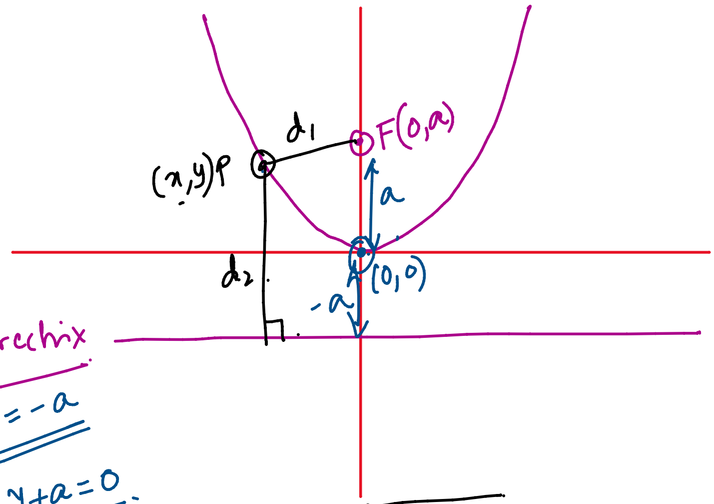
16 December 2023 17:00

Conic Sections

- Circle
- Parabola
- Ellipse
- Hyperbola.



directrix

$y=-a$

$y+a=0$

$0\times x+1\times y+a=0$

Parabola → locus of a point whose distance from a fixed point (focus) = distance from a fixed line (directrix)

$d_1 = d_2$

$\frac{d_1}{d_2} = e \text{ (eccentricity)} = 1$

$d_1 = \sqrt{x^2+(y-a)^2}$.

$d_2 = \frac{|(y+a)|}{1} = |y+a|$

distance of a point from a line

$P(\alpha,\beta)$

$d = \frac{|a\alpha+b\beta+c|}{\sqrt{a^2+b^2}}$.

$ax+by+c=0$

$d_1 = d_2$

$d_1^2 = d_2^2$

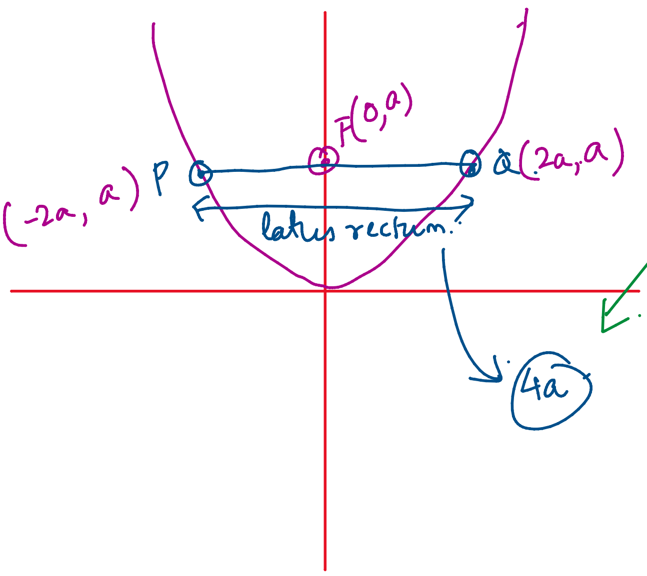
$x^2+(y-a)^2 = (y+a)^2$

$x^2 = (y+a)^2 - (y-a)^2$

$\boxed{x^2 = 4ay}$

a → focal length (distance of the focus from the origin)

