GRAPHS 23 11 23

Thursday, November 23, 2023 5:28 PM

Graphs & formations

→ e^x, $\log x$, $|x|$

→ $\sin x$, $\cos x$, $\tan x$

Transformations ⇒ $e^x \to e^{2x}$

$e^x \Rightarrow e^{x/2}$

→ $|x| \longrightarrow |x+2|$

$|x| \longrightarrow |x-2|$

⇒ Graph of $y = f(x)$

$|y| = |f|x||$

1. The number of real solutions of the equation $e^{|x|}-|x|=0$ is

(a) 0 (b) 1 (c) 2 (d) None of these

2. The number of real solutions of the equation $3^{-|x|}-2^{|x|}=0$ is

(a) 0 (b) 1 (c) 2 (d) 3

3. The number of solutions of $3^{|x|}=|2-|x||$ is

(a) 0 (b) 2 (c) 4 (d) infinite

4. The total number of solutions of the equation $|x-x^2-1|=|2x-3-x^2|$ is

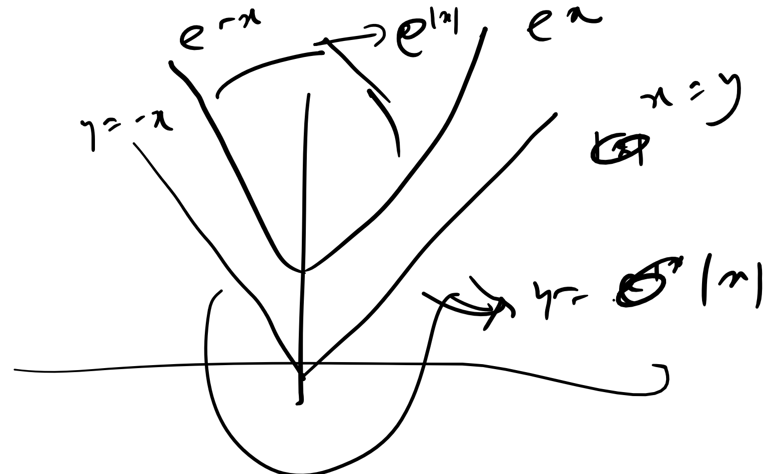
(a) 0 (b) 1

(c) 2 (d) infinitly many

① $e^{|x|}-|x|=0$

$e^{|x|}=|x|$

No intersection

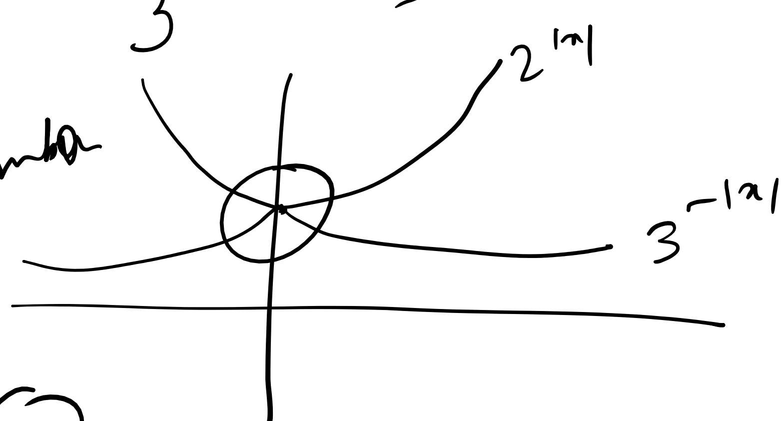


No sol^n So, no common sol^n

② $3^{-|x|}=2^{|x|}$

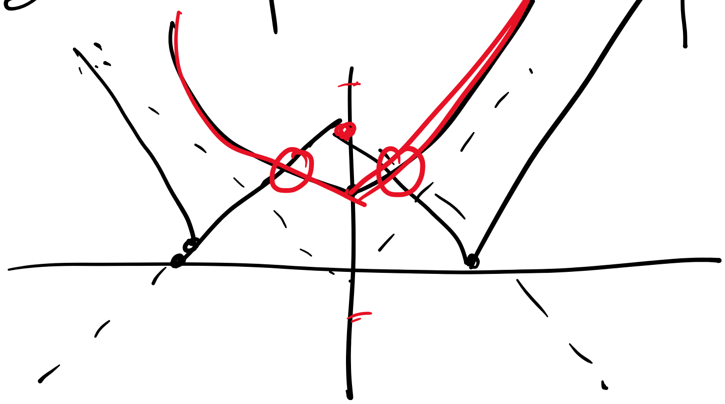
$a^{|x|}$

1 intersection



Sol^n → 1

(3) $3^{|x|} = |2-|x||$



$3^4 = |2-4|$

$81 = |-2|$

$\neq 2$ ✗

$3^3 = 2-3$ ✗

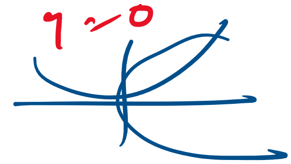
$3^x = 2-x$

$3^1 = 2-1$

$3 = 1$ ✗

$3^2 = 2-2$

$9 = 0$



(4) $\underset{f(x)}{|x - x^2 - 1|} = \underset{g(x)}{|2x - 3 - x^2|}$

$y = x - x^2 - 1$

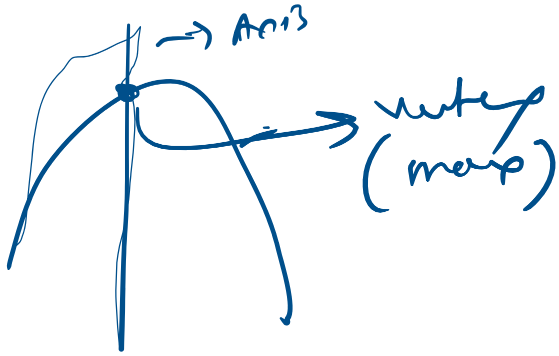
$\Rightarrow (x - \frac{1}{2})^2 = -(y + \frac{3}{4})$ ← Parabola

