24 February 2024 07:52

$\cos x = \frac{e^{ix}+e^{-ix}}{2}$

$\sin x = \frac{e^{ix}-e^{-ix}}{2i}$

$\boxed{\cos^2 x + \sin^2 x = 1}$

Derivative of Hyperbolic Functions

$\boxed{\sinh x = (e^{x} - e^{-x})/2.}$ $\boxed{\cosh x = (e^{x} + e^{-x})/2.}$ $\boxed{\cosh^2 x - \sinh^2 x = 1}$

$\sinh^2 x = \frac{1}{4}\left(e^{2x} + e^{-2x} - 2\right)$

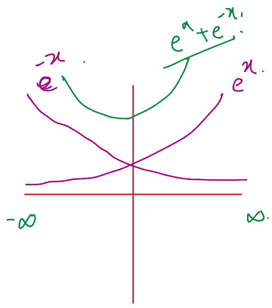
$\cosh^2 x = \frac{1}{4}\left(e^{2x} + e^{-2x} + 2\right)$

Assume arcsinhx = y, then we have x = sinh y.

$\operatorname{arc}\cos x = y$

$\Downarrow$

$x = \cos y$



Function	Derivative	Domain
sinhx	coshx	$-\infty < x < \infty$
coshx	sinhx	$-\infty < x < \infty$
tanhx	$\operatorname{sech}^2 x$	$-\infty < x < \infty$
cothx	$-\operatorname{csch}^2 x$	$x \neq 0$
sechx	-sechx tanhx	$-\infty < x < \infty$
cschx	-cschx cothx	$x \neq 0$
arcsinhx	$1/\sqrt{(x^2+1)}$	$-\infty < x < \infty$
arccoshx	$1/\sqrt{(x^2-1)}$	$x > 1$
arctanhx	$1/(1-x^2)$	$\lvert x\rvert < 1$
arccothx	$1/(1-x^2)$	$\lvert x\rvert > 1$
arcsechx	$-1/x\sqrt{(1-x^2)}$	$0 < x < 1$
arccschx	$-1/\lvert x\rvert\sqrt{(1-x^2)}$	$x \neq 0$

$y = \sinh x = \frac{e^{x}-e^{-x}}{2}$

$\frac{dy}{dx} = \frac{1}{2}\left[e^{x} - (-e^{-x})\right]$

$= \frac{1}{2}\left(e^{x} + e^{-x}\right)$

$= \cosh x$

$\coth x = \frac{\cosh x}{\sinh x}$

$= \frac{e^{x}+e^{-x}}{e^{x}-e^{-x}} \neq 0$

$e^{x} \neq e^{-x}$

$e^{2x} \neq 1 \qquad 2x \neq 0$

$x \neq 0$

$\cosh^2 y - \sinh^2 y = 1$

$\cosh^2 y = 1 + \sinh^2 y$

$= 1 + x^2$

$y = \operatorname{arcsinh} x \Rightarrow \boxed{x = \sinh y}$

$\frac{d}{dx}(x) = \frac{d}{dx}(\sinh y) = \frac{d}{dy}(\sinh y) \cdot \frac{dy}{dx}$

$1 = \cosh y \cdot \frac{dy}{dx} \Rightarrow \boxed{\frac{dy}{dx} = \frac{1}{\cosh y} = \frac{1}{\sqrt{1+x^2}}}$

Leibniz Rule

$$(uv)^n = \sum_{r=0}^{n} {}^nC_r\, u^{n-r} v^r.$$

$$(f(x)\,.\,g(x))^n = \sum_{r=0}^{n} {}^nC_r\, f^{(n-r)}(x).g^r(x)$$

Product rule $y = uv$. $u = f(x)$ $v = g(x)$

$y' = u'v + v'u$.

$y' = u'v + v'u$.

