Q. The value of $\int_1^a [x]\, f'(x)\, dx$, $a > 1$ and $[x]$ denotes the greatest integer fn is:

(a) $a f(a) - \{f(1) + f(2) + \cdots + f([a])\}$

(b) $[a] f(a) - \{f(1) + f(2) + \cdots + f([a])\}$

(c) $[a] f([a]) - \{f(1) + f(2) + \cdots + f(a)\}$

(d) $a f([a]) - \{f(1) + f(2) + \cdots + f(a)\}$.

Let $a = k + h$, where $[a] = k$, $0 \le h < 1$.

$$\therefore \int_1^a [x]\, f'(x)\, dx$$

$$= \int_1^2 [x] f'(x)\, dx + \int_2^3 [x] f'(x)\, dx + \cdots + \int_{k-1}^{k} [x] f'(x)\, dx + \int_k^{k+h} [x] \cdot f'(x)\, dx$$

$$= \int_1^2 1 \cdot f'(x)\, dx + \int_2^3 2 f'(x)\, dx + \cdots + \int_{k-1}^{k} (k-1) f'(x)\, dx + \int_k^{k+h} k \cdot f'(x)\, dx.$$

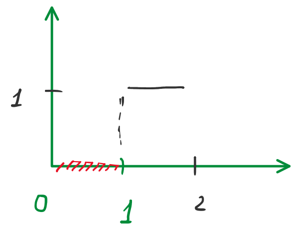
$3[f(4) - f(3)]$

$$= [f(2) - f(1)] + 2[f(3) - f(2)] + \cdots + (k-1)[f(k) - f(k-1)] + k[f(k+h) - f(k)].$$

$$= \{-f(1) - f(2) - f(3) - \cdots - f(k)\} + \overset{[a]}{k}\, f(\overset{a}{k+h}).$$

$$= [a] f(a) - \{f(1) + f(2) + \cdots + f([a])\}$$

Eg: $\int_0^{1.8} [x]\, dx$



$\cdot [x] = \begin{cases} 0, & 0 \le x < 1 \\ 1, & 1 \le x < 2 \end{cases}$

$$\int_0^{1.8} [x]\, dx = \int_0^1 [x]\, dx + \int_1^{1.8} [x]\, dx$$

$$= \int_0^1 0\, dx + \int_1^{1.8} 1\, dx$$

Q. Let $f(x) = \begin{cases} x|x|, & x \le -1 \\ [x+1] + [1-x], & -1 < x < 1. \\ -x|x|, & x \ge 1. \end{cases}$

Then: $\int_{-2}^{2} f(x)\,dx =$ (a) $-\frac{8}{3}$ (b) $-\frac{7}{3}$ (c) $\frac{7}{3}$ (d) $\frac{8}{3}$.

$$f(x) = \begin{cases} -x^2, & -2 \le x \le -1 \\ 1, & -1 \le x < 0 \\ 2, & x = 0 \\ 1 & 0 < x \le 1 \\ -x^2, & 1 \le x \le 2 \end{cases}$$

$[x+1] + [1-x], \; -1 < x < 1$

$-1 \le x < 0 \Rightarrow 0 \le x+1 < 1$

$[x+1] = 0.$

$1 \ge -x > 0.$

$2 \ge 1-x > 1 \Rightarrow 1 < (1-x) \le 2$

$[1-x] = 1.$

$f(x)$ is an Even fn.

$$\int_{-2}^{2} f(x)\,dx = 2\int_{0}^{2} f(x)\,dx$$

$$= 2\left[\int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx\right].$$

$$= 2\left[\int_0^1 1\,dx + \int_1^2 -x^2\,dx\right]$$

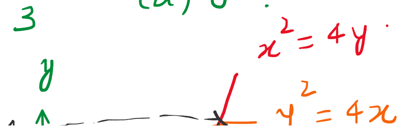
$$= 2\left\{1 - \left[\frac{x^3}{3}\right]_1^2\right\}$$

$$= 2\left\{1 - \frac{8}{3} + \frac{1}{3}\right\}$$

$$= 2\left\{1 - \frac{7}{3}\right\} = 2\left\{-\frac{4}{3}\right\} = -\frac{8}{3}.$$