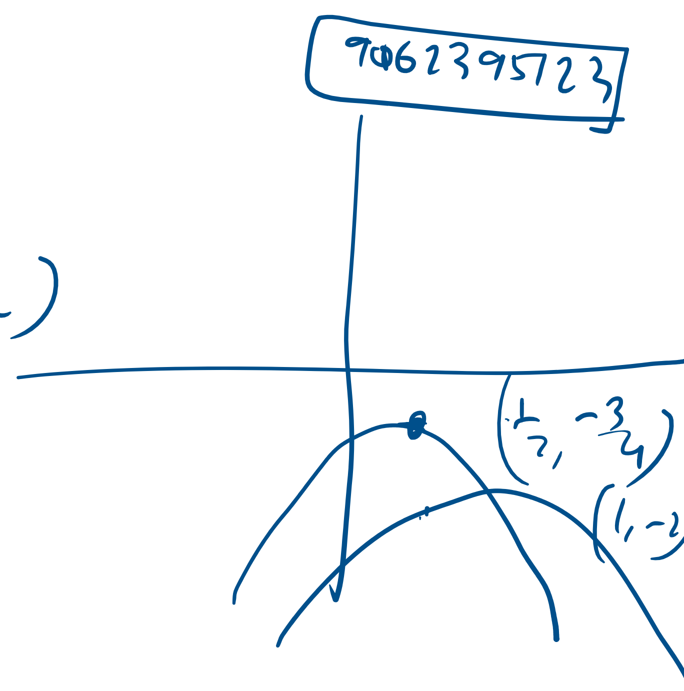
vertex $(-\frac{1}{2}, -\frac{3}{4})$

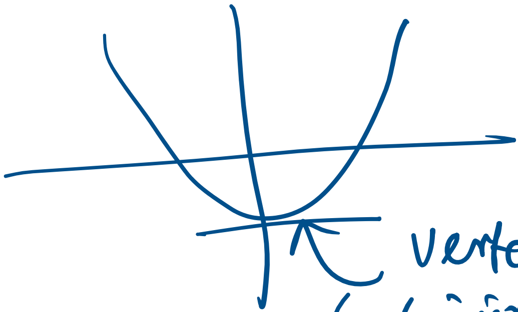
and, $g(x) = 2x - 3 - x^2$

$(x-1)^2 = -(y+2)$

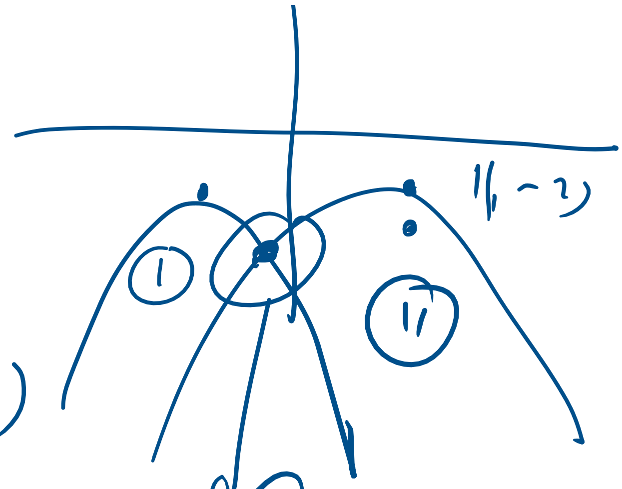
Vertex $(1, -2)$







vertex (minimum value)



$(1, -2)$

I

II

0 pt of intersection

$(m^2 + m)$

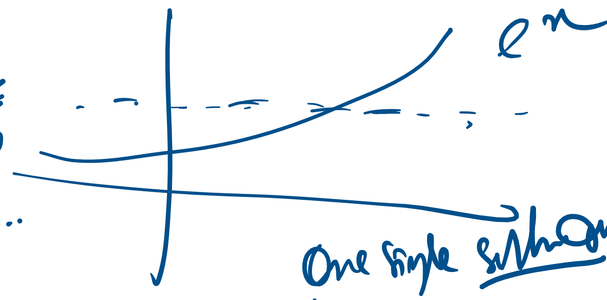
$\boxed{m < -1}$

st line parallel to x axis..

5. The equation $e^x = m(m+1)$, $m < -1$ has
