28 August 2023 07:02 PM

1. AM $\geq$ GM $\geq$ HM (Prove it for any 'n' observations)

Let $x_1, x_2, \ldots, x_n$ be a set of 'n' observations.

then let AM, GM and HM resp be denoted by (A) (G) and (H) such that,

$$A = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

$$G = (x_1 \cdot x_2 \cdots x_n)^{1/n} \quad \text{and} \quad H = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}$$

Considering only two observations say x_1 and x_2 then,

$$\left(\sqrt{x_1} - \sqrt{x_2}\right)^2 \geq 0$$

or, $$\left(x_1 + x_2 - 2\sqrt{x_1 \cdot x_2}\right) \geq 0$$

$$x_1 + x_2 \geq 2\sqrt{x_1 x_2}$$

or,

or, $$\boxed{\left(\frac{x_1 + x_2}{2}\right) \geq \left(\sqrt{x_1 \cdot x_2}\right)}$$

or, (AM $\geq$ GM) when $n=2$. — (1)

Similarly consider x_3 and x_4, we have

$$\boxed{\frac{x_3 + x_4}{2} \geq \sqrt{x_3 \cdot x_4}}$$

If we consider two quantities $\left(\frac{x_1 + x_2}{2}\right)$ and $\left(\frac{x_3 + x_4}{2}\right)$

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from eq (1) AM $\geq$ GM for $n=2$

$$\therefore \frac{\left(\frac{x_1+x_2}{2}\right)+\left(\frac{x_3+x_4}{2}\right)}{2} \geq \sqrt{\left(\frac{x_1+x_2}{2}\right)\left(\frac{x_3+x_4}{2}\right)}$$

$$\frac{x_1+x_2+x_3+x_4}{4} \geq \sqrt{(x_1\cdot x_2)^{1/2}\,(x_3\cdot x_4)^{1/2}}$$

$$\frac{x_1+x_2+x_3+x_4}{4} \geq (x_1\cdot x_2\cdot x_3\cdot x_4)^{1/4}$$

$$\boxed{\text{AM} \geq \text{GM} \quad \text{for } n=4 \text{ obs.}}$$

Proceeding this way it can be shown that AM $\geq$ GM whenever $n = 2, 4, 8, 16$ ..... any form i.e. $n = 2^m$ where m is any integer.

Let us consider $2^{m-1} < n < 2^m$

We consider $2^m = N$ say values consist of 'n' obs $(x_1, x_2, \ldots, x_n)$ and further $(N-n)$ values each equal to A

i.e, $A = \frac{(x_1+x_2+\cdots+x_n)}{n}$

N obs.

~~concept~~

$N \rightarrow (N-n) \Rightarrow A, A, \ldots, A$

$\hookrightarrow n \Rightarrow x_1, x_2 \cdots x_n$

$N: \underbrace{x_1, x_2, x_3 \cdots x_n}_{n}, \underbrace{A, A, \ldots, A}_{(N-n)}$

$A =$ mean of $(x_1, x_2, \ldots, x_n)$

$$n$$

$A = \text{mean of } (x_1, x_2, \ldots, x_n)$

ie $\underbrace{x_1, x_2, \ldots x_n}_{n \text{ obs}}, \underbrace{A, A, \ldots, A}_{(N-n) \text{ obs}}$ — N obs.

$\therefore$ Am of these 'N' obs will be

$$\frac{(x_1 + x_2 + \cdots + x_n) + A + A + \cdots + A}{N} = \frac{(nA) + (N-n)A}{N}$$

$$= \frac{\cancel{nA} + \cancel{N}A - \cancel{nA}}{\cancel{N}}$$

$$= A \quad \text{— } (2).$$

GM for 'N' observations will be,

$$(x_1 \cdot x_2 \cdot x_3 \cdots x_n \cdot A \cdot A \cdots A)^{1/N}$$

$$= \left(G^n \cdot A^{N-n}\right)^{1/N} \quad \text{— } (3)$$

We have prooved above that Am $\geq$ GM for $N = 2^m$

[from (2) and (3)]

$$A \geq \left(G^n A^{N-n}\right)^{1/N}$$

$$\Rightarrow A^N \geq G^n \cdot A^{N-n}$$

$$\Rightarrow A^{\cancel{N}-\cancel{N}+n} \geq G^n$$

$$\Rightarrow A^n \geq G^n$$

$\Rightarrow \boxed{A \geq G}$ → GM of 'n' obs.

