~~If all~~

The ory of Equations

$y = f(x)$ — mono variable

$y = f(x_1, x_2) \rightarrow$ Dual variable

Result

$0 \le x \le 2$

$y = 7x^2 + 4$ → Domain

$y = 4$
$y = 32$ ← Range

An

$y = x^3 + 3x^2 + 7x$ ← Degree of the equation

Cubic

3 ~~different~~ distinct roots

1, 2, 2

Real imaginary

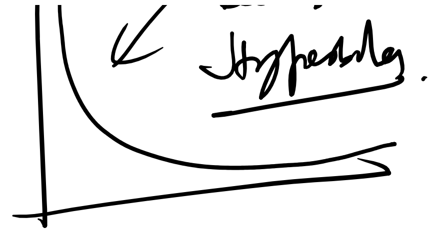
$y = x^{10} - 7x^3 + 8x^4$

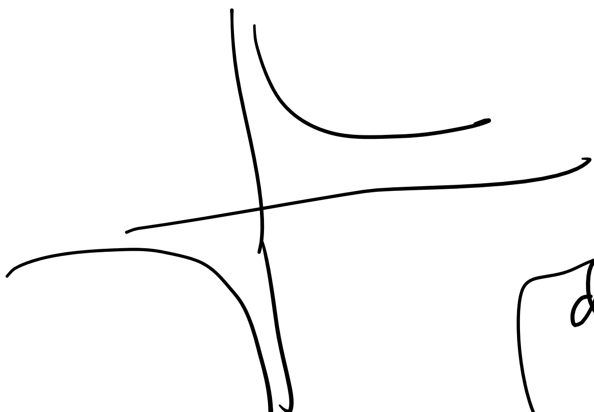
2 → quadratic
3 → Cubic
4 → bi-quadratic
5 → Penta

$y = 2x + 3$ → curve
→ Curve with Zero Curvature.

$y = \frac{1}{x}$ $\boxed{xy = 1}$ → ← Rectangular Hyperbola..

$y = \frac{1}{x}$ $\quad \boxed{xy = 1} \rightarrow$





$-x = -\frac{1}{y}$

$xy = -1$

$x > 0 \quad y < 0$

$y > 0 \quad x < 0$

$\boxed{\text{degree} = \text{no of Roots}}$

Descarte's Rule of Sign

$y\, f(x) = +10x^4 - 5x^3 + 10x^2 + 7x + 9$ $\quad$ 3

characteristic of Roots < Real / Imaginary $\quad >< 0$

$+x \rightarrow$ Total sign change $\rightarrow$ +ve Real Root $\rightarrow$ 2

$y(-x) = 10x^4 + 5x^3 + 10x^2 - 7x + 9 \longrightarrow$ 2

Quadratic $\rightarrow$ 4 $\qquad 2 + 2 \rightarrow 4$

Bi-Quadratic → 4   2+2 → 4

All Roots are Real   2 +ve
2 -ve

$y = (x^9 - 7x^8 + 8x^7 - 7x^6 + 4x^5 + 5x^4 - 3x^3 + 18x^2 - 100)$

~~6~~ 7 +ve Real

$x \to (-x) \to (-x^9 - 7x^8 - 8x^7 - 7x^6 - 4x^5 + 5x^4 + 3x^3 + 18x^2 - 100)$

2 -ve Real Roots

$y = x^{10}$ +ve ~~6~~
-ve ~~2~~ 1
$9 - 7 = 2$ Imaginary Roots.

