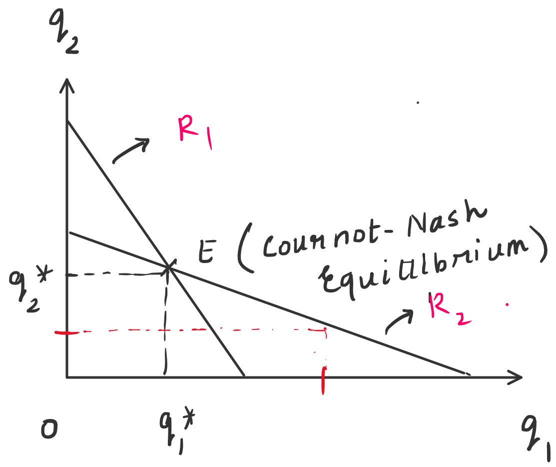
For finding $q_1^*, q_2^*$ solve:

$R_1: \; 2b \cdot q_1 + b \cdot q_2 = (a - c_1) \; \ldots (i)$

$R_2: \; b q_1 + 2b q_2 = (a - c_2) \; \ldots (ii)$




$$q_1^* = \frac{\begin{vmatrix} (a-c_1) & b \\ (a-c_2) & 2b \end{vmatrix}}{\begin{vmatrix} 2b & b \\ b & 2b \end{vmatrix}} \qquad q_2^* = \frac{\begin{vmatrix} 2b & (a-c_1) \\ b & (a-c_2) \end{vmatrix}}{\begin{vmatrix} 2b & b \\ b & 2b \end{vmatrix}}$$

$R_1: 2b q_1 + b q_2 = (a - c_1)$

$2b \cdot dq_1 + b \cdot dq_2 = 0$

$\left| \frac{dq_2}{dq_1} \right| = \frac{2b}{b} = 2$.

$$q_1^* = \frac{a - 2c_1 + c_2}{3b}. \qquad q_2^* = \frac{a - 2c_2 + c_1}{3b}.$$

$$p^* = a - b(q_1^* + q_2^*) = a - \cancel{b}\frac{[2a - c_1 - c_2]}{3\cancel{b}}$$

$$= \frac{3a - 2a + c_1 + c_2}{3} = \frac{a + c_1 + c_2}{3}.$$

$q_1^* > 0$ iff $a > (2c_1 - c_2)$

$q_2^* > 0$ iff $a > (2c_2 - c_1)$

$\therefore \; q_1^* > 0$ & $q_2^* > 0$ iff $a > \operatorname{Max}\{2c_1 - c_2, \, 2c_2 - c_1\}$.

(ii) If $c_1 = c_2 = c > 0$. Find the $\pi$ of both the firms & plot the reactions fns.

$R_1: \; 2b \cdot q_1 + b q_2 = (a - c)$

$R_2: \; b \cdot q_1 + 2b q_2 = (a - c)$.

$q_1^* = \frac{a - c}{3b} = q_2^*$

$\pi_1^* = p^* q_1^* - c \cdot q_1^*$

$= (p^* - c) \cdot q_1^*$

$= \left( \frac{a + 2c}{3} - c \right) \left( \frac{a - c}{3b} \right)$

$= \left( a - c \right) \left( a - \right.$

$$q_1^* = \frac{a-c}{3b} = q_2^*$$

$$p^* = \frac{a+2c}{3}.$$

$$\pi_1^* = \frac{(a-c)^2}{9b} = \pi_2^*$$

$$= \left(\frac{a-c}{3}\right)\left(\frac{a-c}{3b}\right)$$

$$= \frac{(a-c)^2}{9b}.$$

Note: Cournot price level < Monopoly price level.
> competitive price level.

(iii) If both the firms join to form a cartel, find the output levels of the firms. [Assume $c_1 = c_2 = c$.].

Cartel: Two or firms joining together to behave as a single entity.
Optimal production levels will be decided based on joint profit maximization.

Firm I: $R_1 = p.q_1$ , $C_1 = c.q_1$ .
Firm II: $R_2 = p.q_2$ , $C_2 = c.q_2$

$\therefore$ Joint profit $\pi = R_1 + R_2 - C_1 - C_2$.

$$\pi = p.q_1 + p.q_2 - c.q_1 - c.q_2$$

$$\pi = p(q_1 + q_2) - c(q_1 + q_2).$$

$$\pi = [a - b(q_1+q_2)].(q_1+q_2) - c.(q_1+q_2)$$

$\therefore$ Firm I chooses $q_1$ to max $\pi$.
Firm II chooses $q_2$ to max $\pi$.

F-I: $\frac{\partial \pi}{\partial q_1} = 0 \Rightarrow -b(q_1+q_2) + a - b(q_1+q_2) - c = 0$ .... (i)

F-I : $\frac{\partial \pi}{\partial q_1} = 0 \Rightarrow -b(q_1+q_2) + a - b(q_1+q_2) - c = 0$ ---- (i)

F-II : $\frac{\partial \pi}{\partial q_2} = 0 \Rightarrow -b(q_1+q_2) + a - b(q_1+q_2) - c = 0$ --- (ii)

(i) : $(a-c) = 2b(q_1+q_2)$

$$\boxed{q_1 + q_2 = \left(\frac{a-c}{2b}\right)} \Rightarrow$$ combination of $(q_1, q_2)$ that should be produced for joint $\pi$-maximization.

HW : $q_1 = q_2 = \left(\frac{q_1+q_2}{2}\right)$ ?