$y+a=0$ → directrix



$x^2 = 4ay$. ← $y=a$

$x^2 = 4a^2$ $\quad x = \pm 2a$

$4a$

Parametric form of a parabola

$$x^2 = 4ay$$

$x = f(t)$ $\quad$ $y = g(t)$

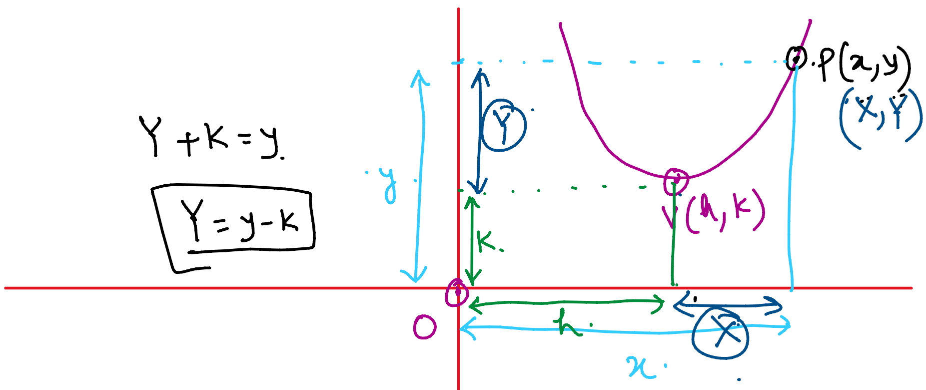
$$\boxed{y = at^2}$$

$$x^2 = 4a \cdot at^2$$

$$x^2 = 4a^2t^2$$

$$\boxed{x = 2at}$$

Generic form of a parabola



$Y + K = y$

$$\boxed{Y = y - K}$$

Shift of origin

$$X^2 = 4aY$$

$$(x-h)^2 = 4a(y-k)$$

$$\downarrow$$

$$x^2 - 2hx + h^2 = 4ay - 4ak$$

$$\boxed{x^2 - 2hx - 4ay + (h^2+4ak) = 0}$$

$h + X = x$

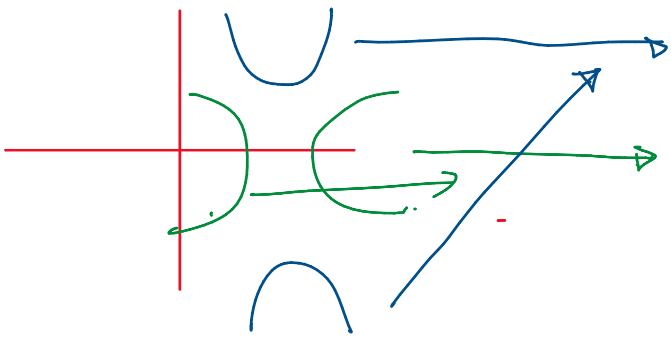
$$\boxed{X = x - h}$$

$y = f(x)$

shift the origin from $(0,0)$ to (h,k)
replace y with $(y-k)$ and
x with $(x-h)$

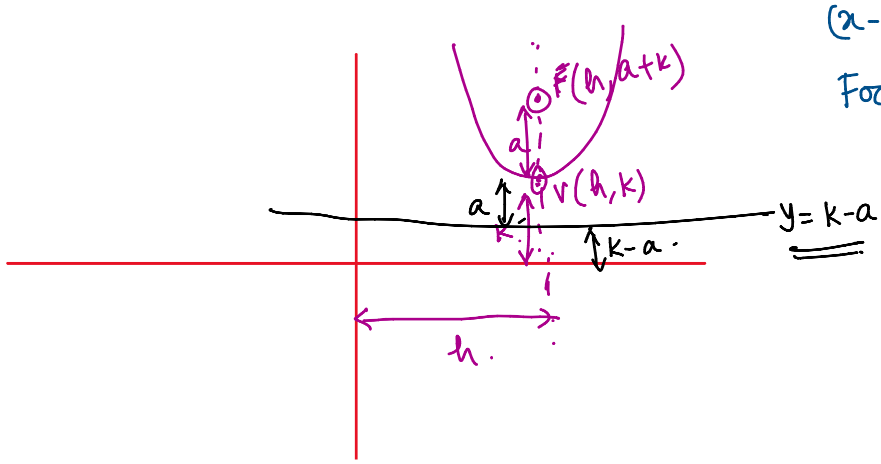
$$\boxed{y - k = f(x-h)}$$

Std eqn of a parabola



$$ax^2 + bx + cy + d = 0$$

$$ay^2 + by + cx + d = 0$$

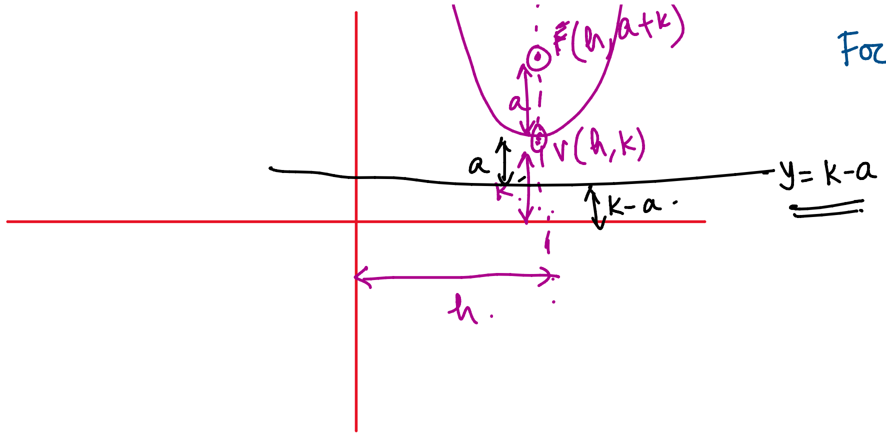


$$(x-h)^2 = 4a(y-k)$$

Focus $= (h, a+k)$

directrix

$$y = k - a$$



Focus = $(h, a+k)$

directrix

$y = k-a$

$2x^2 + 4x - 6y + 3 = 0$ $\qquad$ $2y^2 + 4y - 6x + 3 = 0$

Vertex, Focus, Directrix

$\rightarrow$ i.e. $(x-h)^2 = 4a(y-k)$

$$x^2 + 2x - 3y + \frac{3}{2} = 0$$

$$(x^2 + 2x + 1) - 1 = 3y - \frac{3}{2}$$

$$(x+1)^2 = 3y - \frac{3}{2} + 1$$

$$(x+1)^2 = 3y - \frac{1}{2} = 3\left(y - \frac{1}{6}\right) = 4 \times \left(\frac{3}{4}\right)\left(y - \frac{1}{6}\right)$$