$ax^2 + bx + c = 0$ α, β

$\boxed{D = b^2 - 4ac}$

$D \geq 0$ Real Roots
$D < 0$ Complex Roots
$D > 0$ Real / distinct Roots

$a_1x^2 + b_1x + c_1 = 0 \to D_1$
$a_2x^2 + b_2x + c_2 = 0 \to D_2$

$D_1 + D_2 \geq 0$ D_1 and $D_2 \geq 0$
then at least

$D \angle 0$, $D_2 > 0$ / < 0

then

$D_1 < 0$, $D_2 > 0$

> 0 $\quad$ < 0

D → Perfect Square $\quad$ 16, 25, 49, 64

then Rational

$ax^2+bx+c=0$ $\quad$ α, β $\quad$ $\alpha < \beta$

$$\alpha = \frac{-b+\sqrt{D}}{2a} \qquad \beta = \frac{-b-\sqrt{D}}{2a}$$

$$\alpha+\beta = -b/a \qquad \alpha\beta = c/a$$

$$\alpha - \beta = \frac{\sqrt{D}}{a}$$

$$\alpha^3+\beta^3 = -\left(\frac{b^3-3abc}{a^3}\right)$$

$$\alpha^2+\beta^2 = \frac{b^2-2ac}{a^2}$$

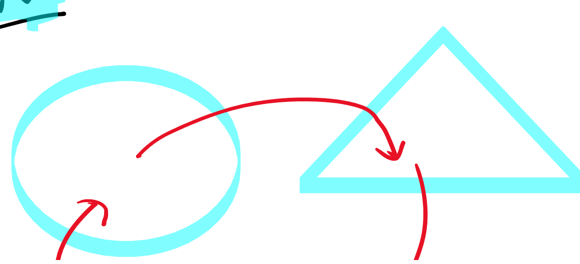
$$\alpha^3-\beta^3 = \frac{\sqrt{D}(D+3ac)}{a^3}$$

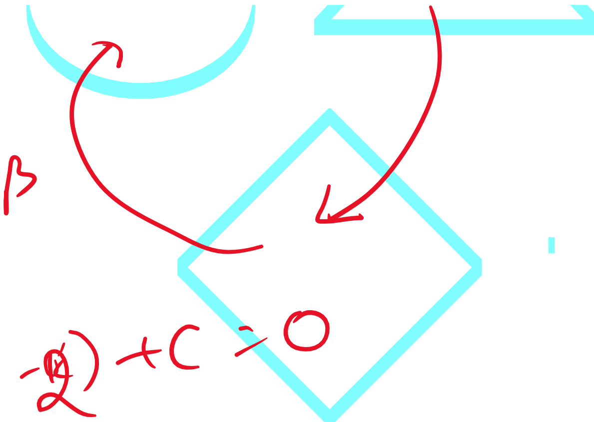
$$\alpha^2-\beta^2 = \frac{-b\sqrt{D}}{a^2}$$

$$\alpha^4+\beta^4 = (\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2 = \frac{b^4+2a^2c^2-4acb^2}{a^4}$$

$$\alpha^4-\beta^4 = \frac{-b\sqrt{D}(b^2-2ac)}{a^4}$$

TRANSFORMATION





eg!! $ax^2+bx+c=0$ α, β

(i) $\alpha+2, \beta+2$

$a(x-2)^2+b(x-2)+c=0$

(ii) Similarly

If $\alpha-2, \beta-2$

then $\boxed{a(x+2)^2+b(x+2)+c=0}$

(iii) $ax^2+bx+c=0$ α, β

5 times $5\alpha, 5\beta$ $a(x/5)^2+b(x/5)+c=0$

(iv) 1/5 times Replace with (5) $\rightarrow a(5x)^2+b(5x)+c=0$

(v) $-\alpha, -\beta$ $\boxed{ax^2-bx+c=0}$

(vi) $\frac{1}{\alpha}, \frac{1}{\beta}$ $x \rightarrow 1/x$

$a\left(\frac{1}{x^2}\right)+b\left(\frac{1}{x}\right)+c=0$

$\boxed{a+bx+cx^2=0}$

(vii) $p\alpha+q, p\beta+q$

$x \rightarrow \frac{x-q}{p}$

$x \rightarrow 5x$ scale change

$x \rightarrow x+2$ origin change

$x \rightarrow 3x+9$

$x \to \frac{x-a}{b}$

$x \to (3x+9)$

$$\boxed{a\left(\frac{x-a}{b}\right)^2 + b\left(\frac{x-a}{b}\right) + c = 0}$$

$3(x+9) \to 3x+27$ X

Scale change

comm. forms

α^n, β^n . $x \to x^{1/n}$

$\alpha^{1/n}, \beta^{1/n}$ $x \to x^n$

$a(x^{1/n})^2 + b(x^{1/n}) + c = 0$

$a(x^n)^2 + b(x^n) + c = 0$

Some Special Cases ..