   (a) no real root
   (b) exactly one real root
   (c) two real roots
   (d) None of the above

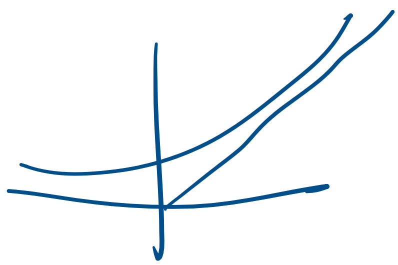
6. The number of real solutions of the equation $1 - x = [\cos x]$ is
   (a) 1 (b) 2
   (c) 3 (d) 4

7. The number of roots of the equation $1 + 3^{x/2} = 2^x$ is
   (a) 0
   (b) 1
   (c) 2
   (d) None of the above



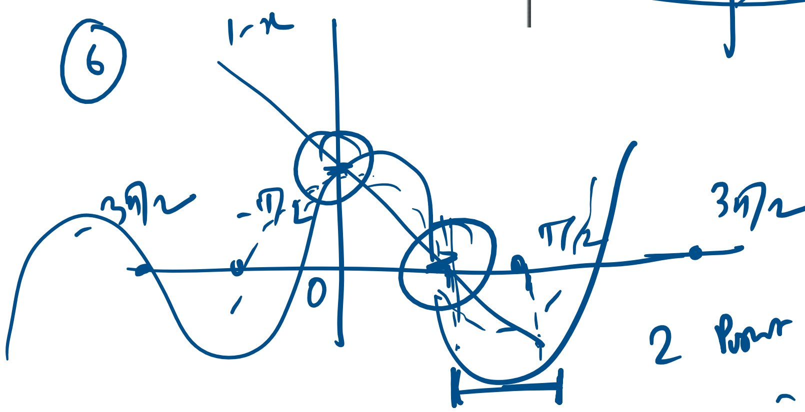
e^x

One single solution..



