STAT 11-1-2024

Thursday, January 11, 2024 6:43 PM

Statistics ..

- Probability
- Distributions

Theory

Integrals — 2D, 3D

Beta & Gamma

P/C

WLLN

SLLN

Sample Space

P →

IID

Converge to mean → almost surely ( p → 1 )

Sample ∞

Keep on taking averages of independent tossing of a coin

→ Ave → Expected value

1/2

$S \to \infty$ 1, 2, 3, 4 . . . .

$S_{n_1} = 2n$ 2, 4, 6, 8 . . .

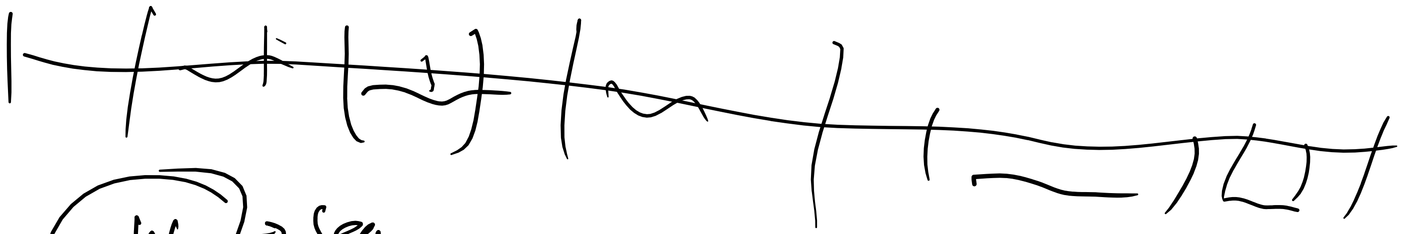
$S_{n_2} = n$

SLLN → IID

Ave → Converges to mean almost surely

Are → Converge to ...

for every Subsequence



W → Seqns

S → Seqt Subseqns

$a_n > a_{n+1}$

$a_n \geq a_{n+1}$

Wider Sense WLLN

MATHEM...

iid R.V

0. A fair die is rolled four times independently. For $i = 1, 2, 3, 4$, define

$$Y_i = \begin{cases} 1, & \text{if 6 appears in the } i^{\text{th}} \text{ throw,} \\ 0, & \text{otherwise.} \end{cases}$$ 1/6

5/6

Then $P(\max\{Y_1, Y_2, Y_3, Y_4\} = 1)$ equals ________.

$Y_i = \text{Bin}\left(\overset{n}{1}, \overset{p}{\tfrac{1}{6}}\right)$

multiple max

$-\infty$ $\infty$

$y = \min\{a, b\}$ $a = b$

max val of $y$ ??

$\cancel{\frac{dy}{da}}$ =

$$Y_i = \text{Bin}\left(1, \tfrac{1}{6}\right)$$

$i = 1, 2, 3, 4$

$$P\left(\max\{Y_1, Y_2, Y_3, Y_4\} = 1\right)$$

$$= 1 - P\left(\max(Y_1, Y_2, Y_3, Y_4) = 0\right)$$

$$= 1 - P\left(Y_i = 0,\ i = 1, 2, 3, 4\right)$$

$$= 1 - \left[P(Y_1 = 0)\right]^4$$

$$= 1 - \left(\tfrac{5}{6}\right)^4 = 1 - \frac{625}{1296} = \frac{671}{1296}$$

$$= 0.51$$

5 7

$y = \min\{5, 7\}$

$y = 5$

$\min y = \text{iff}$ $a > b$

$H_0 \to$ null hypothesis $\to$ tendency to prove / Given statement.

Uniform

2. Let $X$ be a sample observation from $U(\theta, \theta^2)$ distribution, where $\theta \in \Theta = \{2, 3\}$ is the unknown parameter. For testing $H_0 : \theta = 2$ against $H_1 : \theta = 3$, let $\alpha$ and $\beta$ be the size and power, respectively, of the test that rejects $H_0$ if and only if $X \geq 3.5$. Then $\alpha + \beta$ equals ________.

PDF of $X$ $\quad f(x|\theta) = \dfrac{1}{\theta^2 - \theta}$ if $\theta < x < \theta^2$

$= 0$ otherwise

Value of $(\alpha + \beta)$

$$\alpha + \beta = P(\text{Reject } H_0 | H_0) + P(\text{Reject } H_0 | H_1)$$

size $\qquad$ Power

$$= P(X \geq 3.5 | \theta = 2) + P(X \geq 3.5 | \theta = 3)$$

$$= \int_{3.5}^{\infty} f(x|2)\,dx + \int_{3.5}^{\infty} f(x|3)\,dx$$

$$= \int_{3.5}^{4} f(x|2)\,dx + \int_{3.5}^{9}$$

$$\int^{4} \tfrac{1}{2}\,dx + \int_{3.5}^{9} \tfrac{1}{\ }\,dx$$

Thank Both

$$= \int_{3.5}^{4} \frac{1}{2} dx + \int_{3.5}^{9} \frac{1}{6} dx$$

$$= \frac{4-3.5}{2} + \frac{9-3.5}{6}$$

$= 7/6 = 1.167$

$y = x_1^2 x_2^2$

$x_1 + x_2 \le 10$

$\boxed{9062395123}$

3 Let $X$ be a random variable with the probability mass function (PMF)

$$P(X = n) = \begin{cases} \frac{1}{10}, & \text{if } n = 1, 2, \ldots, 10, \\ 0, & \text{otherwise.} \end{cases}$$

Then $E(\max\{X, 5\})$ equals ______.

