AP, GP, HP

AP: series whose difference between 2 consecutive terms is constant

$a, a+d, a+2d, \dots$

$a \to t_1$, $d \to$ common diff.

$t_n = a + (n-1)d$

a, b, c are in AP $\to$ $b = \frac{a+c}{2}$

$S_n = \frac{n}{2}[t_1 + t_n] = \frac{n}{2}[2a + (n-1)d]$

$= n\, t_m$ when n is odd

$= n\left(\frac{t_m + t_{m+1}}{2}\right)$ when n is even.

odd no of terms.

1, 2, 3, 4, 5

$\downarrow$ avg of the series.

n is odd $t_m = t_{\left(\frac{n+1}{2}\right)} =$ avg.

$t_m = \frac{t_1 + t_n}{2}$

n is even

$t_m = t_{\frac{n}{2}}$, $t_{m+1} = t_{\frac{n}{2}+1}$

GP: the ratio of any 2 consecutive terms is constant.

$a, ar, ar^2, \dots$ $t_n = ar^{n-1}$ $S_n = a \cdot \frac{r^n - 1}{r - 1}$

a, b, c are in GP $\to$ $b = \sqrt{ac}$

Infinite GP: $a, ar, ar^2, \dots$ to ∞ $|r| < 1$ $-1 < r < 1$

$S = \frac{a}{r-1}$

If a, b, c, ... are in AP then their reciprocals are in HP

$\frac{1}{a} + \frac{1}{c} = \frac{2}{b}$

What is the sum of all 3 digit numbers that leave a remainder of '2' when divided by 3?

1. 897
2. 1,64,850
3. 1,64,749
4. 1,49,700

$100 - 999$ $R\left(\frac{100}{3}\right) = 1$ $R\left(\frac{101}{3}\right) = 2$

$101, 104, 107, \dots, 998 \to t_n$

$t_n = t_1 + (n-1)d$

$n = \left(\frac{t_n - t_1}{d}\right) + 1 = \frac{998 - 101}{3} + 1 = \frac{897}{3} + 1 = 299 + 1 = 300$

$S_n = \frac{300}{2}[101 + 998] = \frac{300}{2}[1099] = 150 \times 1099 = 164850$

The sum of third and ninth term of an A.P is 8. Find the sum of the first 11 terms of the progression.

$t_3 + t_9 = 8$ $t_1 = a$, $d = d$

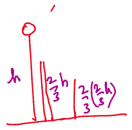
$a + 2d + a + 8d = 8$

$2a + 10d = 8$

$a + 5d = 4$

$S_{11} = \frac{11}{2}[2a + (11-1)d] = \frac{11}{2}[2a + 10d] = \frac{11 \times 8}{2} = 44$

If a rubber ball consistently bounces back ⅔ of the height from which it is dropped, what fraction of its original height will the ball bounce after being dropped and bounced four times without being stopped?



$h, \frac{2}{3}h, \frac{2}{3}\left(\frac{2}{3}h\right), \dots$

$h, \left(\frac{2}{3}\right)h, \left(\frac{2}{3}\right)^2 h, \dots$

$a = t_1$, r.

$t_5 = ar^4 = \left(\frac{2}{3}\right)^4 h = \frac{16}{81}h$.

$\frac{16}{81}$.

Calculate the total distance travelled by the ball,

$D = h + 2\times\left(\frac{2}{3}\right)h + 2\times\left(\frac{2}{3}\right)^2 h + 2\times\left(\frac{2}{3}\right)^3 h + 2\times\left(\frac{2}{3}\right)^4 h$.

$= h\left[1 + 2\times\frac{2}{3}\left(1 + \frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3\right)\right] = h\left[1 + \frac{4}{3}\left(\frac{1 - \left(\frac{2}{3}\right)^4}{1 - \left(\frac{2}{3}\right)}\right)\right]$

$= h\left[1 + \frac{4}{3}\left(\frac{\frac{81-16}{81}}{\frac{1}{3}}\right)\right] = h\left[1 + \frac{4}{3}\left\{\frac{65}{27}\right\}\right] = h\left[1 + \frac{260}{81}\right]$

$= h\left[\frac{341}{81}\right]$

The ball continues to bounce till it comes to rest. Find the total distance travelled.

$D = h + 2\times\left(\frac{2}{3}\right)h + 2\times\left(\frac{2}{3}\right)^2 h + \dots$ to ∞.