$e^x = x$ 0

6



$1-x$

$3\pi/2$ $-\pi/2$ 0 $\pi/2$ $3\pi/2$

2 Points of Possible intersection

$\pi/2$ $5\pi/2$ 0

$|x|$ 0 to 1

0 1 to 2

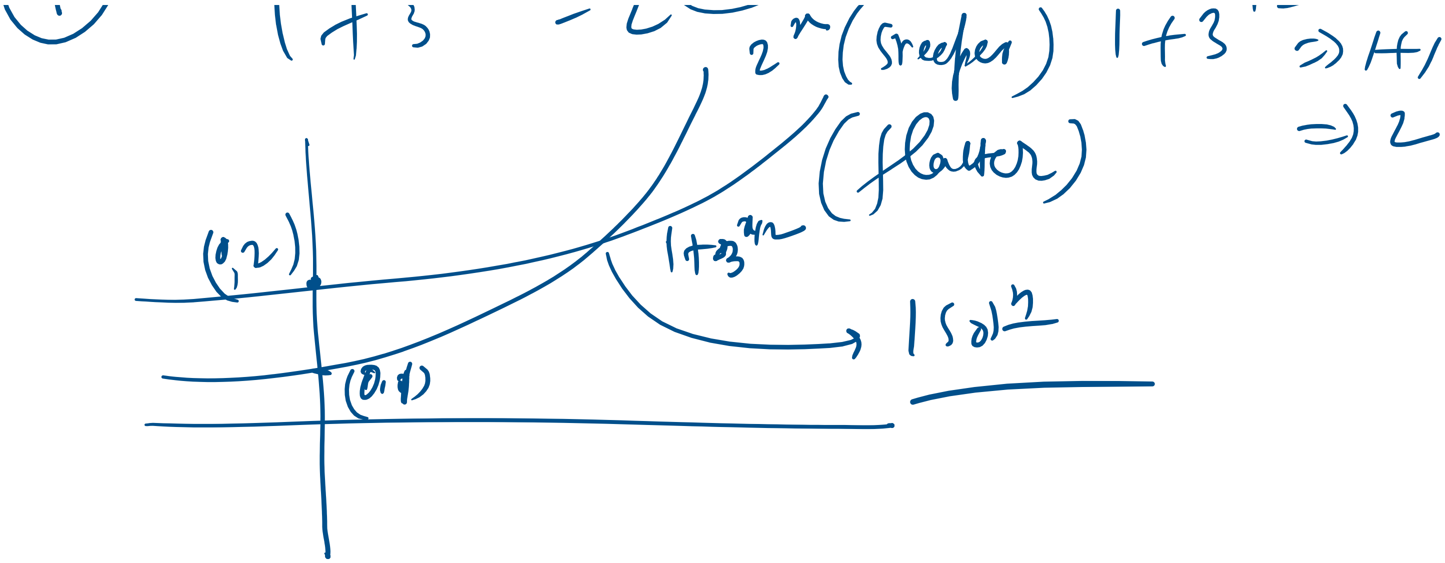
2 sol^n

7

$1 + 3^{x/2} = 2^x$

2^x $2^0 = 1$

, 2^x (steeper) $1 + 3^{0/2} \Rightarrow 1+1$



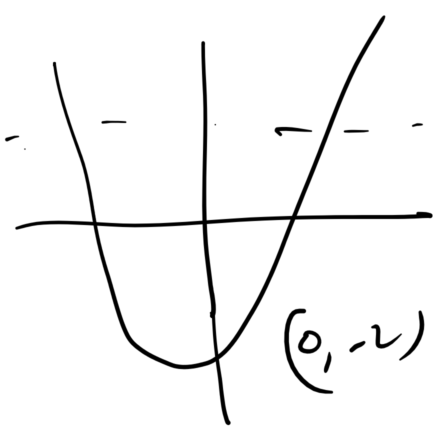
8. The equation $x^2 - 2 = [\sin x]$, where $[\cdot]$ denotes the greatest integer function, has
(a) infinity many roots
(b) exactly one integer root
(c) exactly one irrational root
(d) exactly two roots

fn

9. Consider the function $f(x) = \begin{cases} x - [x] - \frac{1}{2}, & \text{if } x \notin I \\ 0, & \text{if } x \in I \end{cases}$,
where $[\cdot]$ denotes greatest integer function and I is the set of integers, then $g(x) = \max\{x^2, f(x), |x|\}, -2 \le x \le 2$ is defined as
(a) $x^2, -2 \le x \le -1$
(b) $1 - x, -1 < x \le -\frac{1}{4}$
(c) $\frac{1}{2} + x, -\frac{1}{4} < x < 0$
(d) $1 + x, 0 \le x < 1$

