Composite Mean in case 2 sets of data

set 1 : n_1 $\bar{x}_1$, σ_1 Total obs : $N = n_1 + n_2$

set 2 : n_2 $\bar{x}_2$ σ_2 composite group : σ

Grand mean $\bar{X} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1+n_2}$

or, $N\bar{X} = n_1\bar{x}_1 + n_2\bar{x}_2$

for more than 2 sets

$$\boxed{\sum_i n_i \bar{X} = \sum_i n_i \bar{x}_i}$$

Composite variance, $\sigma^2 = \dfrac{n_1\sigma_1^2 + n_2\sigma_2^2}{N} + \dfrac{n_1 d_1^2 + n_2 d_2^2}{N}$

Here $d_1 = \bar{x}_1 - \bar{x}$

$d_2 = \bar{x}_2 - \bar{x}$

For more than 2 sets of obs.

$$N\sigma^2 = \sum_i n_i\sigma_i^2 + \sum n_i d_i^2$$

$$\boxed{\text{where } d_i = \bar{x}_i - \bar{x}}$$

or we can use the following formula after Simplifying the above formula in case two sets of observations:

$$\boxed{N\sigma^2 = n_1\sigma_1^2 + n_2\sigma_2^2 + \frac{n_1 n_2}{N}(\bar{x}_1 - \bar{x}_2)^2}$$

Q In a batch of $\overset{N}{10}$ children, the I.Q. of a dull boy is 36 below the average I.Q. of the other children. Show that the standard deviation of I.Q. for all the children (σ) cannot be less than 10.8. If this standard deviation is actually 11.4, determine what the s.d. will be, when the dull boy is left out.

Soln

Composite Group

$N = 10$

grand mean $= \bar{x}$

composite s.d $= \sigma$

Let $n_1 \rightarrow$ no. of other students $= 9$

$n_2 \rightarrow$ no. of dull student $= 1$

Let 'a' be the avg I.Q of other students

ie $\bar{x}_1 = a$, σ_1

$\therefore \bar{x}_2 = a - 36$ (avg I.Q of dull boy)

$\sigma_2 = 0$

$\therefore \bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{N}$

$= 9a + (a-36)$

$$= \frac{9a + (a-36)}{10} \qquad \therefore \bar{x}_2 \qquad \sigma_2 = 0$$

$$= \frac{10a - 36}{10}$$

$$\bar{x} = a - 3.6$$

Here, $d_1 = \bar{x}_1 - \bar{x} = a - (a - 3.6) = 3.6$

$d_2 = \bar{x}_2 - \bar{x} = (a-36) - (a-3.6)$

$= -32.4$

using the formula of composite variance,

$$N\sigma^2 = n_1\sigma_1^2 + n_2\sigma_2^2 + n_1 d_1^2 + n_2 d_2^2$$

$$10\sigma^2 = 9\sigma_1^2 + 0 + 9(3.6)^2 + (-32.4)^2$$

$$\sigma^2 = 0.9\sigma_1^2 + 116.64$$

$$\sigma^2 \geq 116.64 \qquad (\because \sigma_1^2 \geq 0)$$

$$\sigma \geq \sqrt{116.64}$$

or, $\sigma \geq 10.8$ (Proved)

If $\sigma = \underline{\underline{11.4}}$ then $\sigma_1 = ?$

$$\sigma^2 = 0.9\sigma_1^2 + 116.64$$

or, $(11.4)^2 = 0.9\sigma_1^2 + 116.64$

or, $129.96 - 116.64 = 0.9\sigma_1^2$

or, $129.96 - 116.64 =$ [illegible]

or, $\delta_1^2 = \dfrac{129.96 - 116.64}{0.9}$

or, $\delta_1^2 = 14.8$

$\therefore \delta_1 = \sqrt{14.8} = 3.847$ (ans)

Q2. For a group of 50 boys the mean score and the s.d of scores on a test are 59.5 and 8.38.
For a group of 40 girls the same results are 54.0 and 8.23. So find the mean and S.D of combined group of 90 children.

$\bar{X} = 57.05$

$\bar{x}_1 = 59.5$

$n_1 = 50$

$\delta_1 = 8.38$

$\bar{x}_2 = 54.0$

$n_2 = 40$

$\delta_2 = 8.23$

$N = 90$

$\bar{X} = 57.05$

$$N\delta^2 = n_1\delta_1^2 + n_2\delta_2^2 + n_1 d_1^2 + n_2 d_2^2$$

$$90 \times \delta^2 = 50 \times (8.38)^2 + 40 \times (8.23)^2$$

$[59.5 - 57.05]^2$

$$90 \times \sigma^2 = 50 \times (8.38)^2 + 40 \times (8.23)^2 + 50 \times [59.5 - 57.05]^2 + 40 \times [54 - 57.05]^2$$

$$\sigma^2 = 76.59 \quad \therefore \sigma = \sqrt{76.59} = 8.75 \text{ (ans)}.$$

Q3 - The mean and s.d of 20 items is found to be 10 and 2 resp. At the time of checking it was found that one item 8 was incorrect. Calculate the mean & s.d if

(i) the wrong item is omitted.

& if (ii) it is replaced by 12.

$\bar{x} = 10 \qquad \sigma = 2 \qquad n = 20$

or $\frac{1}{n} \Sigma x_i = 10$

or, $\Sigma x_i = 10 \times n$

$= 10 \times 20$

$\boxed{\Sigma x_i = 200}$

$\sigma^2 = 4$

$\frac{1}{n} \Sigma x_i^2 - \bar{x}^2 = 4$

$\frac{1}{20} (\Sigma x_i^2) = 4 + 10^2$

$\Sigma x_i^2 = 104 \times 20$

$\boxed{\Sigma x_i^2 = 2080}$

(i) when the incorrect obs 8 is omitted.

Now $n = 19$

corrected $\Sigma x_i = 200 - 8 = 192$ ✓

Now $n = 19$

corrected $\Sigma x_i = 200 - 8 = 192$ ✓

corrected $\Sigma x_i^2 = 2080 - 8^2 = 2016$ ✓

$\therefore$ corrected mean $= \frac{1}{19} \times 192 = 10.105$

corrected $\sigma^2 = \frac{1}{19} \times 2016 - (10.105)^2$

$= 3.9889$

$\therefore \sigma = \sqrt{3.9889} = \underline{1.997 \text{ (cm)}}$.

$\frac{1}{n}\Sigma$ E

mean $\frac{1}{n}\Sigma x_i$

$E(x)$

$\frac{1}{n}\Sigma x_i^2 - \left(\frac{1}{n}\Sigma x\right)^2$

$E(x^2) - \{E(x)\}^2$

x	$P(x)$

$E(x) = ?$

$V(x) = ?$