$x^n = 0$

$\alpha = \beta = 0$ $\boxed{b = c = 0}$

Q $x^2 + 2(m-1)x + (m+5) = 0$

Roots are equal in value / opposite in sign ..

$x = \frac{1}{y}$

$\boxed{xy = -1}$ X

$\boxed{xy = 1}$ ✓

$D = b^2 - 4ac = 4(m-1)^2 - 4(m+5)$

$= 4(m^2 - 3m - 4)$

$D = 4(m-4)(m+1)$

$a = 1 > 0$

(1) $\boxed{b = 0}$ $\boxed{D > 0}$

$2(m-1) = 0$

$\boxed{m = 1}$

ϕ

$(-\infty, -1) \cup (4, \infty)$

$4(m-4)(m+1) > 0$

$m \in \phi$

$\boxed{m > 4}$ (5) $m > 0$ $m > -1$

$m+1 > 0$

$m > -1$

No value of m

Reciprocals

(ii) Roots are Reciprocals

$\boxed{a = c}$, $\boxed{D \geq 0}$

$m + 5 = 1$

$m = -4$

$4(m-4)(m+1) \geq 0$

$(-\infty, -1] \cup [4, \infty)$

(iii) $\boxed{m = -4}$ ✓

$\boxed{5, -5}$ Roots are opposite in sign

~~$a = c, D$~~ $\boxed{a > 0}$ $\boxed{c < 0}$ $\boxed{D > 0}$

$1 > 0$

$\boxed{(-\infty, -1)} \cup (4, \infty)$

$m + 5 < 0$

$\boxed{m < -5}$

-1 to $-\infty$

-5 to $-\infty$

-5 to $-\infty$

$(-\infty, -5)$

$D \geq 0 \to (-\infty, -1] \cup [4, \infty)$ $m \in$

$D > 0 \to (-\infty, -1) \cup (4, \infty)$

$\boxed{9062395723}$

At least 1 root is Positive

+ve roots $= 1, 2$

Add up 2 cases $\to a > 0, c < 0, D > 0$

$\boxed{m = (-\infty, -5)}$

(i) Roots are opposite in sign

(ii) Both roots are positive

$\to$ $\boxed{a > 0}$ $\boxed{b < 0}$ $\boxed{c > 0}$ $\boxed{D \geq 0}$

$m \in (-\infty, -1] \cup [4, \infty)$

$m / 4, m > -5,$

$m \in (-5, -1]$

$m \in ($

$m < 4, \; m > -5,$ $\quad m \in (-5, -1]$

$(-\infty, -5) \cup (-5, -1]$

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Condition of Common Roots

$ax^2 + bx + c = 0 \qquad a'x^2 + b'x + c' = 0$

α Common Root

$$\begin{vmatrix} a & b \\ a' & b' \end{vmatrix} \times \begin{vmatrix} b & c \\ b' & c' \end{vmatrix} = \begin{vmatrix} c & a \\ c' & a' \end{vmatrix}^2$$

Required Condition that there is a Common Root

Both Roots are Common

$$\boxed{\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}}$$

Special Tips

$0 < x^2 < a^2 \qquad 0 < |x| < a$

$\rightarrow -a < x < a$

$\boxed{x \in (-a, a)}$ $\quad x \neq 0$

$a^2 \le x^2 \le b^2 \qquad a \le |x| \le b$

for $a^2 \le x^2 \le b^2$ $\quad a \le |x| \le$

$-b \le x \le -a$

$a \le x \le b$

$x \in [-b, -a] \cup [a, b]$

$a > b$

$-a < -b$

$(x-a)(x-b) > 0$

$a < b$

$x < a$ or $x > b$

$\boxed{x \in (-\infty, a) \cup (b, \infty)}$