10. If $f(x)$ is defined on $[-2, 2]$ and is given by
$f(x) = \begin{cases} -1, & -2 \le x < 0 \\ x - 1, & 0 < x \le 2 \end{cases}$ and $g(x) = f|x| + |f(x)|$,
$g(x)$ is defined as
(a) $-x, -2 \le x \le 0$
(b) $x, -2 \le x \le 0$
(c) $0, 0 < x \le 1$
(d) $2(x-1), 1 < x \le 2$

8



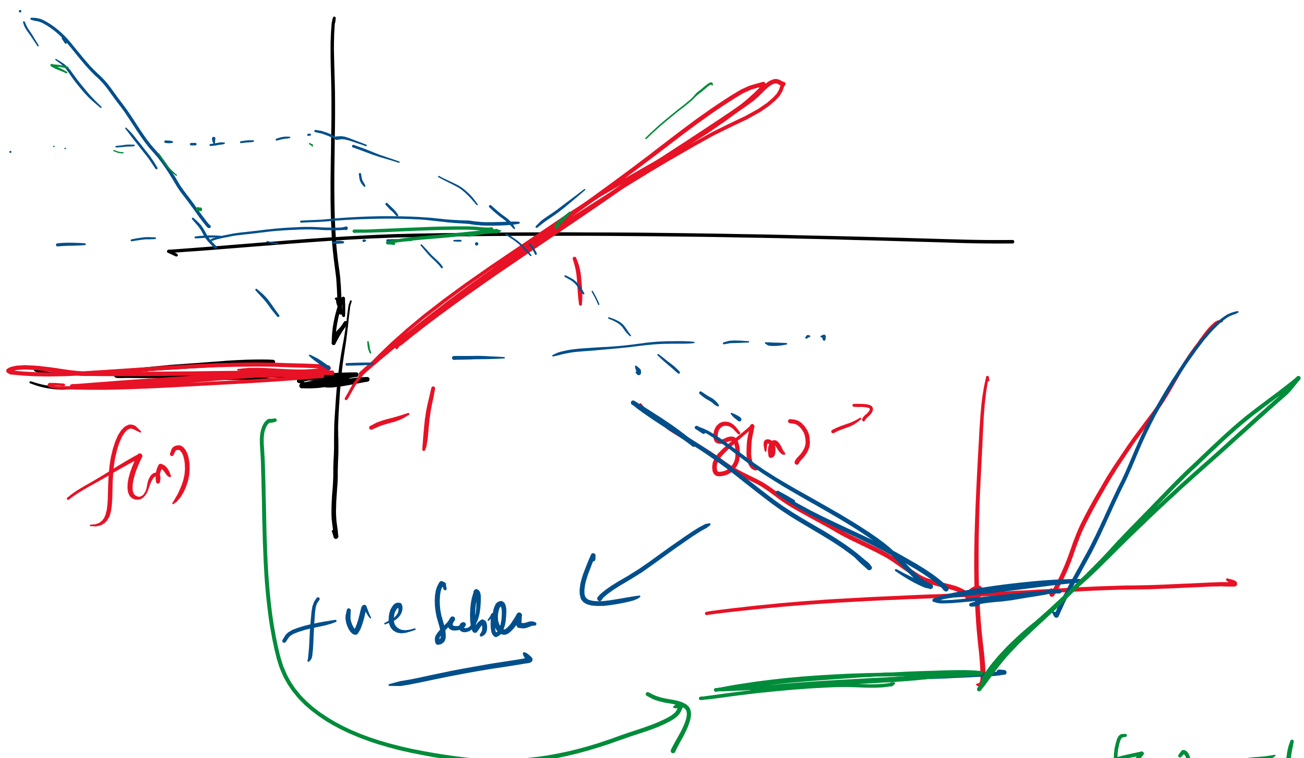
$f(x) = [\ln x]$
$[\sin x]$
$[\sec x]$

10

$f(x) = \begin{cases} -1 & -2 \le x < 0 \\ x - 1 & 0 < x \le 2 \end{cases}$

$g(x) = f(|x|) + |f(x)|$

$$g(x) = f(|x|) + |f(x)|$$



$f(x) = -1 \quad -2 \le x \le 0$

