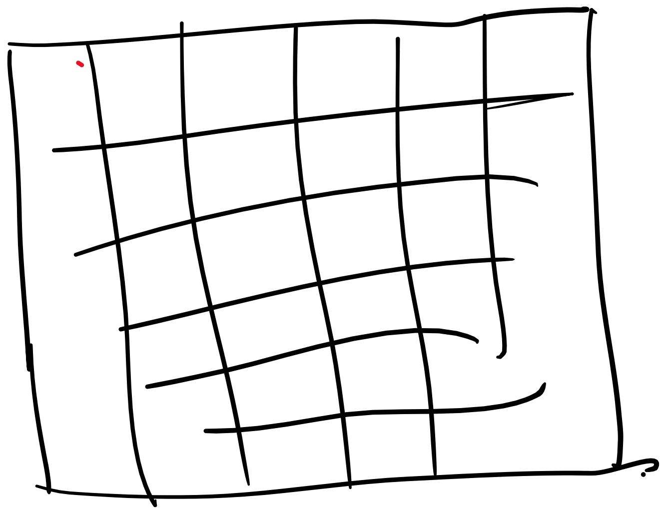
P AND C 4.4.4

04 April 2024 17:37

Chess Board

8x8

Putnam 151 117

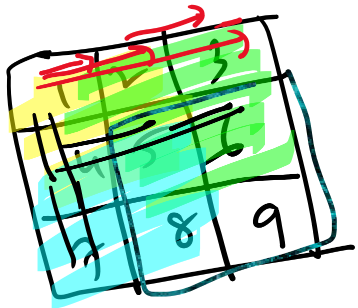


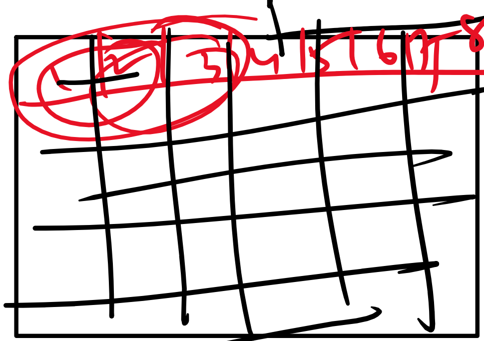
$1^2+2^2+3^2$

$9+4+1=14$

(8x4)

13x10





How to know R in R ?

8x4 $({}^8C_2 \times {}^4C_2)$

8x8 Square in sq

8x8 → squares → $1^2+2^2+\dots+8^2$

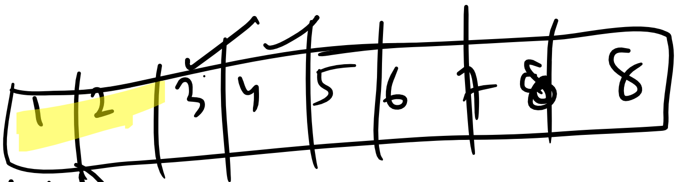
$= \frac{8(8+1)(2.8+1)}{6}$

$= 204$

${}^8C_2 \times {}^8C_2$

$\frac{8!}{2!6!} \times \frac{8!}{2!6!}$ … → 784

→ $\frac{8!}{2!6!}$ → $\frac{8\times7\times6!}{6!2!}$ → $\frac{8\times7\times6!}{6!2!}$ → 28^2 > 28



8×1 + (7×1)

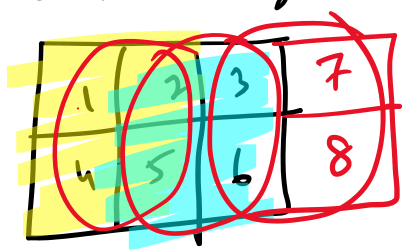
(7+6+5+4+3+2+1) × 8

12
123
1234
12345
123456
12345678
12
12

23
234
2345
234567
2345678
8

How many Squares in 10×7 Rectangle?

8

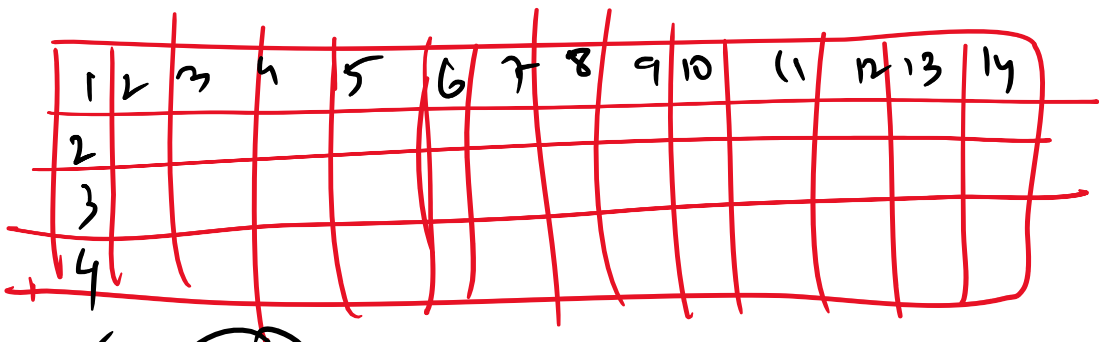


4×2 + (3×1)