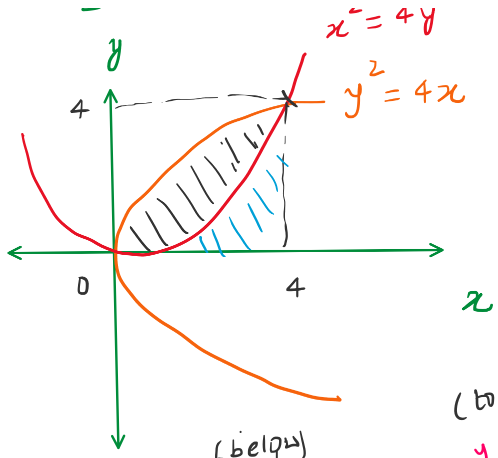
Q. Area of the region bounded by the curves: $y^2 = 4x$ and $x^2 = 4y$ is: (a) $\frac{32}{3}$ (b) $\frac{16}{3}$ (c) $\frac{8}{3}$ (d) 0.

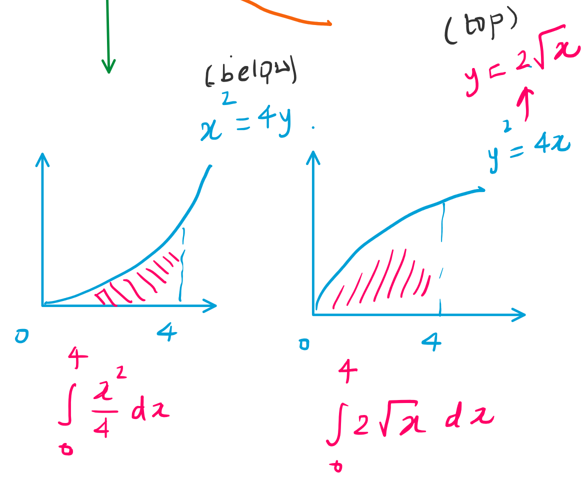
$A = \int^{4} 2\sqrt{x}\,dx - \int^{4} x^2\,d$



$$A = \int_0^4 2\sqrt{x}\, dx - \int_0^4 \frac{x^2}{4}\, dx$$

$$= \frac{16}{3}.$$





Q. The area of the region :

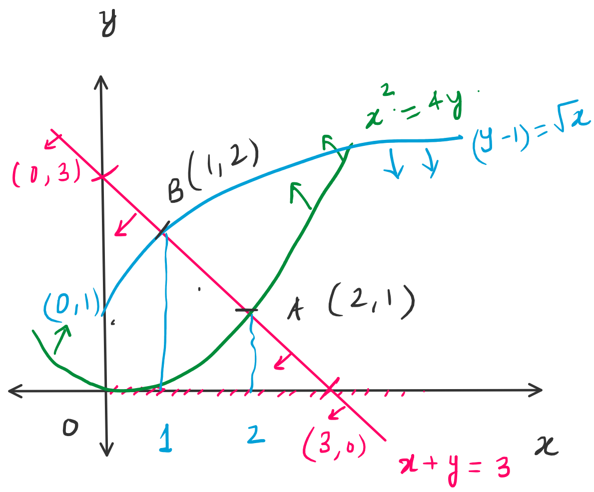
$$\{(x, y) : x \geq 0,\ x + y \leq 3,\ x^2 \leq 4y \text{ and } y \leq 1 + \sqrt{x}\} \text{ is:}$$

(a) $5/2$ (b) $59/12$ (c) $3/2$ (d) $7/3$.

$y = 1 + \sqrt{x}$.

or, $(y-1) = \sqrt{x} \Rightarrow x = 0, y = 1$

$y = \sqrt{x}$



HW.

$$A = \int_0^1 (\ -\ )\, dx + \int_1^2 (\ -\ )\, dx.$$