$\quad\;\; = x - 1 \quad 0 \le x \le 2$

$$g(x) \Rightarrow -x \qquad -2 \le x \le 0$$

$$\Rightarrow 0 \qquad 0 < x < 1$$

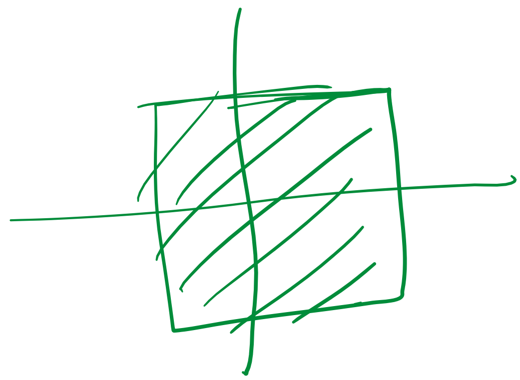
$$\Rightarrow 2(x-1) \qquad 1 < x \le 2$$

2 ways to do it

(i) enter some via diagram

(ii) dont remember

some..



Derivatives and Graphs

151 $y = x^3 - 3x$

$$\frac{dy}{dx} = 3x^2 - 3 = 0 \Rightarrow 3x^2 = 3 \Rightarrow x = \pm 1$$

$$\frac{d^2y}{dx^2} = 6y$$

at $x = +1$ $\quad \frac{d^2y}{dx^2} > 0$. mini

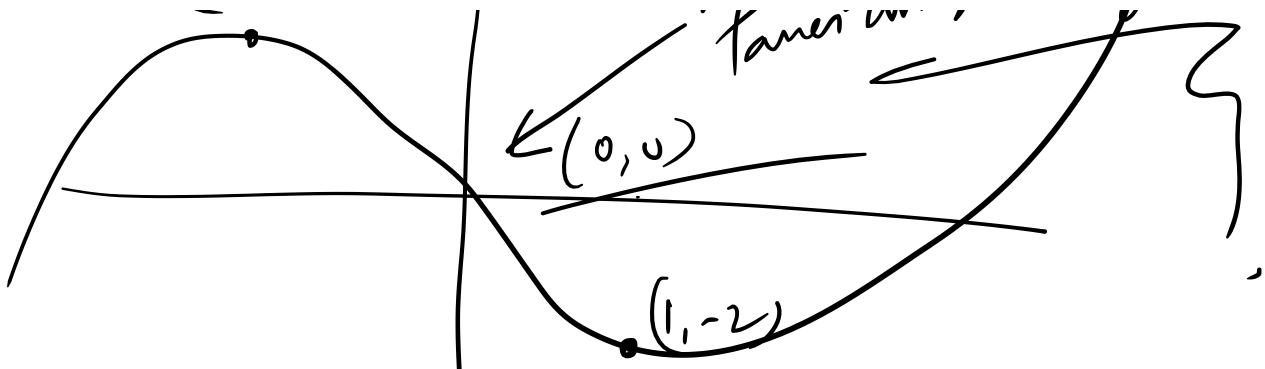
at $x = -1$ $\quad \frac{d^2y}{dx^2} < 0$ max