$= h\left[1 + \frac{4}{3}\left\{1 + \left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)^2 + \dots \infty\right\}\right]$

$= h\left[1 + \frac{4}{3} \cdot \frac{1}{1 - \frac{2}{3}}\right] = h\left[1 + \frac{4}{3} \cdot 3\right] = 5h$.

x, 17, $3x - y^2 - 2$ and $3x + y^2 - 30$ are four consecutive terms of an increasing arithmetic sequence. The sum of the four numbers is divisible by:

$d > 0$ $\to$ $t_4 > t_3 > t_2 > t_1$

A. 2
B. 3
C. 5

$S = x + 17 + 3x - \cancel{y^2} - 2 + 3x + \cancel{y^2} - 30$

$S = \boxed{7x - 15}$

the four numbers is divisible by:

A. 2
B. 3
C. 5
D. 7
E. 11

$a>0$ → $t_4>t_3>t_2>t_1$

$S = x+17+3x-y^2-2+3x+y^2-30$

$S = \boxed{7x-15}$

$17-x = 3x-y^2-2-17$

$\boxed{y^2-4x=-36 \quad (1)}$

$3x-y^2-19 = 3x+y^2-30-(3x-y^2-2)$

$3x-y^2-19 = 2y^2-28$

$3x-3y^2=-9$

$\boxed{x-y^2=-3 \quad (2)}$

$S = 13\times 7-15$
$= 91-15$
$= 76$

(1) + (2)

$-3x=-39$

$\boxed{x=13}$

If log 2, log (2^x -1) and log (2^x + 3) are in A.P, then x is equal to ____

1. $\frac{5}{2}$
2. $\log_2 5$
3. $\log_3 2$
4. $\frac{3}{2}$

a, b, c are in AP

$a+c = 2b$.

$\log 2+\log(2^x+3) = 2\log(2^x-1)$

$\log(a^b) = b\log a$

$\log(ab) = \log a+\log b$

$\log\{2(2^x+3)\} = \log[(2^x-1)^2]$

$\boxed{2^x = a}$

$2\cdot 2^x+6 = (2^x-1)^2$

$a>0$.

$2a+6 = (a-1)^2 = a^2-2a+1$

$a^2-4a-5=0$

$(a-5)(a+1)=0$

$a=5, \cancel{-1}$.

$a=5$

$2^x=5$

$x\log 2 = \log 5$

$x = \frac{\log 5}{\log 2} = \log_2 5$

Q7. If $t_1+t_5+t_{10}+t_{20}+t_{24} = 225$, Find the sum of first ~~24th~~ 23rd term of that A.P?

$t_1 = a \qquad cd = d$.

$t_1+t_5+t_{10}+t_{20}+t_{24} = 225$

$a+a+4d+a+9d+a+19d+a+23d = 225$.

$5a+55d = 225$

$\boxed{a+11d = 45}$

$S_{23} = \frac{24}{2}[2a+(22)d] = 12\times 2\times(a+11d)$

$= 24\times 45 = 1080$.

Imp

Q10. If ratio of sum of 1st n^{term} of 2 different AP = $\frac{2n-3}{3n+1}$ Find the ratio of 9th term of those 2 A.P?

→ ratio of nth term = $\frac{2n-3}{3n+1}$ find the ratio of the sums of n terms.

$t_{n1} = a_1+(n-1)d_1 \qquad t_{n2} = a_2+(n-1)d_2$

$AP_1:\ a_1,\ cd=d_1 \qquad AP_2:\ a_2,\ cd=d_2$

$S_{n1} = \frac{n}{2}[2a_1+(n-1)d_1] \qquad S_{n2} = \frac{n}{2}[2a_2+(n-1)d_2]$

$\frac{2n-3}{3n+1} = \frac{t_{n1}}{t_{n2}} = \frac{a_1+(n-1)d_1}{a_2+(n-1)d_2}$

$\frac{S_{m1}}{S_{m2}} = \frac{\frac{m}{2}[2a_1+(m-1)d_1]}{\frac{m}{2}[2a_2+(m-1)d_2]}$

$\boxed{\frac{S_{n1}}{S_{n2}} = \frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2} = \frac{2n-3}{3n+1}}$