PDF = Contin var $\int$

PMF = Discr var $\sum$

$$E\{\max\{X,5\}\} = \sum_{n=1}^{10} (\max(X,5)) P(X=n)$$

for 1 to 5 → 5

for 6 to 10 → X

$$= \sum_{n=1}^{10} \max(n,5) \frac{1}{10}$$

$$= \frac{1}{10}\left(\sum_{1}^{5} \max(n,5) + \sum_{n=6}^{10} \max(n,5)\right]$$

$$= \frac{1}{10}\left(\sum 5 + \sum n\right)$$

$$= \frac{1}{10}(25 + 40)$$

$= \underline{6.5}$

9062395123

$n < 30$

$n > 30$

4 Let $X_1, X_2, X_3$ and $X_4$ be i.i.d. discrete random variables with the probability mass function

$$P(X_1 = n) = \begin{cases} \frac{3^{n-1}}{4^n}, & \text{if } n = 1, 2, \ldots, \\ 0, & \text{otherwise.} \end{cases}$$

Then $P(X_1 + X_2 + X_3 + X_4 = 6)$ equals ______.

$$P(X_1 = n) = \frac{1}{4}\left(\frac{3}{4}\right)^{n-1} \quad \text{if } n = 1, 2 \ldots$$

$= 0$, other.

Which implies $X_i$ are iid geometric distribution parameter $1/4$

$p \in (0,1)$

Which implies $X_i$ are iid geometric ... $p \in (0,1)$

Since the sum of $n$ → iid GRV

then negative Binomial distribution

$Y = X_1 + X_2 + X_3 + X_4 \sim NB(4, \frac{1}{4})$ $\quad$ ⑥ $10 \to 6$ ?? $n = ?$

Then PMF of $Y$ is given by

$$P(Y=y) = {}^{y-1}C_{4-1} \left(\frac{1}{2}\right)^4 \cdot \left(\frac{3}{4}\right)^{y-4}, \quad y = 4, 5, \ldots$$

$= 0$, otherwise

the required probability is

$$P(Y=6) = {}^{6-1}C_{4-1} \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^{6-4} = {}^5C_3 \cdot \left(\frac{1}{4}\right)^4 \cdot \left(\frac{3}{4}\right)^2$$

⑤ Let $X$ and $Y$ be two discrete random variables with the joint moment generating function

$$M_{X,Y}(t_1, t_2) = \left(\frac{1}{3}e^{t_1} + \frac{2}{3}\right)^2 \left(\frac{2}{3}e^{t_2} + \frac{1}{3}\right)^3, \quad t_1, t_2 \in \mathbb{R}.$$

Then $P(2X + 3Y > 1)$ equals __________.

$$M_X(t) = M_{X,Y}(t, 0) = \left(\frac{1}{3}e^t + \frac{2}{3}\right)^2, \quad t \in \mathbb{R}$$

It's direct to see that

$X \sim Bin(2, \frac{1}{3}) \quad Y \sim Bin(3, \frac{2}{3})$

$M_{X,Y}(t_1, t_2) = M_X(t_1) M_Y(t_2)$

$t_1, t_2 \in \mathbb{R} \to X, Y$ are independent R.V.

Required Probability is

$$P(2X+3Y > 1) = 1 - P(2X+3Y \le 1)$$
$$= 1 - P(2X+3Y = 0 \text{ or } 2X+3Y = 1)$$
$$= 1 - [P(2X+3Y=0) + P(2X+3Y=1)]$$
$$= 1 - [P(X=0, Y=0) + 0]$$
$$= 1 - (1-\tfrac{1}{3})^2 (1-\tfrac{2}{3})^3$$
$$= 1 - (\tfrac{2}{3})^2 (\tfrac{1}{3})^3$$

$= 1 - \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^3$

$= 1 - \frac{4}{9} \times \frac{1}{27}$

$= 1 - \frac{4}{243} = \frac{239}{243}$

$= 0.98$

49. Let $X_1, X_2, \ldots, X_n$ be a random sample from the distribution with the probability density function

$$f(x) = \frac{1}{4}e^{-|x-4|} + \frac{1}{4}e^{-|x-6|}, \quad x \in \mathbb{R}.$$

Then $\frac{1}{n}\sum_{i=1}^{n} X_i$ converges in probability to ____________.