Now, $y = x^3 - 3x$

@ $x = 1 \quad y = -2 \quad (1, -2)$

$x = -1 \quad y = 2 \quad (-1, 2)$

$(-1, 2)$ Passes through origin



How to know $y = x^3 - 3x$

@ $x = 0$ $y = 0^3 - 3 \cdot 0$
$= 0$

Uses of derivative

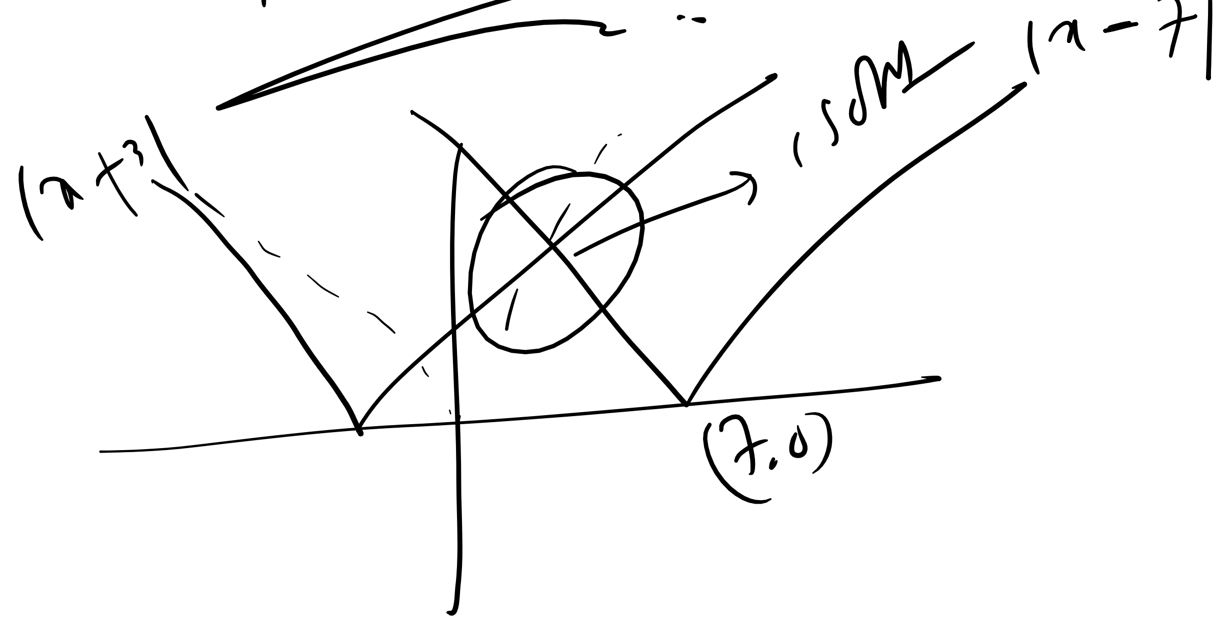
Find any curve
Solve the equation $\frac{d^2y}{dx^2}, \frac{dy}{dx} = 0$

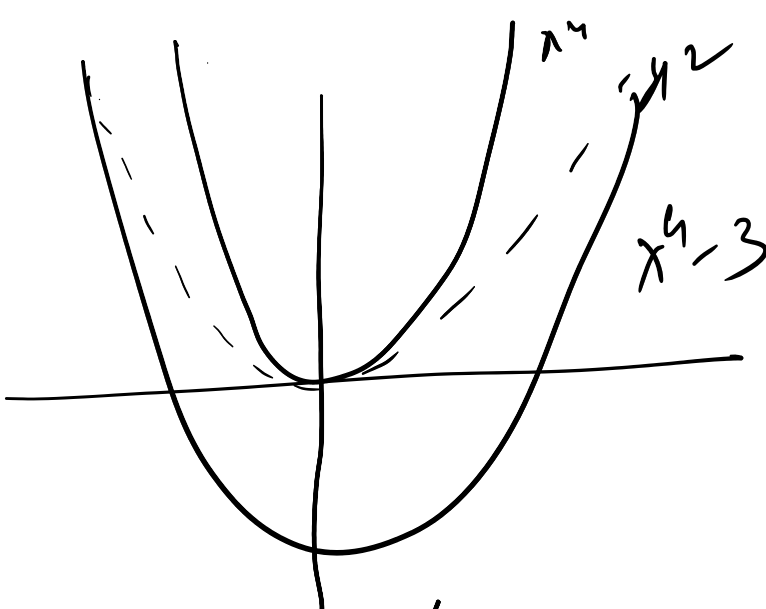
Find the max, min
check whether passes through origin.