$\frac{t_{m1}}{t_{m2}} = \frac{a_1+(m-1)d_1}{a_2+(m-1)d_2}$ || form

$\frac{S_{m1}}{S_{m2}} = \frac{2a_1+(m-1)d_1}{2a_2+(m-1)d_2}$ || this form

$\frac{2a_1+(n-1)d_1}{2}$

$\frac{a_1+\left(\frac{n-1}{2}\right)d_1}{\ldots} = \frac{2n-3}{\ldots}$

$\frac{n-1}{2} = m-1$

$\frac{2a_1+2(n-1)d_1}{\ldots} = \frac{2n-3}{3n+1}$

$$\frac{S_{n1}}{S_{n2}} = \frac{\ldots}{2a_2+(n-1)d_2} = \frac{\ldots}{3n+1}$$

$$\frac{t_{m1}}{t_{m2}} = \frac{\ldots}{a_2+(m-1)d_2} \quad || \text{ form}$$

$$\frac{\frac{2a_1+(n-1)d_1}{2}}{\frac{2a_2+(n-1)d_2}{2}}$$

$$\frac{a_1+\left(\frac{n-1}{2}\right)d_1}{a_2+\left(\frac{n-1}{2}\right)d_2} = \frac{2n-3}{3n+1}$$

$$\boxed{\frac{n-1}{2} = m-1}$$

$$n-1 = 2m-2$$

$$\boxed{n = 2m-1}$$

$$\frac{t_{m1}}{t_{m2}} = \frac{a_1+(m-1)d_1}{a_2+(m-1)d_2} = \frac{4m-2-3}{6m-3+1} = \frac{4m-5}{6m-2}.$$

$$\frac{t_{9_1}}{t_{9_2}} = \frac{36-5}{54-2} = \frac{31}{52}$$

$$\frac{S_{m1}}{S_{m2}} = \boxed{\frac{2a_1+(m-1)d_1}{2a_2+(m-1)d_2}} \quad || \text{ this form}$$

$$\boxed{\frac{2a_1+2(n-1)d_1}{2a_2+2(n-1)d_2}} = \frac{2n-3}{3n+1}$$

$$m-1 = 2n-2$$

$$2n = m+1$$

$$\boxed{n = \frac{m+1}{2}}$$

$$\frac{S_{m1}}{S_{m2}} = \frac{2a_1+(m-1)d_1}{2a_2+(m-1)d_2} = \frac{m+1-3}{\frac{3}{2}(m+1)+1} = \frac{m-2}{\frac{3m+3+2}{2}}$$

$$= \frac{2m-4}{3m+5}$$

$$\frac{S_{n1}}{S_{n2}} = \frac{2n-4}{3n+5}$$

Q2. $5 + 55 + 555 + \ldots\ldots\ldots t_{100}$, Find the Sum?

$$S_{100} = 5(1+11+111+\cdots \text{ to } 100 \text{ terms})$$

$$= \frac{5}{9}(9+99+999+\cdots \text{ to } 100 \text{ terms})$$

$$= \frac{5}{9}\left[(10-1)+(10^2-1)+(10^3-1)+\cdots \text{ to } 100 \text{ terms}\right]$$

$$= \frac{5}{9}\left[(10+10^2+10^3+\cdots \text{ to } 100 \text{ terms}) - 100\right].$$

$10^1 = 10$

$10^2 = 100$

$10^3 = 1000$

$10^{100} = 1000000\ldots0$ 101 terms

$$= \frac{5}{9}\times\left[10\left(\frac{10^{100}-1}{10-1}\right) - 100\right] = \frac{5}{9}\left[10\times\frac{\overbrace{999\ldots9}^{100\ 9s}}{9} - 100\right]$$

$$= \frac{5}{9}\left[10\times(\underbrace{1111\ldots1}_{100\ 1s}) - 100\right] = \frac{5}{9}\left[\underbrace{111\ldots10}_{100\ 1s+0} - 100\right]$$

$$= \frac{5}{9}\times\underbrace{111\ldots1}_{98\ 1s}010 \quad \cancel{= 5\times123\ldots}$$

$$\begin{array}{r} 1111110 \\ -\ \ \ \ 100 \\ \hline 1111010 \end{array}$$

$$\frac{11111010}{9}$$

Q4. Find the value of $25^{\left[\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots\ldots\infty\right]}$?

$$S = \frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\cdots\infty = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{1}{2}$$

$$S = 25^{\frac{1}{2}} = 5$$

Q5. A ball is thrown from a height of 5000 meter on a ground the ball is bounced 4/5 times of its every last bounce then calculate the total distance covered by the ball till it stopped?