$$(x+1)^2 = 4 \times \left(\frac{3}{4}\right)\left(y - \frac{1}{6}\right)$$

Vertex $= (h, k)$

$= \left(-1, \frac{1}{6}\right)$

$h = -1 \quad a = \frac{3}{4} \quad k = \frac{1}{6}.$

$\frac{3}{4} + \frac{1}{6} = \frac{11}{12}.$

$\frac{1}{6} - \frac{3}{4} = -\frac{7}{12}.$

Focus $= (h, a+k)$

$= \left(-1, \frac{11}{12}\right)$

Directrix : $y = k - a.$

$y = \frac{1}{6} - \frac{3}{4}$

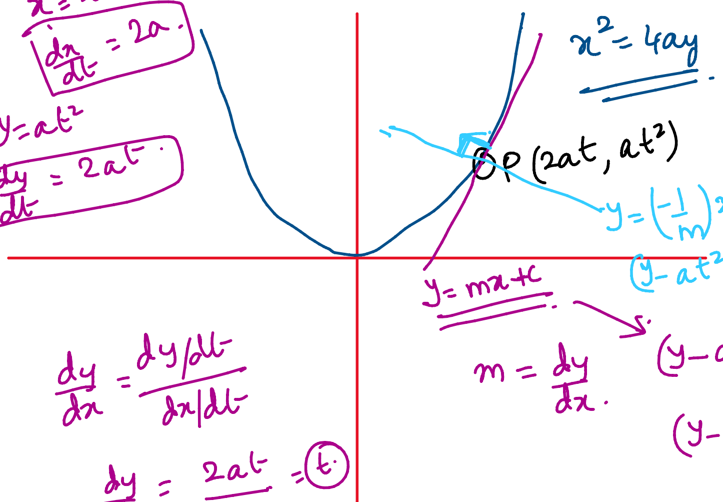
$y = -\frac{7}{12}$

$x = 2at$

$\boxed{\frac{dx}{dt} = 2a.}$

$y = at^2$

$\boxed{\frac{dy}{dt} = 2at.}$



slope of the tangent

$x = 2at \quad y = at^2$

Find the equation of the tangent and normal at P

$(y - at^2) = \left(\frac{-1}{m}\right)(x - 2at)$

$m = \frac{dy}{dx}.$

$\rightarrow (y - at^2) = m(x - 2at)$

$(y - at^2) = \frac{dy}{dx}(x - 2at)$

$y - at^2 = t(x - 2at)$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$m = \frac{dy}{dx} = \frac{2at}{2a} = t$$

$m = \frac{dy}{dx} = \frac{2at}{2a} = t$

$y - at^2 = t(x - 2at)$

$y = tx - 2at^2 + at^2$

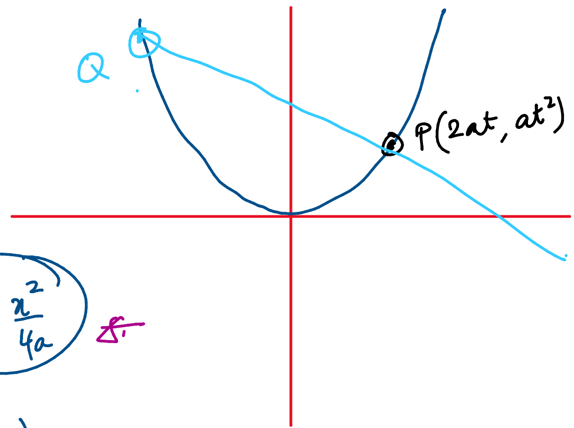
$\boxed{y = tx - at^2}$

Normal

$y - at^2 = -\frac{1}{t}(x - 2at)$

$y = -\frac{x}{t} + 2a + at^2.$

$\boxed{y = -\frac{x}{t} + a(t^2+2)}$



$x^2 = 4ay.$ $\quad y = \frac{x^2}{4a}$

$\frac{x^2}{4a} = -\frac{x}{t} + a(t^2+2)$ $\quad \Rightarrow \quad t = 2, \quad a = 1$

$\frac{x^2}{4} = -\frac{x}{2} + 6.$

$x^2 = -2x + 24$

$x^2 + 2x - 24 = 0$

$(x+6)(x-4) = 0.$

$\boxed{x = 4, -6}$

$y = \frac{x^2}{4}$

$\boxed{y = 4, 9}$

$P(4,4) \quad Q(-6,9)$