Draw $y = |x - 7|$

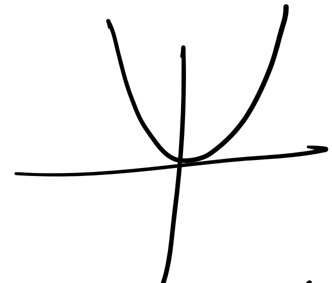
$y = |x+3|$

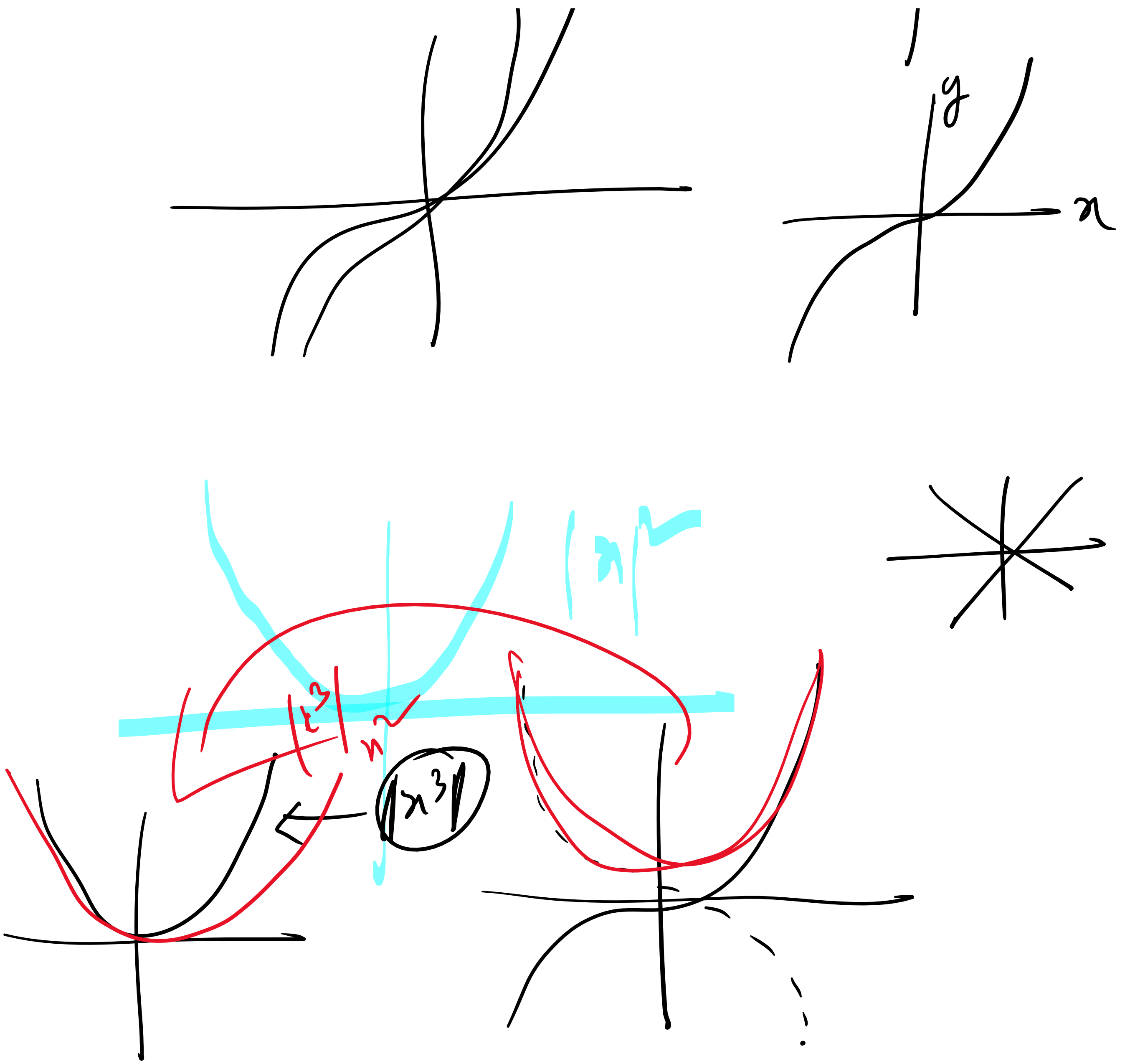
find sol^n points





x^4




y
x
|x³|