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AP / GP

Second term of a GP is 1000 and the common ratio is $r = \frac{1}{n}$ where n is a natural number. P_n is the product of n terms of this GP. $P_6 > P_5$ and $P_6 > P_7$, what is the sum of all possible values of n?

$P_n = t_1 t_2 t_3 \dots t_n$

$\boxed{P_n = P_{n-1} t_n}$

$P_{n-1} = t_1 t_2 t_3 \dots t_{n-1}$

$t_6 = ar^5$ $t_7 = ar^6$

$ar^5 > 1$ $ar^6 < 1$

$P_6 > P_5$

$P_5 \cdot t_6 > P_5$

$\boxed{t_6 > 1}$

$P_6 > P_7$

$P_6 > P_6 \cdot t_7$

$\boxed{t_7 < 1}$

$t_2 = 1000$

$ar = 1000$

$ar \cdot r^4 > 1$

$1000 \cdot r^4 > 1$

$\boxed{r^4 > \frac{1}{1000}}$

$\frac{1}{n^4} > \frac{1}{1000}$

$n^4 < 1000$

$6^4 = 1296$

$5^4 = 625$

$\boxed{n < 6} \rightarrow \boxed{n \le 5}$

$ar \cdot r^5 < 1$

$\boxed{r^5 < \frac{1}{1000}}$

$\frac{1}{n^5} < \frac{1}{1000}$

$4^5 = 1024$

$n^5 > 1000$

$\boxed{n \geqslant 4}$

$n = 4$ or 5

$\boxed{4 + 5 = 9}$

Let the n^{th} term of AP be defined as t_n, and sum up to 'n' terms be defined as S_n. If $|t_8| = |t_{16}|$ and t_3 is not equal to t_7, what is S_{23}?

$t_1 = a$ $cd = d$

$t_8 = a + 7d$ $t_{16} = a + 15d$ $t_3 = a + 2d$ $t_7 = a + 6d$

$t_3 \ne t_7$

$a + 2d \ne a + 6d$

$d \ne 0$

$|a + 7d|^2 = |a + 15d|^2$

$\cancel{a^2} + 14d + 49d^2 = \cancel{a^2} + 30d + 225d^2$

$176d^2 = -16d$

$\boxed{d = \frac{-16}{176} = -\frac{1}{11}}$

$-\left(a - \frac{7}{11}\right) = a - \frac{15}{11}$

$-a + \frac{7}{11} = a - \frac{15}{11}$

$2a = \frac{22}{11} = 2$

$\boxed{a = 1}$

$S_{23} = \frac{23}{2}\left[2a + 22d\right]$

$= 23(a + 11d)$

$= 23(a - 1)$

$= 0$

Consider a, b, c in a G.P. such that |a + b + c| = 15. The median of these three terms is a, and b = 10. If a > c, what is the product of the first 4 terms of this G.P.?

$b, a, c \rightarrow 10, a, c \rightarrow \boxed{a^2 = 10c}$

GP.

$a = \sqrt{10c}$

$t_1 = 10 \quad cr = r$

$a = 10r$

$c = 10r^2$.

$|\sqrt{10c} + 10 + c| = 15$

$|10r + 10 + 10r^2| = 15$.

$10r^2 + 10r + 10 = \pm 15$.

$2r^2 + 2r + 2 \pm 3 = 0$

$2r^2 + 2r + 5 = 0 \qquad 2r^2 + 2r - 1 = 0$.

$r = \frac{-2 \pm \sqrt{12}}{4} = \frac{-2 \pm 2\sqrt{3}}{4} = \frac{-1 \pm \sqrt{3}}{2}$

$8 - \frac{5}{4} = \frac{27}{4}$.

$0 < r < 1 \Rightarrow r = \frac{\sqrt{3} - 1}{2}$.

$r^2 = \frac{4 - 2\sqrt{3}}{4} = \frac{2 - \sqrt{3}}{2}$.

$r^6 = \frac{1}{8}\left[(2 - \sqrt{3})^3\right]$.

$= \frac{1}{8}\left[8 - 3\sqrt{3} - 12\sqrt{3} + 18\right]$

$= \frac{1}{8}\left[26 - 15\sqrt{3}\right]$

$P_4 = 10 \cdot 10r \cdot 10r^2 \cdot 10r^3 = 10^4 r^6$

$= \frac{10^3}{8} \cdot 10 \cdot (26 - 15\sqrt{3}) = 1250(26 - 15\sqrt{3})$

Sequence P is defined by $p_n = p_{n-1} + 3$, $p_1 = 11$, Sequence Q is defined as $q_n = q_{n-1} - 4$, $q_3 = 103$. If $p_k > q_{k+2}$, what is the smallest value k can take?

