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Random variables

Data → Attributes → Qualitative Data (Honesty, intelligence, sex of new born baby)

Data → variables → Quantitative Data (No. of books, family members etc.)

Quantitative Data → Discrete variables

Quantitative Data → Continuous variables

1. A random variable is a real-valued function defined over a sample space. It takes a definite set of values with definite probabilities.

Ex: throwing a dice in game of Ludo

$$S = \{\underbrace{1, 2, 3, 4, 5, 6}_{\text{real numbers}}\}$$ r.v

⇒ each r.v have a corresponding probability which is 1/6

2. A r.v which can assume a finite or a countably ...ite number of values is called a

2. A r.v which can assume a finite or a count[illegible] infinite number of values is called a discrete r.v.

For ex: throwing a dice
tossing a coin etc.

3. Again a random variable that can take an uncountable infinite value is continuous random variable.

* ——

(Mean / Average)

\# **Expectation** and Variance of discrete r.v.

Probability distribution?

Tabular presentation of r.v with corresponding probabilities.

Ex: Throwing a dice

r.v (X)	$P(X=x)$
1	$1/6$
2	$1/6$
3	$1/6$
4	$1/6$
5	$1/6$
6	$1/6$
	$\Sigma P = 1$

Probability distribution.

Formulas:

(1) Expectation of a discrete r.v x is

$$E(x) = \sum x p$$

$$= 1 \times \tfrac{1}{6} + 2 \times \tfrac{1}{6} + 3 \times \tfrac{1}{6} + 4 \times \tfrac{1}{6} + 5 \times \tfrac{1}{6} + 6 \times \tfrac{1}{6}$$

$$= \tfrac{1}{6} + \tfrac{2}{6} + \tfrac{3}{6} + \tfrac{4}{6} + \tfrac{5}{6} + \tfrac{6}{6}$$

$$= \tfrac{21}{6} = \tfrac{7}{2} = 3.5 \text{ (ans)}$$

Properties:

(1) $x = b$

$E(x) = b$

(2) $x = by$

$E(x) = bE(y)$

(3) $x = a + by$

$E(x) = a + bE(y)$

(4) $x = y + z$

$E(x) = E(y) + E(z)$

(5) $x = yz$

$E(x) = E(y)E(z)$

$$\boxed{\begin{aligned} &\text{mean, } \bar{x} = \frac{1}{n}\sum x_i \\ &\qquad E(x) \\ &V(x) = \frac{1}{n}\sum (x_i - \bar{x})^2 \\ &\qquad E(x - E(x))^2 \end{aligned}}$$

(2) Variance $V(x) = E\left(x - E(x)\right)^2$

$$= E\left(x^2 - 2xE(x) + E^2(x)\right)$$

$$= E\left(x^2 - 2xE(x) + E^2(x)\right)$$

$$= E(x^2) - \underbrace{2E(x)E(x)}_{2E^2(x)} + E^2(x)$$

$$\boxed{V(x) = E(x^2) - E^2(x)}$$

Ex: Rolling a die:

r.v (x)	p(x)
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

$E(x) = \frac{7}{2} = 3.5$

$\sum xp$

$$V(x) = E(x^2) - [E(x)]^2$$

Now, $E(x^2) = \sum x^2 p = 1^2 \times 1/6 + 2^2 \times 1/6 + 3^2 \times 1/6 + 4^2 \times 1/6 + 5^2 \times 1/6 + 6^2 \times 1/6$

$$E(x^2) = \frac{1}{6} + \frac{4}{6} + \frac{9}{6} + \frac{16}{6} + \frac{25}{6} + \frac{36}{6}$$

$$= \frac{61}{6}$$

$$\therefore V(x) = E(x^2) - E(x)^2$$

$$= \frac{61}{6} - \frac{7}{2}$$

$$= \frac{61}{6} - \frac{21}{6} = \frac{40}{6} = \frac{20}{3} = 6.66$$ (ans)

Properties:

① $x = c$

$V(x) = 0$

② $x = by$

$V(x) = b^2 V(y)$

③ $x = a + by$

$V(x) = b^2 V(y)$

Questions:

① If a fair coin is tossed twice, the number of heads obtained (x) will have the following probability distribution

Value of (X) No. of heads	0	1	2	Total
$P(X=x)$	$1/4$	$1/2$	$1/4$	01

Calculate the expectation $E(x)$ and variance $V(x)$.

sample of tossing a coin twice

$S = \{HT, TH, TT, HH\}$

$x \rightarrow$ No. of heads $E = \{1, 0, 2\}$

Ans

$$E(x) = \Sigma xp$$
$$= 0 \times \frac{1}{4} + 1 \times \frac{1}{2} + 2 \times \frac{1}{4}$$
$$= 1$$

$$V(x) = E(x^2) - [E(x)]^2$$

Now $E(x^2) = \Sigma x^2 p = 0^2 \times \frac{1}{4} + 1^2 \times \frac{1}{2} + 2^2 \times \frac{1}{4}$

$$= 0 + 1/2 + 4/4$$
$$= 3/2$$

$\therefore V(x) = E(x^2) - E(x)^2 = \frac{3}{2} - 1^2 = \frac{3}{2} - 1$
$= 1/2$ (ans)

② Find the expectation and variance of the following probability distribution

Value of X	0	1	2	3	Total
$P(X=x)$	1/56	15/56	15/28	5/28	01

[Ans) $E(x) = 15/8$
$v(x) = \frac{225}{448}$]

(3) If $E(x) = 3$, $E(y) = 5$ then find $E(3x - 5y + 16)$

$x = a + by$
$E(x) = a + bE(y)$

Ans $E(3x - 5y + 16)$
$= 3E(x) - 5E(y) + 16$
$= 3 \times 3 - 5 \times 5 + 16$
$= 9 - 25 + 16$
$= -16 + 16 = 0$ (ans)

(4) If $E(x) = 4$, $Var(x) = 9$ then $E(x^2) = ?$

Soln $\boxed{V(x) = E(x^2) - E(x)^2}$

or, $E(x^2) = V(x) + E(x)^2$
$= 9 + 4$
$E(x^2) = 13$ (ans)

(5) If $E(x) = 5$ and $E(x(x-1)) = 44$ what is $V(1 - 2x)$?

$y = a + bx$] Here let $y = 1 - 2x$

Property : $y = a + bx$
$v(y) = 0 + b^2 v(x)$

Here let $y = 1 - 2x$
or $v(y) = v(1-2x)$
$= \underline{4 v(x)}$

Given $E(x) = 5$ ✓, $\underline{E(x(x-1))} = 44$

or, $E(x^2 - x) = 44$
or, $E(x^2) - E(x) = 44$

simplification

or, $E(x^2) = 44 + E(x)$

or, $\boxed{E(x^2) = 44 + 5 = 49}$ ✓

$\therefore v(x) = E(x^2) - E(x)^2 = 49 - 5^2$
$= 49 - 25 = 24$

$\therefore v(y) = v(1-2x) = 4 v(x) = 4 \times 24 = 96$

(ans)

Next class

✓ Bivariate Probab Distribution
✓ pdf and pmf