21 November 2023 07:40 PM

Measures of Dispersion

Absolute Measures

1. Range
2. Interquartile range
3. Mean / Median deviation
4. Variance
5. Standard deviation

Relative Measures

1. Coefficient of variance CV
2. " " Q.D.

⋮

10, 20, 50, 100, 900

1. Range → Max − Min value

$900 - 10 = \underline{\underline{890}}$

2. Mean deviation about Mean (without frequency)

$$M.D(\bar{x}) = \frac{1}{n} \sum_{i=1}^{n} \left| x_i - \bar{x} \right|$$

with frequency: n obs: $x_1, x_2 \dots x_n$

f : $f_1, f_2 \dots f_n$

$f : f_1, f_2 \ldots f_n$

Tot frequency, $N = \sum_{i=1}^{n} f_i$

$$M.D(\bar{x}) = \frac{1}{\Sigma f_i} \sum_{i=1}^{n} |x_i - \bar{x}| f_i$$

3. Variance and s.d (without frequency)

$$\text{Variance, } \sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$= \frac{1}{n} \sum_{i=1}^{n} \left(x_i^2 - 2 \cdot x_i \bar{x} + \bar{x}^2 \right)$$

$$= \frac{1}{n} \sum_{i=1}^{n} x_i^2 - \frac{2\bar{x}}{n} \sum_{i=1}^{n} x_i + \frac{1}{n} \sum_{i=1}^{n} \bar{x}^2$$

$$= \frac{1}{n} \Sigma x_i^2 - 2\bar{x}^2 + \frac{\cancel{n}}{\cancel{n}} \bar{x}^2$$

$$\text{Variance, } \boxed{\sigma^2 = \frac{1}{n} \Sigma x_i^2 - \bar{x}^2}$$

$\therefore$ standard deviation, $\sigma = \sqrt{\text{Variance}}$

$$\boxed{\sigma = \sqrt{\frac{1}{n} \Sigma x_i^2 - \bar{x}^2}}$$

or, $$\boxed{\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2}}$$

1: if all observations are constant, then variance is 0.

Let $x_i = c$ for all $i = 1, 2, \ldots, n)$

then mean, $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{n}\sum_{i=1}^{n} c$

$= \frac{\cancel{n}c}{\cancel{n}}$

$\boxed{\bar{x} = c}$

variance, $\sigma^2 = \frac{1}{n}\sum x_i^2 - \bar{x}^2$

$= \frac{1}{n}\underbrace{(c^2 + c^2 + \cdots + c^2)}_{n \text{ times}} - c^2$

$= \frac{1}{\cancel{n}} \times \cancel{n}c^2 - c^2 \quad = c^2 - c^2 = 0$

(ans)

2: if $y = a + bx_i$

then what is the variance of y

(a and b are constants).

Variance of y, $\sigma_y^2 = \frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2$

and $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{1}{n}\sum_{i=1}^{n}(a + bx_i)$

$= \frac{1}{n}\sum a + b\frac{1}{n}\sum x_i$

$$= \frac{\cancel{n} a}{\cancel{n}} + b\bar{x}$$

$$\boxed{\bar{y} = a + b\bar{x}}$$

$$\sigma_y^2 = \frac{1}{n}\sum (y_i - \bar{y})^2$$

$$= \frac{1}{n}\sum \left(a + bx_i - (a + b\bar{x})\right)^2$$

$$= \frac{1}{n}\sum \left(b(x_i - \bar{x})\right)^2$$

$$= \frac{b^2}{n}\sum (x_i - \bar{x})^2$$

$$\boxed{\sigma_y^2 = b^2 \cdot \sigma_x^2}$$

$y = a + bx$

Result $\boxed{\begin{array}{l} \bar{y} = a + b\bar{x} \\ \sigma_y^2 = b^2\sigma_x^2 \end{array}}$

With frequency : $\boxed{(N = \sum f_i) \text{ and } \bar{x} = \frac{1}{N}\sum x_i f_i}$

variance, $\sigma^2 = \frac{1}{N}\sum_{i=1}^{n} (x_i - \bar{x})^2 f_i$

or, $\sigma^2 = \frac{1}{N}\sum_{i=1}^{n} x_i^2 \cdot f_i - (\bar{x})^2$

$s.d = \sqrt{\text{variance}}$.

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Q S.d calculated from set of 32 obs is 5. If sum of observation is 80, what is the sum of squares of these observations?

