Trigonometry

Wednesday, May 17, 2023 12:00 PM

$x \to ?$

$\sin x$,

$1^\circ = \frac{\pi}{180}$ radian

$\pi/180 \to 1^\circ$

$1 \to \frac{180}{\pi}^\circ$

$\pi/8 \to \left(\frac{180}{\pi} \times \frac{\pi}{8}\right)^\circ =$

$45/2 \overline{)45}\,(22\frac{1}{2}$

$\underline{44}$

1

$\sin(10\sqrt{3})$

$\mathbb{N} = \{1, 2, 3, \dots\}$

$\mathbb{W} = \{0, 1, 2, 3, \dots\}$

$\mathbb{Q} = \{$ set of all fractions in p/q form, where $q \neq 0, p, q \in \mathbb{Z}\}$

$\mathbb{Z} = -\mathbb{N} \cup \{0\} \cup \mathbb{N}$

$\mathbb{Q}^c = \{$not rational$\}$

$\mathbb{R} = \mathbb{Q} \cup \mathbb{Q}^c$

$\mathbb{C}$ = Complex number

sin, cos, tan real-valued functions

$f(x) = \frac{1}{1-x}$

Angles of a triangle are in AP. $\frac{\text{least angle (degree)}}{\text{greatest angle (radian)}} = \frac{60}{\pi}$. Find angles.

$(a-d)^\circ, a^\circ, (a+d)^\circ$ $\quad a = 60^\circ$

greatest $= 60 + d = \frac{\pi}{180}(60+d)$ rad

$\therefore \frac{60-d}{\pi/180\,(60+d)} = \frac{60}{\pi} \Rightarrow \frac{60-d}{60+d} = \frac{1}{3} \Rightarrow 180 - 3d = 60 + d \Rightarrow d = 30^\circ$

Angles of a quadrilateral are in AP, greatest angle is 120°. Find the $\angle$s (in rad)

$a-3d, a-d, a+d, a+3d$, $\therefore a = 90^\circ$ (sum of 4 angles of a quadrilateral $= 360^\circ$)

$\Rightarrow 90 + 3d = 120^\circ \Rightarrow d = 10^\circ$

$\therefore$ 4 angles $\to$ $60^\circ, 80^\circ, 100^\circ$ & 120° . In radians $\to \frac{\pi}{3}, \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{2\pi}{3}$

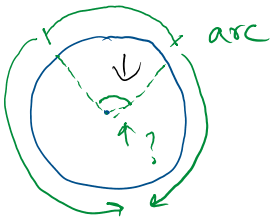
The minute hand of a clock is 10 cm long. How far does the tip of the hand move in 20 mins?

$2\pi(10)$ cm distance covered in 60 mins

60 mins $\to 20\pi$ cm

20 mins $\to \frac{20\pi}{3}$ cm

Circle with radius 21 cm. Has an arc that subtends 60° angle at the center. Find arc length.



Sol1

$1/6$ ($360^\circ \to 42\pi$ cm) $1/6$

$60^\circ \to 7\pi$ cm

Arc length $= \frac{22}{7} \times 7$ cm $= 22$ cm

Sol2

$s = r\theta$, where s = arc length, r = radius, θ = angle at center by the arc, (in rad)

$s = \left(21 \times \frac{\pi}{3}\right)$ cm $= 7\pi$ cm $= 22$ cm

Logarithm

Thursday, May 25, 2023 2:00 PM

(1) $\log_a a = 1$ (2) $\log_a 1 = 0$

(3) $\log_a b \cdot \log_b a$ let $\log_b a = x \Rightarrow b^x = a$

$\log_a b^x = 1$ — (A) $\Rightarrow \log_a b^x = \log_a a = 1$

Using in A $\rightarrow x \log_a b = 1$

$\Rightarrow \log_a b \cdot \log_b a = 1$

(4) $\log_a b^x = x \log_a b$

(5) $\log_b a = \log_b c \cdot \log_c a$ (base changing formula)

(6) $\log_a (bc) = \log_a b + \log_a c$ (7) $\log_a \left(\frac{b}{c}\right) = \log_a b - \log_a c$

(8) $\log_{(a^q)} b^p = \frac{1}{q} \log_a b$ (9) Combining 4 & 8 $\log_{a^q} b^p = \frac{p}{q} \log_a b$

(10) $a^{\log_a b} = b$ $\log_a b = x \Rightarrow a^x = b \rightarrow a^x = b$

(11) $\log a \rightarrow \log_{10} a$, $\ln a \rightarrow \log_e a$

(1) $4^{3/2} = 8$ (2) $(2\sqrt{2})^{-2/3} = 1/2$

$\log_2 4^{3/2} = \log_2 8$

$\Rightarrow 3/2 \log_2 4 = 3 \Rightarrow$

$\Rightarrow \log_2 4 = 2 \Rightarrow$

$\Rightarrow \log_2 2^2 = 2 \Rightarrow$

$\Rightarrow 2 \times 1 = 2 \Rightarrow$

True

$(2 \cdot 2^{1/2})^{-2/3} = 1/2 = 2^{-1}$

$2^{-2/3} \cdot 2^{-1/3} = 2^{-1}$

$\log_2 (2^{-2/3} \cdot 2^{-1/3}) = -1$

$\log_2 2^{-2/3} + \log_2 2^{-1/3} = -1$

$-2/3 + -1/3 = -1$ (True)

(3) $\log_{5\sqrt{5}} 5 = ?$

(4) $\log_{100} 0.1 = ?$

$\log_{5 \cdot 5^{1/2}} 5$

$= \log_{5^{(1+1/2)}} 5$

$= \log_{5^{3/2}} 5$

$= 2/3$

$\log_{100} 0.1 = \log_{10^2} 10^{-1} = -1/2$

(1) $3^{\log_3 9^{-1/2}} = 3^{-\log_3 \sqrt{9}} = 3^{\log_3 1/3} = 1/3$

(2) $2^{2-\log_2 5} = 2^2 \cdot 2^{-\log_2 5} = 4\, 2^{\log_2 5^{-1}} = 4 \cdot 1/5 = 4/5$

(3) $10^{\log m + \log n} = mn$

(4) $2^{\log_{2\sqrt{2}} 15} = 2^{\log_{2^{3/2}} 15} = 2^{2/3 \log_2 15} = 2^{\log_2 15^{2/3}} = (15)^{2/3}$

(1) $3^{-1/2 \log_3 9}$ (2) $2^{2-\log_2 5}$

$[a^m \cdot a^n = a^{m+n}]$

$a^{m+n} = a^m \cdot a^n$

$a^{m-n} = a^m \cdot a^{-n}$

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