12 August 2023 12:57

Let $f : R \to R$ be defined as $f(x) = x^3 + x - 5$. If $g(x)$ is a function such that $f(g(x)) = x$, $\forall x \in R$, then $g'(63)$ is equal to ______.

(A) $\frac{1}{49}$ (B) $\frac{3}{49}$

(C) $\frac{43}{49}$ (D) $\frac{91}{49}$

$g'(63) = ?$

$f^{-1} f[g(x)] = f^{-1}(x)$.

$g(x) = f^{-1}(x)$ $\quad g \to f^{-1} \Rightarrow f \to g^{-1}$

$g[f(x)] = x$.

Differentiate both sides wrt x (applying Chain Rule)

$g'[f(x)] f'(x) = 1$

$f(x) = x^3 + x - 5$

$f'(x) = 3x^2 + 1$

$f'(4) = 49$.

$f(x) = 63$. $\quad x^3 + x - 5 = 63$.

$x^3 + x - 68 = 0 \Rightarrow x = 4$

$g'(63) f'(4) = 1$

$g'(63) \cdot 49 = 1$

Let $f : \mathbb{R} \to \mathbb{R}$ satisfy $f(x + y) = 2^x f(y) + 4^y f(x), \forall x, y \in \mathbb{R}$. If $f(2) = 3$, then $14 \cdot \frac{f'(4)}{f'(2)}$ is equal to ______.

$f(x+y) = 2^x f(y) + 4^y f(x)$

$x = 2$ $\quad f(y+2) = 2^2 f(y) + 4^y f(2)$

diff both sides wrt y.

$f'(y+2) = 2^2 f'(y) + f(2) 4^y \ln 4$. — ①

$\frac{d}{dy}(4^y) = 4^y \cdot \ln 4$

$f'(y+2) = 4 f'(y) + 3 \cdot 4^y \cdot 2\ln 2$.

$f'(y+2) = 4 f'(y) + 6 \cdot 4^y \ln 2$.

$y = 2$ $\quad f'(4) = 4 f'(2) + 96 \ln 2$.

$y = 2$ $\quad f(x+2) = 2^x f(2) + 4^2 f(x)$.

diff both side wrt x.

$f'(x+2) = f(2) 2^x \ln 2 + 4^2 f'(x)$ — ②

$f'(x+2) = 3 \cdot 2^x \ln 2 + 16 f'(x)$

$x = 2$ $\quad f'(4) = 12 \ln 2 + 16 f'(2)$

$16 f'(2) + 12 \ln 2 = 4 f'(2) + 96 \ln 2$.

$12 f'(2) = 84 \ln 2$.

$f'(2) = 7 \ln 2$ — ③.

$f'(4) = 28 \ln 2 + 96 \ln 2$.

$f'(4) = 124 \ln 2$. — ④

$14 \times \frac{f'(4)}{f'(2)} = \frac{124}{7} \times 14^{2}$

248

$14 \times \frac{1}{f'(2)} = \frac{\ }{7} \times 14$

$= 248$

$12f'(2) = 84 \ln 2.$

$f'(2) = 7 \ln 2$ —③.

$f'(4) = 28 \ln 2 + 96 \ln 2.$

$f'(4) = 124 \ln 2.$ —④

If $y(x)=(x^{x^x})$, $x>0$ then $\frac{d^2x}{dy^2}+20$ at $x = 1$ is equal to: 18

$[f(x)]^{g(x)} = y. \rightarrow$ take log.

$y = x^{x^x}$ ⇒ $\boxed{\ln y = x^x \ln x}$

diff both sides wrt y.

$\frac{1}{y} = \left[\frac{d}{dy}(x^x)\right]\ln x + x^x\left[\frac{d}{dy}(\ln x)\right]$

$\frac{1}{y} = (\ln x)\left[\frac{d}{dx}(x^x)\right]\frac{dx}{dy} + x^x\left[\frac{d}{dx}(\ln x)\right]\frac{dx}{dy}.$

$\frac{1}{y} = (\ln x)\cdot x^x(\ln x+1)\left(\frac{dx}{dy}\right) + x^{x-1}\left(\frac{dx}{dy}\right)$

$z = x^x.$

$\ln z = x \ln x.$

$\frac{1}{z}\frac{dz}{dx} = \ln x + x\cdot\frac{1}{x}.$

$= \ln x + 1$

$\frac{dz}{dx} = x^x(\ln x + 1)$

$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - v'u}{v^2}.$

$\frac{dx}{dy} = \frac{1}{y\left[\boxed{x^x \ln x}(\ln x+1) + x^{x-1}\right]} = \frac{1}{y\left[(\ln y)(\ln x+1) + x^{x-1}\right]}$