11

(14×4) +
(13×3) +
(12×2) +
(11×1)
+ (10×0)

8+



57+39+24+11 = 130

Sq in Sq $\frac{n(n+1)(2n+1)}{6}$

Sq in Sq

$\frac{\dots}{6}$

Sq in Rect — $ab + (a-1)(b-1) + \dots$

Rect in Rect — $^aC_2 \times {}^bC_2$ (R ≠ C)

Rect in Sq — $({}^aC_2)^2$ R = C = a

78! ends with how many 0's ?

GIF

$\left[\frac{78}{5}\right] + \left[\frac{78}{25}\right] + \frac{78}{125}$

→ 15 + 3

→ 18

78!

→ 1 × 2 × 3 × 4 × 5 × ... 10 ... 78

↓

0

15!

→ 1.2.3.4.5 ... 10 → 1

12 ... 15 → 1

Total 3

Number of divisors

+ Sum of divisors

Sum of proper divisors ...

Even divisors

odd

12

108

1.108
2.54
3.36
4.27
6.18
9.12

12

$108 \to 9 \times 12 \to 3^2 \times 3 \times 2^2$

$\to 3^3 \times 2^2$

1, 108

Trivially 1 & 108

Number of divisor $(3+1)\times(2+1)$

$4\times3=12$

Sum of divisor → 1 -

3808

19 - 8 → 11

$2\times2\times2\times7\times11$

$2^3\cdot7\cdot11^1$

$(3+1)\times(2+1)\times(2+1)\times(1+1)$

→ $4\times3\times3\times2$

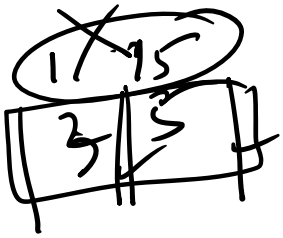
→ 12×6

→ 72

all Sum of divisor

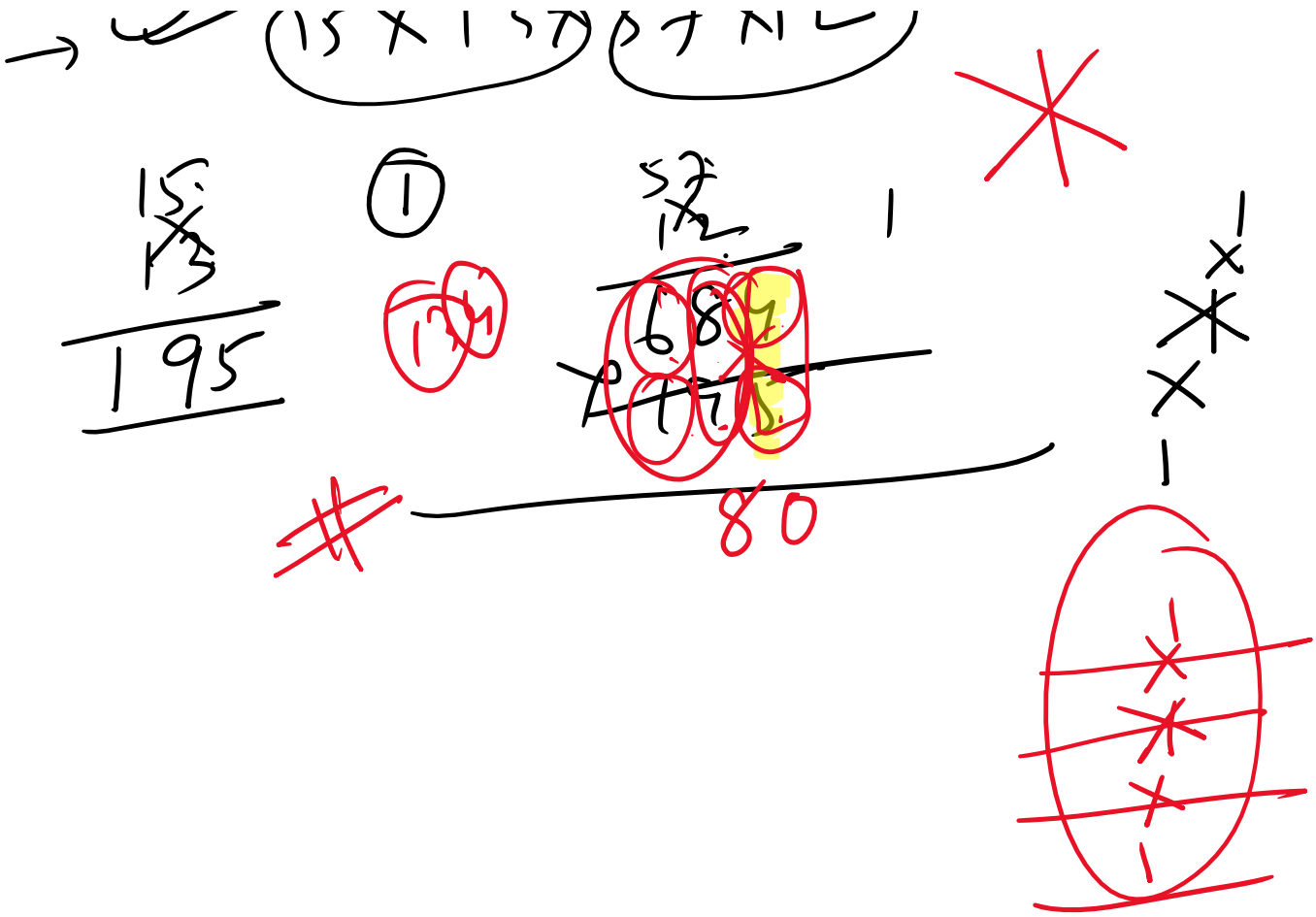
$(2^3+2^2+2^1+2^0)\times(3^2+3^1+3^0)\times(7^0+7^1+7^2)\times(11^1+11^0)$

