$$\sum (Y_i - \bar{Y})^2 = \sum (\hat{Y}_i - \bar{\hat{Y}})^2 + \sum e_i^2 .$$

$$TSS = ESS + RSS .$$

$$d.f: \ (n-1) = (1) + (n-2) \qquad [\text{d.f is additive}]$$

$$ESS = \sum (\underbrace{\hat{Y}_i - \bar{\hat{Y}}})^2 = \sum (\hat{\beta} x_i)^2 = \hat{\beta}^2 \sum x_i^2 .$$

Here, $\hat{Y}_i = \hat{\alpha} + \hat{\beta} X_i$ --- (i)

Mean: $\bar{\hat{Y}} = \hat{\alpha} + \hat{\beta} \bar{X}$

$\therefore (\hat{Y}_i - \bar{\hat{Y}}) = \hat{\beta}(X_i - \bar{X}) = \hat{\beta} x_i$

In the model, we estimate 2 parameters: $\hat{\alpha}, \hat{\beta}$. But only $\hat{\beta}$ contributes in ESS.

$\therefore$ d.f of ESS = 1.

$$RSS = \sum e_i^2 = e_1^2 + e_2^2 + \dots + e_n^2$$

$\therefore$ No. of terms = n

No. of restrictions = 2 [$\sum e_i = 0$ & $\sum e_i X_i = 0$ from NE's]

$\therefore$ df of RSS = $(n-2)$

$$TSS = \sum (Y_i - \bar{Y})^2 = (Y_1 - \bar{Y})^2 + (Y_2 - \bar{Y})^2 + \dots + (Y_n - \bar{Y})^2$$

$\therefore$ No. of terms = n

$\therefore$ No. of restrictions = 1 [$\because \sum (Y_i - \bar{Y}) = 0$] .

$\therefore$ df of TSS = $(n-1)$

ANOVA: TSS = ESS + RSS $\rightarrow$ Total unexplained variation in Y.

TSS: Total variation in Y.

ESS: Total variation in Y explained by the model (through X).

$$\Rightarrow \quad 1 = \frac{ESS}{TSS} + \frac{RSS}{TSS} .$$

$\frac{RSS}{TSS} \rightarrow$ unexplained prop.

$\frac{ESS}{TSS} \rightarrow$ Prop of variation in Y

$\frac{ESS}{TSS}$

↳ Prop of variation in Y explained by the model (through X)

Higher the value of $\left(\frac{ESS}{TSS}\right)$, better is the model fit on the given data.

$$\text{Goodness of Fit Measure} = \frac{ESS}{TSS} = \frac{\hat{\beta}^2 \sum x_i^2}{\sum y_i^2}$$

$$= \left(\frac{\sum y_i x_i}{\cancel{\sum x_i^2}}\right)^2 \cdot \frac{\cancel{\sum x_i^2}}{\sum y_i^2} \quad \left(\because \hat{\beta} = \ldots\right)$$

$$= \frac{\left(\sum y_i x_i\right)^2}{\left(\sum x_i^2\right)\left(\sum y_i^2\right)}$$

$$= \left\{\frac{\sum y_i x_i}{\sqrt{\sum x_i^2}\sqrt{\sum y_i^2}}\right\}^2 = r_{XY}^2$$

$\therefore$ Goodness of Fit Measure $= r^2$.

Note that: $0 \leq r^2 \leq 1$. Higher value of r^2, better fit is the model to the data.

Interpretation of estimated parameters:

True Model / PRF: $Y_i = \alpha + \beta X_i + u_i$

Estimated Model / SRE: $\hat{Y}_i = \hat{\alpha} + \hat{\beta} X_i$ → slope coeff.

Interpret $\hat{\beta}$: $\frac{\partial \hat{Y}_i}{\partial X_i} = \hat{\beta}$ $[\Delta \hat{Y}_i$ due to $\Delta X_i]$

↳ represents change in $\hat{Y}$ due to change in X.

↳ If X changes by 1 unit, then on an avg, how much Y will change.

→ If X changes by 1 unit, then on an avg, how much Y will change.

Q. Consider a firm producing output(q) using only Labour (L) using the prodn fn: $q = \bar{K} L^{\beta}$. [$\bar{K}$ is the given level of capital in the short run]. A sample of size 'n' is collected on output level & corresponding labour units.

Can we fit the prodn fn to the given data?

q	L
q_1	L_1
q_2	L_2
⋮	⋮
q_n	L_n

$$q = \bar{K} L^{\beta}$$

Take log: $\underbrace{(\ln q)}_{Y} = \underbrace{(\ln \bar{K})}_{\alpha} + \beta \underbrace{(\ln L)}_{X}$

Model to be estimated: $Y = \alpha + \beta X + u$, $Y = \ln q$, $X = \ln L$

Estimated model: $\hat{Y} = \hat{\alpha} + \hat{\beta} X$: $\hat{\beta}$ is an estimate of β.

$\hat{\alpha}$ is an estimate of $\alpha = \ln \bar{K}$

$\therefore$ Estimate of $\bar{K} = e^{\hat{\alpha}}$