$y^2 = (u'v + v'u)' = (u'v)' + (v'u)' = u^2v + u'v' + v^2u + v'u'$

$\boxed{= u^2v + 2u'v' + v^2u}$

$y^3 = (u^2v + 2u'v' + v^2u)'$

$= (u^2v)' + 2(u'v')' + (v^2u)' = u^3v + u^2v' + 2(u^2v' + u'v^2) + v^3u + v^2u'$

$= \boxed{u^3v + 3u^2v' + 3u'v^2 + v^3u}$.

$(a+b)^2$, $(a+b)^3$. $\Rightarrow$ $(a+b)^n = a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \dots + b^n$.

$= \sum_{r=0}^{n} {}^nC_r\, a^{n-r} b^r$.

$\frac{d}{dx} e^{nx} = ne^{nx}$.

$e^{ax+b} = e^{ax} . e^b$.

$= e^b e^{ax}$

$y = e^{ax+b} \sin x$.
(u, v.)

$u' = a e^{ax} . e^b = a e^{ax+b}$. $v' = \cos x$.

$y'' = \sum_{r=0}^{2} {}^2C_r\, u^{2-r} v^r$. $= {}^2C_0 u^2 v^0 + {}^2C_1 u'v' + {}^2C_2 u^0 v^2$

$= u^2 + 2u'v' + v^2$.

$= e^{2(ax+b)} + 2a e^{ax+b} \cos x + \sin^2 x$.

$y = (ax+b)^n \sin x$.
(u, v.)

$y' = u'v + v'u = n(ax+b)^{n-1} a \sin x + (ax+b)^n \cos x$.

$= (ax+b)^{n-1} [na \sin x + (ax+b)\cos x]$

$y'' = u^2 + 2u'v' + v^2$.

$= (ax+b)^{2n} + 2na(ax+b)^{n-1} \cos x + \sin^2 x$.

$$y'' = u^2 + 2u'v' + v^2.$$
$$= (ax+b)^{2n} + 2na(ax+b)^{n-1}\cos x + \sin^2 x.$$

L'Hospital's Rule Formula

if $\lim_{x \to c} f(x) = \lim_{x \to c} g(x) = 0$ or $\pm\infty$, $g'(x) \neq 0$ for all x in I with $x \neq c$, and

$\lim_{x \to c} \frac{f'(x)}{g'(x)}$ exists, then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}.$$

$$\lim_{x \to 0} \frac{x(e^x - 1)}{1 - \cos x}$$

- Reduction formulae, derivations and illustrations of reduction formulae of the type $\int \sin^n x\, dx$, $\int \cos^n x\, dx$, $\int \tan^n x\, dx$, $\int \sec^n x\, dx$, $\int (\log x)^n dx$,

$$\int \sin^5 x \cos^3 x dx$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$ Taylor series.

$$\boxed{i = \sqrt{-1}}$$

$$i^2 = -1$$

$$i^3 = i^2 \cdot i = -i$$

$$i^4 = 1$$

$$e^{i\theta} = 1 + \frac{i\theta}{1!} + \frac{i^2\theta^2}{2!} + \frac{i^3\theta^3}{3!} + \frac{i^4\theta^4}{4!} + \cdots$$

$$= 1 + \frac{i\theta}{1!} - \frac{\theta^2}{2!} - \frac{i\theta^3}{3!} + \frac{\theta^4}{4!} + \cdots$$

$$= \boxed{\left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + \cdots\right)} + i\boxed{\left(\frac{\theta}{1!} - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots\right)}$$

$$= \underbrace{\left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + \cdots\right)}_{\cos\theta} + i \underbrace{\left(\frac{\theta}{1!} - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots\right)}_{\sin\theta}$$

$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta := \text{cis}\,\theta}$$

$$e^{-i\theta} = \cos(-\theta) + i\sin(-\theta) = \cos\theta - i\sin\theta.$$

$$\boxed{e^{-i\theta} = \cos\theta - i\sin\theta}$$

$$e^{i\theta} + e^{-i\theta} = 2\cos\theta.$$

$$\left[\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}\right] \checkmark$$

$$e^{i\theta} - e^{-i\theta} = 2\sin\theta\, i$$

$$\left[\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}\right] \checkmark$$