↳ AM of 'n' obs

$\therefore$ AM $\geq$ GM for 'n' observations → (4)

We shall now prove GM $\geq$ HM for 'n' obs.

Let us consider 'n' observations $\frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3}, \ldots, \frac{1}{x_n}$

then AM of these values $= \dfrac{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \cdots + \frac{1}{x_n}}{n}$

$$= \frac{1}{\dfrac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}}$$

$$= \frac{1}{H}$$

H → HM of 'n' obs.

→ (5)

Again for these 'n' obs

$$GM = \left(\frac{1}{x_1} \cdot \frac{1}{x_2} \cdots \frac{1}{x_n}\right)^{1/n}$$

$$= \left(\frac{1}{x_1 \cdot x_2 \cdots x_n}\right)^{1/n}$$

$$= \left(\frac{1}{G^n}\right)^{1/n}$$

$$= \frac{1}{G} \rightarrow (6)$$

So since AM $\geq$ GM, in this case from (5) and (6)

$$\frac{1}{H} \geq \frac{1}{G}$$

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$$\boxed{G \geq H} \longrightarrow (7)$$

$\therefore$ from eq (4) and eq (7) we have proved that $\boxed{A \geq G \geq H}$ for 'n' obs.

(Proved)

If x_1 and x_2 are two positive values of a variable, prove that their geometric mean is equal to geometric mean of their arithmetic mean and Harmonic mean.

That is : prove that $\boxed{GM = \sqrt{AM \times HM}}$

Proof : Let A, G, H be the AM GM and HM of the two values x_1 and x_2. Then by definition we know

$$A = \frac{x_1 + x_2}{2} \quad \text{—— (1)}$$

$$G = \sqrt{x_1 \cdot x_2} \quad \text{—— (2)}$$

$$H = \frac{2}{\frac{1}{x_1} + \frac{1}{x_2}} = \frac{2}{\frac{x_1 + x_2}{x_1 x_2}} = \frac{2x_1x_2}{x_1 + x_2} \quad \text{—— (3)}$$

$$\frac{1}{x_1}+\frac{1}{x_2} \qquad x_1 x_2 \quad (3)$$

$$\therefore A \times H = \left(\frac{\cancel{x_1+x_2}}{\cancel{2}}\right) \times \left(\frac{\cancel{2}x_1 x_2}{\cancel{x_1+x_2}}\right)$$

$$A \times H = x_1 x_2$$

$$A \times H = G^2 \quad (\text{from } 2)$$

$$\therefore \sqrt{A \times H} = G$$

that GM of two obs x_1, x_2 = GM of AM and HM.

(Hence proved).

Formula

(a) $$\boxed{\text{Median} = x_l + \frac{\frac{N}{2} - CF}{f_m} \times i}$$

Here x_l = lower boundary of median class

N = Total frequency

CF = Cummulative frequency preceding the median class

f_m = frequency of the median class

i = Class width of the " "

(b) $$\boxed{\text{Mode} = x_l + \frac{f_0 - f_{-1}}{2f_0 - f_{-1} - f_1} \times i}$$

Here x_l = lower boundary of modal class

f_0 = frequency of " "

f_{-1} = " preceding modal class

modal class'

f_1 = frequency succeeding to the modal class

i = class width

Ⓒ Relation between Mean, Median and Mode.

$$\boxed{\text{Mean} - \text{Mode} = 3(\text{Mean} - \text{Median})}$$

Q Find the median by interpolation or from ogive or using formula:

Weekly wages	0-20	20-40	40-60	60-80	80-100
Number of workers	40	51	64	38	7

ogive ⇒ graphical representation of cummulative frequency.

Calculation of cummulative frequency.

