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Method of Interpolation

① From the following data, calculate

(a) (Less than type) cummulative frequencies.

(b) the number of caseses between 112 and 134.

(c) Number of cases Less Than 112

(d) Number greater than 134.

class limit	90-100	100-110	110-120	120-130	130-140	140-150	150-160
Frequency	16	22	45	60	50	24	10

Ans

Cummulative Frequency Distribution

Class Boundary	Frequency	Cummulative frequency (Less than)
90-100	16	16
100-110	22	38
110-120	45	83
120-130	60	143
130-140	50	193
140-150	24	217
150-160	10	227 = N
	$\Sigma f = 227 = N$	

CB x (cases)	Cummulative frequency (less than type) (no.)
90	0
100	16
110	38 ← x
112 →	
120	83
130	143 ← y
134 → 140	193
150	217
160	227

(b) Using interpolation method.

$$\frac{112-110}{120-110} = \frac{x-38}{83-38}$$

or, $$\frac{\cancel{2}}{\cancel{10}\,5} = \frac{x-38}{45}$$

or, $$x-38 = \frac{\cancel{45}}{\cancel{5}}\,9$$

or, $$\boxed{x = 38+9 = 47}$$

And, $$\frac{134-130}{140-130} = \frac{y-143}{193-143}$$

$$\Rightarrow \boxed{y = 163}$$

$\therefore$ No. of cases between 112 and 134 are

163 − 47

∴ No. of cases between 112 and 134

$163 - 47$

$= \boxed{116 \text{ (ans)}}$

(d) No. of cases greater than 134 are

$227 - 163 = 64 \text{ (ans)}$

Measures of Central Tendency *

- Average
  1. AM ($\bar{x}$)
  2. GM ($\bar{x}_g$)
  3. HM ($\bar{x}_h$)
- Median
- Mode

Q) What are the desirable properties/characteristics of a good measure of central tendency.

1. rigidly defined
2. based on all observation
3. easily comprehensible
4. easily applied to mathematical/algebraic treatment.

4. easily ... algebraic treatment.

5. easy calculation.

6. Subjected to less sampling fluctuation.

Arithmetic Mean ($\bar{x}$)

1. Simple AM (without frequency)

If $x_1, x_2 \dots x_n$ are 'n' no. of obs then

simple AM, $$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

2. Weighted average ($\bar{x}$)

if $x_1, x_2 \dots x_n$ are 'n' no. of observations and $f_1, f_2 \dots f_n$ are their corresponding frequencies, then

weighted average $$\bar{x} = \frac{1}{N} \sum_{i=1}^{n} x_i f_i$$

where $N = \sum_{i=1}^{n} f_i = f_1 + f_2 + \dots + f_n$
(Total frequency)

Ex: (a) Suppose data is given as 8, 1, 6 $\boxed{n=3}$

then simple AM, $\bar{x} = \frac{8+1+6}{3}$

then simple AM, $\bar{x} = \frac{\quad}{3}$

$= \frac{15}{3} = 5$ (ans)

(b) Now suppose the corresponding frequencies are given as 3, 2, 5

x	f	xf
8	3	24
1	2	2
6	5	30
	$\Sigma f = N = 10$	56

$\therefore$ weighted AM, $\bar{x} = \frac{\Sigma x_i f_i}{\Sigma f_i}$

$= \frac{56}{10}$

$= 5.6$ (ans)

Properties of Arithmetic Mean

1. If all observations are same, then AM is the same value.

Proof Let $x_i = c$ for all value of x $i = 1 (1) n$. and c is any constant.

then AM, $\bar{x} = \frac{c + c + c + \cdots + c \text{ (n times)}}{n}$

$= \sum_{i=1}^{n} c / n = \frac{\cancel{n}}{\cancel{n}} \cdot c$

$\boxed{\therefore \bar{x} = c}$ (Proved)

2. Sum of the deviation of observation from it's mean is zero.

x_i : 1, 2, 3, 4, 5
$\bar{x} = 15/5 = 3$
$x_i - \bar{x} = -2, -1, 0, 1, 2$
$\Sigma(x_i - \bar{x}) = 0$

Proof: Let $x_1, x_2, \dots, x_n$ be 'n' number of observation

then $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ — (1)

Now $\sum_{i=1}^{n}(x_i - \bar{x}) = \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \bar{x}$

$= n\bar{x} - n\bar{x}$

$= 0$ (Proved)

3. If all the observations are changed by origin and scale, then mean of that observation will also be changed by origin and scale.

In other word, AM is dependent on the change in both origin and scale.

Proof Let us change x observation with origin 'a' and scale 'b'

then $y = a + bx$

origin ... ...

then $y = a + bx$

Now $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$

$\bar{y} = \frac{1}{n}\sum_{i=1}^{n}(a + bx_i)$

$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} a + b\frac{1}{n}\sum_{i=1}^{n} x_i$

$\bar{y} = \frac{\cancel{n}a}{\cancel{n}} + b\bar{x}$

$\therefore \boxed{\bar{y} = a + b\bar{x}}$

(Proved)

④ if all observations are repeated same number of times, then simple mean = weighted mean.

Proof Let $x_1, x_2, \ldots, x_n$ be 'n' observation with frequencies $f_1, f_2 \ldots f_n$ respectively such that $f_1 = f_2 = f_3 = \ldots = f_n = k$ (const) (that is frequencies are same).

then $N = \sum_{i=1}^{n} f_i = f_1 + f_2 + \ldots + f_n = \sum_{i=1}^{n} k$

$\boxed{N = nk}$

then ... $i=1, n$

$$\boxed{N = nk}$$

$\therefore$ Weighted mean $= \frac{1}{N} \sum_{i=1}^{n} x_i f_i$

$= \frac{1}{N} \sum x_i \cdot k$

$= \frac{1}{n\cancel{k}} \times \cancel{k} \sum x_i$

$= \frac{1}{n} \sum x_i$ = simple mean

(Proved)

5. Prove that $\sum_{i=1}^{n} (x_i - A)^2$ is the least when $A = \bar{x}$

OR Sum of square of deviation is least around mean.

Proof:

$$x_i - A = (x_i - \bar{x}) + (\bar{x} - A)$$

$$\therefore \sum_{i=1}^{n} (x_i - A)^2 = \sum_{i=1}^{n} \left[(x_i - \bar{x}) + (\bar{x} - A) \right]^2$$

$$= \sum_{i=1}^{n} \left[(x_i - \bar{x})^2 + 2 \cdot (x_i - \bar{x})(\bar{x} - A) + (\bar{x} - A)^2 \right]$$

$$= \sum_{i=1}^{n} (x_i - \bar{x})^2 + 2(\bar{x} - A) \sum (x_i - \bar{x})$$

$$= \sum_{i=1} (x_i - \bar{x})^2 + 2(\bar{x} - A)\left(\sum (x_i - \bar{x})\right) + \sum_{i=1}^{n} (\bar{x} - A)^2$$

$\because \sum (x_i - \bar{x}) = 0$

then $$\sum (x_i - A)^2 = \sum_{i=1}^{n} (x_i - \bar{x})^2 + n(\bar{x} - A)^2 \quad \text{--- (1)}$$

Comparing both sides of eq (1),

$$\sum (x_i - A)^2 \geq \sum_{i=1}^{n} (x_i - \bar{x})^2$$

iff $n(\bar{x} - A)^2 = 0$

or, $(\bar{x} - A)^2 = 0$

or, $\bar{x} - A = 0$

or, $\boxed{\bar{x} = A}$

So the sum of square of deviation is minimum if $\bar{x} = A$ (Proved)