$n = 32$, $\sigma = 5$, $\Sigma x_i = 80$

$\Sigma x^2 = ?$

$\therefore \sigma^2 = \frac{1}{n}\Sigma x^2 - \bar{x}^2$

$\bar{x} = \frac{1}{n}\Sigma x_i = \frac{1}{\cancel{32}_{\cancel{8}\,2}} \times \cancel{80}^{\cancel{20}\,5} = \frac{5}{2}$

$5^2 = \frac{1}{32}\Sigma x^2 - \left(\frac{5}{2}\right)^2$

$25 = \frac{1}{32}\Sigma x^2 - \frac{25}{4}$

$25 = \frac{\Sigma x^2 - 8 \times 25}{32}$

$25 \times 32 = \Sigma x^2 - 200$

$\Sigma x^2 = 800 + 200$

$$\Sigma x^2 = 1000 \quad \text{ans}$$

Q The mean income per month of a friendly society of 25 members is Rs 350 and the s.d is Rs 50. Five more members are admitted to the society and their income in Rs per month are (260, 300, 320, 490, 590). Find mean and sd of income for new group. ($n' = 25+5 = 30$)

Given $n = 25$, $\bar{x} = 350$, $\sigma = 50$

Now $\bar{x} = \frac{1}{n} \Sigma x_i$

or, $350 = \frac{\Sigma x_i}{25}$

or, $\boxed{\Sigma x_i = 25 \times 350 = 8750}$

New $\Sigma x' = 8750 + (260+300+320+490+590)$
$= 10710$

$n' = 30$

$\therefore$ New mean, $\bar{x}' = \frac{1}{n'} \Sigma x'$

$\sigma^2 = \frac{1}{n} \Sigma x_i^2 - \bar{x}^2$

$2500 = \frac{1}{25} \Sigma x_i^2 - (350)^2$

$\Sigma x^2 = 2500 \times 25 + (350)^2 \times 25$

$\Sigma x^2 = 3125000$

New sum of square

$\Sigma x^{2\prime}$

$\therefore$ New mean, $\bar{x}' = \frac{1}{n'} \Sigma x'$

$= \frac{1}{30} \times 10710$

$\boxed{\bar{x}' = 357}$

ans

$\Sigma x^{2\prime}$

$= 3125000 + (260^2 + 300^2 + 320^2 + 490^2 + 590^2)$

$= 3973200$

$\sigma^{2\prime} = \frac{1}{30} \times 3973200 - 357^2$

$\sigma^{2\prime} = 4991$

$\therefore \sigma' = \sqrt{4991}$

$\boxed{= 70.65}$

(ans)

Q The mean and s.d of $n =$ 20 items is found to be 10 and 2 resp. At the time of checking it was found that one item 8 was incorrect. Calculate the mean and s.d if

(i) The wrong item is omitted

(ii) it is replaced by 12.

Given: $n = 20$, $\bar{x} = 10$ and $\sigma = 2$

$\bar{x} = 10$

or $\frac{1}{n} \Sigma x_i = 10$

$\Sigma x_i = n \times 10 = 20 \times 10 = 200$

$\sigma = 2$

$\sigma^2 = 4$

$\frac{1}{n} \Sigma x^2 - \bar{x}^2 = 4$

$\frac{1}{n} \Sigma x^2 - 10^2 = 4$

n
$\Sigma x_i = n \times 10 = 20 \times 10 = 200$

n
$\frac{1}{20} \Sigma x^2 - 10^2 = 4$
$\Sigma x^2 = 80 + 100 \times 20$
$\Sigma x^2 = 2080$

(i) if 8 is omitted then new sum of obs is
$\Sigma x' = 200 - 8 = 192$

and $n' = 19$

$\therefore$ new mean, $\bar{x}' = \frac{1}{n'} \Sigma x'$

$\bar{x}' = \frac{192}{19}$

$\bar{x}' = 10.1$

$\therefore$ new sum of square

$\Sigma x^{2'} = 2080 - 8^2$
$= 2080 - 64$
$\Sigma x_i' = 2016$

$\therefore$ new varian $\sigma^{2'} = \frac{1}{n'} \Sigma x^{2'} - \bar{x}'^2$

$= \frac{1}{19} \times 2016 - 10.1^2$

$= 106.10 - 102.01$

$= 3.9889$

s.d, $\sigma = \sqrt{3.9889} = 1.997$ (ans).

$n = 20$

(ii) $\Sigma x = 200$ $\quad \Sigma x^2 = 2080$

Now replacing 8 with 12, we get

$\Sigma x' = 200 - 8 + 12$
204

$(\Sigma x^2)' = 2080 - 8^2 + 12^2$
$= 2160$

$$\Sigma x' = 200 - 8 + 12$$
$$= 204$$

$$\bar{x}' = \frac{1}{n} \Sigma x'$$

$$\bar{x}' = \frac{204}{20} = 10.2$$

$$(\Sigma x^2)' =$$
$$= 2160$$

$$\sigma^{2\prime} = \frac{1}{n} \Sigma x^{2\prime} - \bar{x}^{\prime 2}$$

$$\sigma^{2\prime} = \frac{1}{20} \times 2160 - \left(\frac{204}{20}\right)^2$$
$$= 3.96$$

$$\sigma = \sqrt{3.96} = 1.990$$

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