Class Boundary (wages)	frequency (N workers)	Cummulative Frequency Less than	Cummulative Frequency More than
0	0	0	200 = N
20	40	40	160
40	51	91	109

20	40	40	160
40	51	91	109
60	64	155	45
80	38	193	7
100	7	200 (N)	0
	$\Sigma f = 200 (N)$		

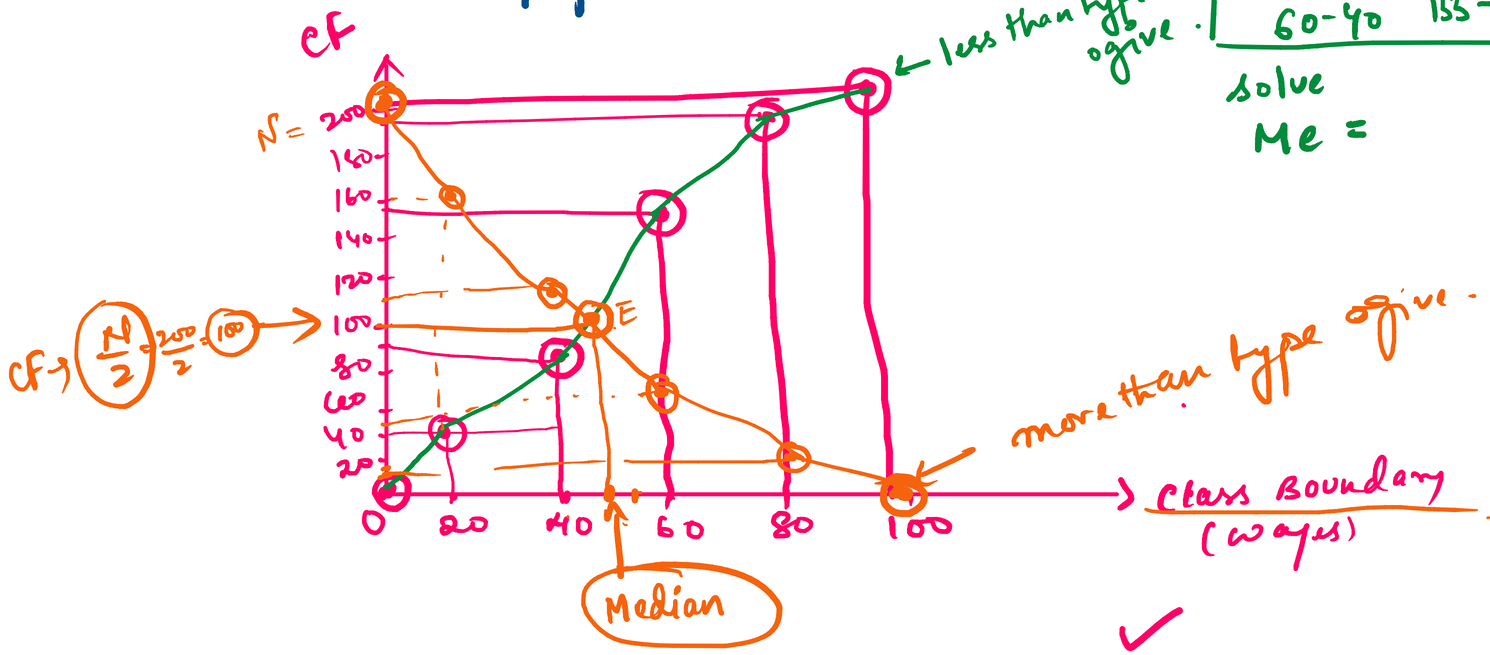
Me

$100 = \frac{N}{2}$

$$\frac{60 - Me}{60 - 40} = \frac{155 - 100}{155 - 91}$$

solve

Me =



Method of interpolation:

$$\frac{60 - Me}{60 - 40} = \frac{155 - 100}{155 - 91}$$

$$\frac{(60 - Me)}{20} = \frac{55}{64}$$

$$Me = \left(60 - \frac{55}{64}\right) \times 20$$

$$\text{Median (Me)} = 43$$

Formula:

$$Me = x_l + \frac{N/2 - cF}{f_m} \times i = 40 + \left[\frac{100 - 91}{64}\right] \times 20$$

$$= \text{Ans}$$

CB	f	CF

Ans

CB	f	CF
0-20	40	40
20-40	51	91
40-60	64 $= f_m$	155
60-80	38	193
80-100	7	200 = N
	$\Sigma f = 200 = N$	

Since $N/2 = 100$ we consider the next class