$t_1 = p_1 \quad cd = 3 \qquad t_1 = q_1 \quad cd = -4$

$t_1 = 11 \quad cd = 3. \qquad q_1 - 8 = 103 \quad q_1 = 111$

$\ldots 111 + (k+1)(-4)$

$t_1 = 11 \quad cd = 3. \qquad a_1 - 8 = 103 \quad a_1 = 111$

$$11 + (k-1)3 > 111 + (k+1)(-4)$$

$$3k + 8 > -4k + 107$$

$$7k > 99$$

$$\boxed{k > 14} \qquad \boxed{k \geq 15} \qquad 15$$

a, b, c and d are in A.P., What can we say about terms bcd, acd, abd and abc?

A. They are also in A.P.

B. They are also in H.P.

C. They are also in G.P.

D. They are not in an A.P., G.P. or H.P.

$$\boxed{b-a = c-b = d-c}$$

$$acd - bcd = cd(b-a)$$

$$abd - acd = ad(b-c)$$

$$abc - abd = bd(c-a)$$

$$\boxed{\frac{1}{bcd}, \frac{1}{acd}, \frac{1}{abd}, \frac{1}{abc}.}$$

$$t_2 - t_1 = \frac{1}{cd}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{\boxed{b-a}}{abcd}.$$

$$t_3 - t_2 = \frac{1}{ad}\left(\frac{1}{b} - \frac{1}{c}\right) = \frac{\boxed{c-b}}{abcd}$$

AP

$$t_4 - t_3 = \frac{1}{ab}\left(\frac{1}{c} - \frac{1}{d}\right) = \frac{\boxed{d-c}}{abcd}$$

If $S_n = n^3 + n^2 + n + 1$, where S_n denotes the sum of the first n terms of a series and $t_m = 291$, then m is equal to?

$$S_n = n^3 + n^2 + n + 1$$

$$S_{n-1} = (n-1)^3 + (n-1)^2 + (n-1) + 1$$

$$t_n = S_n - S_{n-1} = [n^3 - (n-1)^3] + [n^2 - (n-1)^2] + [n - (n-1)]$$

$$= [n^2 + n(n-1) + (n-1)^2] + (2n-1) + 1$$

$$= [n^2 + n^2 - n + n^2 - 2n + 1] + 2n$$

$$t_n = 3n^2 - n + 1$$

$$3m^2 - m + 1 = 291$$

$$3m^2 - m - 290 = 0$$

$$m = \frac{1 \pm \sqrt{1 + 3480}}{6} = \frac{1 \pm 59}{6}$$

$$\boxed{m = 10}$$

290
12
3480

59
59
531
295
3481

Let $\{a_n\}$ be a non-constant arithmetic progression. $a_1 = 1$ and the following holds true: for any $n \geq 1$, the value of $\frac{a_{2n} + a_{2n-1} + \ldots + a_{n+1}}{a_n + a_{n-1} + \ldots + a_1}$ remains constant (does not depend on n). Find a_{15}

$$\frac{a_{2n} + a_{2n-1} + \cdots + a_{n+1}}{a_n + a_{n-1} + \cdots + a_1} = c \text{ (constant)}$$

$$\therefore \quad c + 1 = \frac{\sum_{r=1}^{2n} a_r}{\sum_{r=1}^{n} a_r} = \frac{\frac{2n}{2}[2a_1 + (2n-1)d]}{\frac{n}{2}[2a_1 + (n-1)d]} = \frac{2(2a_1 + (2n-1)d)}{2a_1 + (n-1)d}$$

C+1 is constant

$$\sum_{r=1}^{n} a_r \qquad \frac{n}{2}\left[2a_1 + (n-1)d\right]$$

$$\therefore \frac{c+1}{2} = \frac{2a_1 + (2n-1)d}{2a_1 + (n-1)d}$$

Since c is constant $\therefore \frac{c+1}{2}$ is constant

let $\frac{c+1}{2} = k$

$$\therefore k = \frac{2a_1 + (2n-1)d}{2a_1 + (n-1)d} = \frac{2 + (2n-1)d}{2 + (n-1)d} \quad \text{since } a_1 = 1$$

$\therefore \quad 2k + ndk - dk = 2 + 2nd - d$

or $\quad ndk - 2nd = 2 - d - 2k + dk$

or $\quad nd(k-2) = d(k-1) - 2(k-1)$

or $\quad nd(k-2) = (k-1)(d-2)$

Since k is a constant $\therefore$ RHS is constant but the LHS varies with n. Hence both sides should be $= 0$

$\therefore \quad k = 2$ and $d = 2$

$\therefore \quad a_{15} = a_1 + 14d = 1 + 14 \times 2 = 29$