$\frac{dx}{dy} = \frac{1}{y \ln y \ln x + y \ln y + y x^{x-1}}$

$\frac{d^2x}{dy^2} = \frac{-(\text{sum})}{(\text{den})^2} = -2.$

Sum

$\frac{d}{dy}(y \ln y \ln x) = \ln y \ln x + y\left[\frac{1}{y}\ln x + \ln y \frac{1}{x}\cdot\frac{dx}{dy}\right]$

$\frac{d}{dy}(y \ln y) = \ln y + 1$

$\frac{d}{dy}(y x^{x-1}) = x^{x-1} + y\cdot x^{x-1}\left[\ln x + 1 - \frac{1}{x}\right]\frac{dx}{dy}$

$\frac{dz}{dx} = x^{x-1}\left[\ln x + 1 - \frac{1}{x}\right]$

for $x=1$

$y = 1$

sum $= 1 + 1 = 2.$

den $= 1$

The surface area of a balloon of spherical shape being inflated, increases at a constant rate. If initially, the radius of balloon is 3 units and after 5 seconds,, it becomes 7 units, then its radius after 9 seconds is :

$t=0 \quad r=3$

(A) 9 (B) 10

(C) 11 (D) 12

$t=0, r=3$

$t=5, r=7.$

$t=9, r=?$

$S = 4\pi r^2.$

$\frac{dS}{dt} = \boxed{8\pi r \frac{dr}{dt} = K.}$

$\int 8\pi r\,dr = \int k\,dt$

$8\pi \frac{r^2}{2} = kt + c. \rightarrow 4\pi r^2 = \boxed{k}t + c.$

$c = 36\pi$ —①.

$8\pi \frac{r^2}{2} = kt + c. \rightarrow 4\pi r^2 = kt + c.$

$c = 36\pi$ ·— ①

$4\pi r^2 = kt + 36\pi$

$196\pi = 5k + 36\pi$ · $\quad k = 32\pi$ — ②

$4\pi r^2 = 32\pi t + 36\pi$ ·

$r^2 = 8t + 9. \quad r^2 = 8\times 9 + 9 = 81$

$r = 9$

$f(x) = 4\log_e(x-1) - 2x^2 + 4x + 5$, $x > 1$, which one of the following is NOT correct ?

(A) f is increasing in (1, 2) and decreasing in $(2, \infty)$ ✓

(B) f(x)= –1 has exactly two solutions ✓

(C) f'(e) – f''(2) < 0 ✗

(D) f(x) = 0 has a root in the interval (e, e +1)

$f'(x) > 0 \Rightarrow f(x)\uparrow.$

$f'(x) < 0 \Rightarrow f(x)\downarrow.$

$f''(2) = -4(2 - 4 + 4) = -8$

$f'(x) = 4\left(\frac{1}{x-1}\right) - 4x + 4 = 4\left[\frac{1}{x-1} - x + 1\right]$

$f''(x) = -4\frac{\left[(2x-2)(x-1) - x^2 + 2x\right]}{(x-1)^2}$

$= 4\left[\frac{1 - x^2 + x + x - 1}{x-1}\right] = \frac{-4(x^2 - 2x)}{(x-1)} = \frac{-4x(x-2)}{(x-1)}.$

$f'(x) > 0$ in $(1, 2)$

$f'(x) < 0$ in $(2, \infty)$

$f'(e) = -\frac{4e(e-2)}{(e-1)} < 0$

$= \frac{-4\times 2.7\times 0.7}{1.7}$ $\quad -5$

$4\log_e(x-1) - 2x^2 + 4x + 5 = -1$

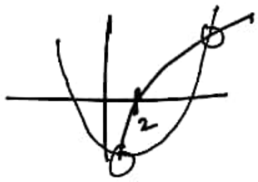
$4\log(x-1) = 2x^2 - 4x - 6.$

$2\log(x-1) = x^2 - 2x - 3 = y.$

$x^2 - 3x + x - 3$

$x(x-3) + 1(x-3)$

$(x-3)(x+1)$



the tangent at the point (x_1, y_1) on the curve $y = x^3 + 3x^2 + 5$ passes through the origin, then (x_1, y_1) does NOT lie on the curve :

(A) $x^2 + \frac{y^2}{81} = 2$ (B) $\frac{y^2}{9} - x^2 = 8$

(C) $y = 4x^2 + 5$ (D) $\frac{x}{3} - y^2 = 2$

The sum of absolute maximum and absolute minimum values of the function

$f(x) = |2x^2 + 3x - 2| + \sin x \cos x$ in the interval

[0, 1] is :

(A) $3 + \dfrac{\sin(1)\cos^2(1/2)}{2}$ (B) $3 + \dfrac{1}{2}(1 + 2\cos(1))\sin(1)$

(C) $5 + \dfrac{1}{2}(\sin(1) + \sin(2))$ (D) $2 + \sin\left(\dfrac{1}{2}\right)\cos\left(\dfrac{1}{2}\right)$

The number of distinct real roots of the equation $x^7 - 7x - 2 = 0$ is

(A) 5 (B) 7 (C) 1 (D) 3