15



→ $(8+4+2+1)\times(9+3+1)\times(49+7+1)\times(11+1)$

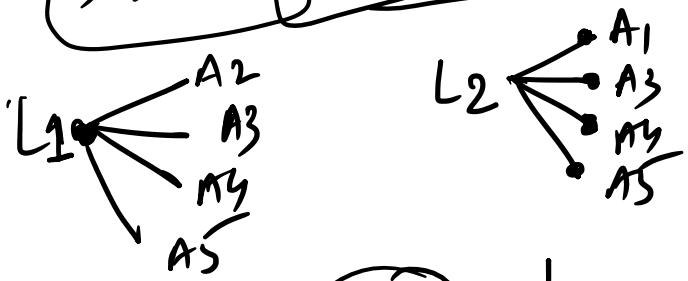
→ $15\times13\times57\times12$



De-Arrangements

L1 → A1
L2 → A2

5 letter 5 Addresses



$$D_n = n!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots + \frac{(-1)^n}{n!}\right)$$

$$D_5 = 5!\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!}\right)$$

or 4 (2 Letter 2 wrong)

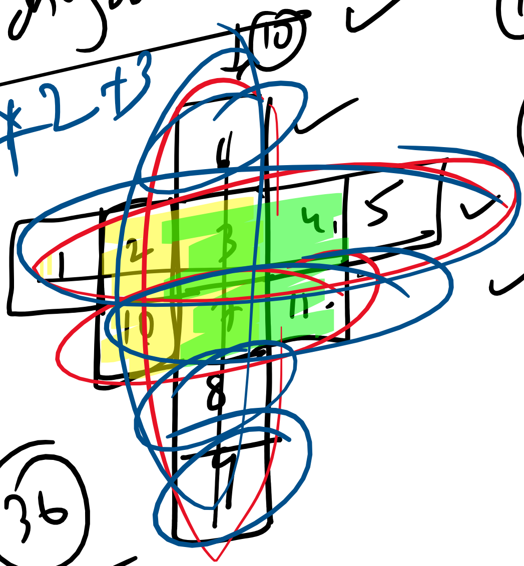
or 4 (2 ...) ... (...)

(D2)

2019 Find Rectangles in a chessboard which are non-squares.

(Rect in Sq — 204)

Overlapping diagram



$10 + 2 + 3$

(3) { 10 7 / 10 7 11 / 7 11

(1) (1)

(36)

(11)

(4) | 1 2 / 1 2 3 / 1 2 3 4 / 1 2 3 4 5

(3) | 2 3 / 2 3 4 / 2 3 5

(2) | 3 4 / 3 4 5

(1) | 4 5

$^5C_2 = \frac{5!}{3! \times 2!} = \frac{5 \times 4 \times 3!}{3! \times 2} = 10$

Sq $(11 + 2) = 13$

$36 - 13 = 23$