxed{\frac{\hat{x}^2}{3} + \frac{\hat{y}^2}{8} = 1}$

$\checkmark$. Given eqn

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

$x = \hat{x}\cos\theta - \hat{y}\sin\theta$ $y = \hat{x}\sin\theta + \hat{y}\cos\theta.$

$$\hat{A}\hat{x}^2 + \hat{B}\hat{x}\hat{y} + \hat{C}\hat{y}^2 + \hat{D}\hat{x} + \hat{E}\hat{y} + \hat{F} = 0.$$ Transformed eqn

$\boxed{B^2 - 4AC = \hat{B}^2 - 4\hat{A}\hat{C}.}$

$\boxed{\hat{B} = 0,}$

$\boxed{B^2 - 4AC = -4\hat{A}\hat{C}.}$

$$\boxed{B^2 - 4AC = -4\hat{A}\hat{C}.}$$

$$\hat{A}\hat{x}^2 + \hat{C}\hat{y}^2 + \hat{D}\hat{x} + \hat{E}\hat{y} + \hat{F} = 0 \longleftrightarrow \frac{(\hat{x}-h)^2}{a^2} + \frac{(\hat{y}-k)^2}{b^2} = 1$$

$\hat{A} = \frac{1}{a^2}$ $\quad \hat{C} = \frac{1}{b^2}$

will be:

i) An ellipse if $\hat{A}\hat{C} > 0$;

ii) A hyperbola if $\hat{A}\hat{C} < 0$;

iii) A parabola if $\hat{A}\hat{C} = 0$.

Condition for conics

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

will be:

i) An ellipse if $B^2 - 4AC = -4\hat{A}\hat{C} < 0$;

ii) A hyperbola if $B^2 - 4AC = -4\hat{A}\hat{C} > 0$;

iii) A parabola if $B^2 - 4AC = -\hat{A}\hat{C} = 0$.

$B^2 - 4AC = -4\hat{A}\hat{C}$

Identifying a Conic in Polar Form

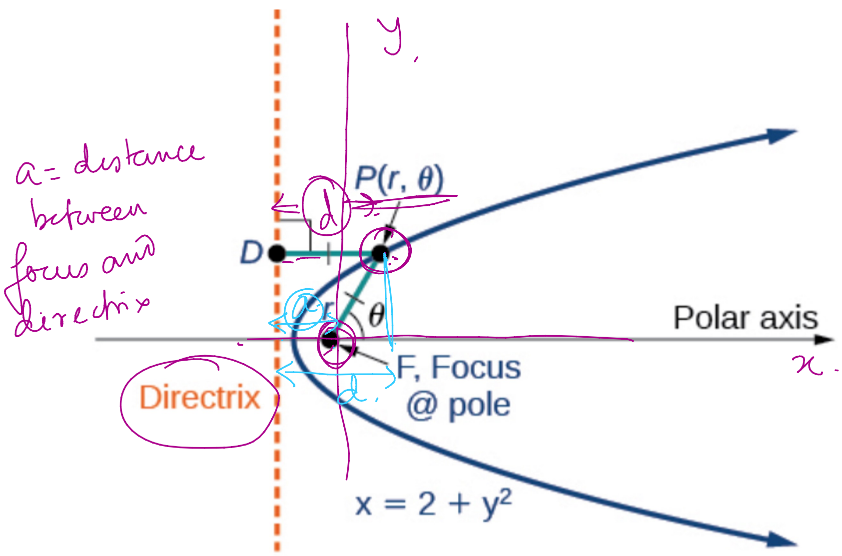


Figure 10.6.2

Any conic is defined wrt (a) a fixed pt. (focus) and (b) a fixed line (directrix)

$\frac{\text{distance from focus}}{\text{'' '' directrix}}$ = constant (eccentricity) e

$e = \frac{r}{d}$

$d = a + r\cos\theta$

$e = \frac{r}{a + r\cos\theta}$

$ea + er\cos\theta = r$

$ea = r(1 - e\cos\theta)$

$$\boxed{r = \frac{ea}{1 - e\cos\theta}}$$

e

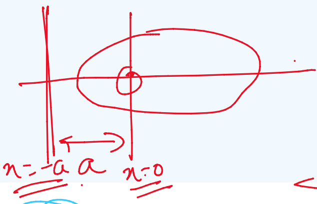
<1 ellipse. $\quad =1$ parabola $\quad >1$ hyperbola.

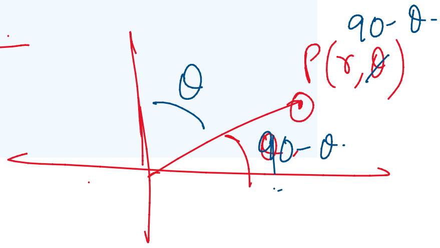
For each of the following equations, identify the conic with focus at the origin, the directrix, and the eccentricity.

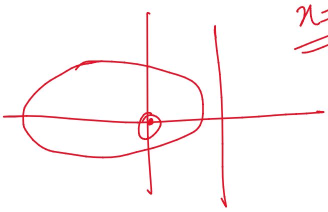
a. $r = \dfrac{6}{3 + 2\sin\theta}$ $\quad = \dfrac{2}{1 + \frac{2}{3}\sin\theta}$.

b. $r = \dfrac{12}{4 + 5\cos\theta}$

c. $r = \dfrac{7}{2 - 2\sin\theta}$







$x = 3$

$ea = 2.$

$e = \frac{2}{3}.$

$r = \dfrac{ea}{1 - e\cos(90 - \theta)}$