11 April 2024 18:47

Q. Let $X_1, X_2, \ldots, X_n$ be a random sample from exponential distribution with mean θ. Which of the following is NOT a sufficient statistic for θ?

(a) $\frac{1}{X_1+X_2+\ldots+X_n}$ Yes

(b) $X_1 + X_2 + \ldots + X_n$ Yes

(c) $\frac{X_n}{X_1+X_2+\ldots+X_{n-1}}$ NO

(d) $(X_n, X_1 + X_2 + \cdots + X_{n-1})$ Yes

2021)

$\frac{1}{\sum x_i}$ is sufficient

$\sum x_i$

PDF $f(x_i) = \frac{1}{\theta} e^{-x_i/\theta}$ $x_i > 0$

Likelihood function

$L = \prod f(x_i) = \prod_{i=1}^{n} \frac{1}{\theta} e^{-x_i/\theta}$

$= \frac{1}{\theta^n} e^{-\frac{1}{\theta}\sum x_i}$

$= g\{\sum x_i, \theta\} h(x)$

function of sample value through parameter θ

$h(x) \to$ sufficient statistic for θ

Cricket IND vs USA

T20 WC 2024

USA WI SL vs AUS 17/8

ISS 1 BSIM 1 0/6 th 2

Consider a sequence of i.i.d observations $X_1, X_2, \ldots$ from $N(0, \sigma^2)$. Which of the following are unbiased and consistent estimators of σ?

(a) $\sqrt{\frac{\pi}{2}}\frac{1}{n}\sum_{i=1}^{n}|X_i|$ T_1 (b) $\sqrt{\frac{1}{n}\sum_{i=1}^{n} X_i^2}$ T_2

(c) $\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2}$ T_3 (d) $\sqrt{\frac{1}{2n}(X_1^2 + X_n^2)}$ T_4

$X_i \sim N(0, \sigma^2)$

$T_1 = \sqrt{\frac{\pi}{2}}\frac{1}{n}\sum_{i=1}^{n}|x_i|$

$E(T_1) = \sqrt{\frac{\pi}{2}}\frac{1}{n}\sum E|x_i|$

$= \sqrt{\frac{\pi}{2}}\frac{1}{n}\cdot n \cdot \sqrt{\frac{2}{\pi}}\sigma$

n var(|x_i|)

(c) $\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i-\overline{X})^2}$ T_3 (d) $\sqrt{\frac{1}{2n}(X_1^2+X_n^2)}$

$V(T_1) = \frac{\pi}{2}\cdot\frac{1}{n^2}\, n\, var|x_i|$ $= \sqrt{\frac{\pi}{2}}\,\frac{1}{n}$

$E(T_1) = \sigma$

$var|x_i| = E(x^2) - (E|x_i|)^2$

$= \sigma^2 - \frac{2}{\pi}\sigma^2$

9 0 6 2 3
9 5 1 2 3

$var(T_1) = \frac{\pi}{2}\cdot\frac{1}{n}\sigma^2\left(1-\frac{2}{\pi}\right)$

$n \to \infty \quad var(T_1) \to 0$

T_1 is consistent estr of σ

$T_2 = \sqrt{\frac{1}{n}\sum X_i^2}$

$T_2^2 = \frac{1}{n}\sum x_i^2$

$E(T_2^2) = \frac{1}{n}\, n\sigma^2 = \sigma^2$

$E(T_2) \neq \sigma$

so, T_2 is not cs of σ

5. Let $X_1, X_2, \ldots, X_n$ be i.i.d. with the common pdf [redacted]
$f(\theta) = \frac{\theta}{x^{\theta+1}}$ for $x > 1$ where $\theta > 1$ is an unknown parameter.
Which of the following estimators of θ are consistent?
(a) $\frac{1}{n}\sum_{i=1}^{n} X_i$ (b) $\frac{1}{n}\sum_{i=1}^{n} log\,(X_i)$
(c) $\frac{n}{\sum_{i=1}^{n} X_i}$ (d) $\frac{n}{\sum_{i=1}^{n} log\,(X_i)}$

a b c d

~~$f(\theta) = \theta$~~

$f(x|\theta) = \frac{\theta}{x^{\theta+1}}$

$E(\log x) = \int^{\infty} \ln x\, f(x)\,dx$

$= \int^{\infty} \ln x \cdot \frac{\theta}{x^{\theta+1}}\,dx$

$= \int_0^{\infty} e^{-\theta}\cdot e^{z}\,dt$ (sub/bon)

$= \frac{1}{\theta}$

(b) $\frac{1}{n}\sum \ln(x_i) \to \frac{1}{\theta}$

(d) $\frac{n}{\sum \ln(x_i)} \to \frac{1}{\frac{1}{\theta}} = \theta$

Convergence in

$E(X_i) = \theta \qquad = \frac{1}{\theta}$

We know, Convergence in prob. implies consistency

9062395123

Gupta & Kapoor fun exam

Kapoor & Saxena

Saxena

1. GUPTA & KAPOOR
2. KAPUR & SAXENA
3. ROHATGI
4. SCARBARROWMAN
5. PARIMAL MUKHERJEE

Sheldon Ross

UPSC

#

12. Let $X_1, X_2, \ldots, X_n$ ($n \geq 2$) be a random sample from a distribution with probability density function f_θ, $\theta > 0$, unknown, where

$$f_\theta(x) = \begin{cases} \frac{2(\theta - x)}{\theta^2}, & 0 \le x \le \theta \\ 0 & \text{otherwise} \end{cases}$$

Let $\bar{X}_n$ be the sample mean and $X_{(n)} = \max\{X_1, X_2, \ldots, X_n\}$. Then which of the following statements is correct?

(a) $X_{(n)}$ is sufficient θ
(b) $X_{(n)}$ is unbiased θ
(c) $3\bar{X}_n$ is unbiased for θ
(d) $3\bar{X}_n$ is sufficient for θ

U/B

$E(\bar{X}_n) = E\left[\frac{1}{n}\sum X_i\right]$

$= \frac{1}{n}\sum E(X_i)$

$E(X_i) = \int_0^\theta x \cdot f(x)\,dx = \frac{2}{\theta^2}\int_0^\theta (\theta x - x^2)\,dx = \frac{\theta^3}{3} \cdot \frac{1}{\theta^2} = \theta/3$

$E[\bar{X}_n] = \frac{1}{n}\sum E(X_i) = \frac{\theta}{3}$

$E[3\bar{X}_n] = \theta$

U/B of θ

$r.[3 \times n] = \alpha$

$\overline{3 \times n} \to$ UB of α

Schaum's outline
∴ pu sched

Archive

- April 2024 (9)
- March 2024 (21)
- February 2024 (5)
- January 2024 (7)
- December 2023 (1)
- November 2023 (3)
- October 2023 (6)
- September 2023 (8)
- August 2023 (47)
- July 2023 (35)
- June 2023 (17)
- May 2023 (9)
- April 2023 (3)
- March 2023 (103)
- July 2022 (31)
- June 2022 (13)
- February 2021 (1)
- October 2020 (1)
- March 2020 (2)