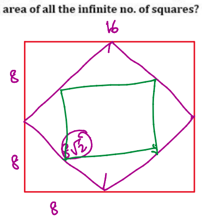
$$S = 5000 + \frac{4}{5}\times2\times5000 + \left(\frac{4}{5}\right)^2\times2\times5000 + \cdots\cdots\infty$$

$$= 5000\left[1+\frac{8}{5}\left\{1+\frac{4}{5}+\left(\frac{4}{5}\right)^2+\cdots+\infty\right\}\right].$$

$$= 5000\left[1+\frac{8}{5}\left\{\frac{1}{1-\frac{4}{5}}\right\}\right] = 5000\left[1+\frac{8}{5}\times5\right] = 5000\times9$$

$$= \underline{45000}$$

Q7. The side of a square is 16 cm infinite no. of squares are made by joining mid points of each side of the squares. Then calculate the total area of all the infinite no. of squares?



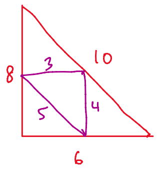
$$\frac{16}{\sqrt{2}} = 8\sqrt{2}. \qquad r = \frac{1}{\sqrt{2}}$$

$$S_{areas} = 16^2 + \left(\frac{16}{\sqrt{2}}\right)^2 + \left(\frac{16}{2}\right)^2 + \cdots + \infty.$$

$$= 16^2\left[1+\frac{1}{2}+\frac{1}{4}+\cdots\cdots\infty\right]$$

$$= 16^2\cdot\frac{1}{1-\frac{1}{2}} = 256\times2 = \underline{512}$$

Q8. The side of a right-angled triangle are 6, 8 & 10 cm respectively. A new right-angled triangle is made by joining the mid-points of ~~all~~ each sides of triangle. This process continues for infinite times then calculate the area of all infinite triangles?



$r = \frac{1}{2}$

$S_{areas} = \frac{1}{2}\times 8\times 6 + \frac{1}{2}\times\frac{1}{2}\times 8\times\frac{1}{2}\times 6 + \frac{1}{2}\times\left(\frac{1}{2}\right)^2\times 8\times\left(\frac{1}{2}\right)^2\times 6 + \dots \infty$

$= 24\left[1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^4 + \dots \infty\right]$

$= 24\left[\frac{1}{1-\frac{1}{4}}\right] = 24\times\frac{4}{3} = 32$

Let $\{a_n\}_1^{100}$ be an arithmetic progression with 100 elements. $a_1 = 5$, $a_2 = 8$ and so on. $\{b_n\}_1^{100}$ also has 100 elements, but $b_1 = 3$, $b_2 = 7$ and so on. Find how many common elements $\{a_n\}$ and $\{b_n\}$ have.

Ans.

a_n : $t_1 = 5$, $d = 3$
$a = 5$

$t_3 = 11$

b_n : $a = 3$, $d = 4$

$t_3 = 11$

n^{th} term of a_n = m^{th} term of b

for $m = 3$
$n = 3 - 1 + 1 = 3$

$5 + (n-1)3 = 3 + (m-1)4$

$3n + 2 = 4m - 1$

$\boxed{4m = 3n + 3}$

$m = \frac{3n+3}{4}$ or $3(n+1) = 4m$

$n + 1 = \frac{4m}{3}$

$n = \frac{4m}{3} - 1$

$\boxed{n = (m-1) + \frac{m}{3}}$

$m = 3k$

$4m - 3n = 3$

AP with $d = 3$

AP with $d = 4$

m	n
3	3
6	7
9	11

3, 3 ; 4, 4

$t_k \leq 100$

$3 + (k-1)4 \leq 100$

$k - 1 \leq \frac{97}{4}$

$k - 1 \leq 24.25$

$k \leq 25.25$

$k = 25$

$t_k = 3 + (k-1)4$

$2x + 3y = 500$ where $x, y \in N$.

① find the sum of all values of x, y.

② find the no of equal values of x and y.

x will have an AP. with $d = 3$

y " " " " " $d = 2$

x	y
297	2
294	4
291	6
⋮	⋮
t_m	$t_n = 2 + (n-1)2$
	$= 2n$

$2x + 3y = 500$
even, even
even

y starts from 2.

$\frac{494}{2} = 297$

$2n \leq 500$

$\boxed{n \leq 250}$

$n = 249$.