SOURAV SIR'S CLASSES

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$$AM \geq GM \geq HM.$$

$$AM = \frac{a+b}{2} \quad GM = \sqrt{ab} \quad HM = \frac{2}{\frac{1}{a}+\frac{1}{b}}.$$

$$\frac{a+b}{2} \geq \sqrt{ab} \Rightarrow a+b \geq 2\sqrt{ab}.$$

multiply both sides by $\sqrt{ab}$

$$(a+b)\sqrt{ab} \geq 2ab$$

$$\sqrt{ab} \geq \frac{2ab}{a+b} \Rightarrow \sqrt{ab} \geq \frac{2}{\frac{1}{a}+\frac{1}{b}}$$

$$GM \geq HM.$$

$$\boxed{\frac{a_1+a_2+a_3+\cdots+a_n}{n} \geq (a_1a_2\cdots a_n)^{\frac{1}{n}} \geq \frac{n}{\frac{1}{a_1}+\frac{1}{a_2}+\cdots\frac{1}{a_n}}}$$

$a_1, a_2, \dots a_n$ are non-negative real nos.

Example: Let a, b and c be nonnegative integers such that $\boxed{a + b + c = 15}$. What is the maximum value of a.b.c + a.b + b.c + c.a?

$$(a+1)(b+1)(c+1) = (ab+a+b+1)(c+1) = abc+ab+ac+a+bc+b+c+1$$

$$= abc+ab+bc+ca+\boxed{(a+b+c)}+1$$

$$= \boxed{abc+ab+bc+ca} + 15+1 = abc+ab+bc+ca+16.$$

$$AM \geq GM.$$

$$\frac{(a+1)+(b+1)+(c+1)}{3} \geq \sqrt[3]{(a+1)(b+1)(c+1)}$$

$$\frac{\boxed{a+b+c}+3}{3} \geq \sqrt[3]{abc+ab+bc+ca+16}$$

$$\frac{15+3}{3} \geq \sqrt[3]{abc+ab+bc+ca+16}$$

$$6 \geq \sqrt[3]{abc+ab+bc+ca+16}$$

$$6^3 \geq abc+ab+bc+ca+16$$

$$216 \geq abc+ab+bc+ca+16$$

$$abc+ab+bc+ca \leq 200$$

: for positive real number x and y, if 2x + 3y = 15, find the maximum value of x^2y.

$$2x+3y = 15$$

$x^2y \to 2\,x$ and $1\,y$.

$$\frac{x+x+3y}{3} \geq \sqrt[3]{x.x.3y}$$

$$x^2y \leq \frac{125}{3}$$

$$2x+3y \geq \sqrt[3]{3x^2y}$$

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$xy \leq \left(\frac{125}{3}\right).$

$$\frac{2x+3y}{3} \geq \sqrt[3]{3x^2y}$$

$$\frac{15}{3} \geq \sqrt[3]{3x^2y}.$$

$$5 \geq \sqrt[3]{3x^2y}.$$

$$125 \geq 3x^2y.$$

Example: If a1, a2, a3 and a4 are positive integers with sum = 16. Find the minimum value of $\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\frac{1}{a_4}\right)$? $= S$.

$$a_1+a_2+a_3+a_4=16.$$

AM $\geq$ HM

$$\frac{a_1+a_2+a_3+a_4}{4} \geq \frac{4}{\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\frac{1}{a_4}}.$$

$$\frac{16}{4} \geq \frac{4}{S}$$

$S \geq 1$ $\therefore$ minimum value of $S = 1$

Example: If three positive real numbers x, y and z are in A.P. such that $\boxed{xyz=4,}$ then what will be the minimum value of y.

$y-x = z-y \Rightarrow \boxed{2y = x+z} \Rightarrow y = \frac{x+z}{2}$

$$\frac{x+y+z}{3} \geq \sqrt[3]{xyz}.$$

$$\frac{2y+y}{3} \geq \sqrt[3]{4}.$$

$$y \geq \sqrt[3]{4}.$$

$\therefore$

$y \geq \sqrt[3]{4}$.

$y_{min} = 2^{2/3}$.

Example: Minimize the expression (x+y)(y+z), where x, y and z are positive real numbers satisfying xyz(x + y + z) = 1.

$$y(x+y+z) = \frac{1}{xz}$$

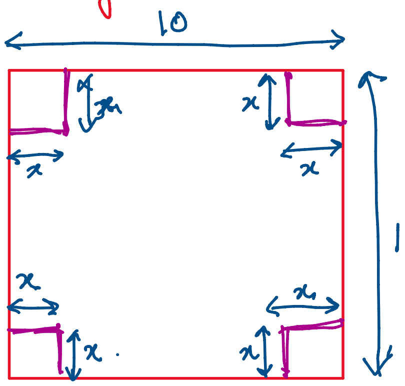
$$\frac{xz + \frac{1}{xz}}{2} \geq \sqrt{xz \cdot \frac{1}{xz}}$$

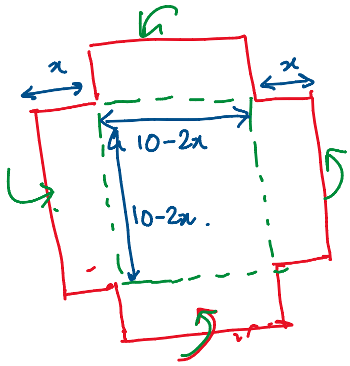
$$xz + \frac{1}{xz} \geq 2$$

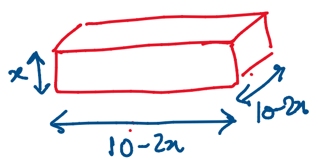
$$xz + y(x+y+z) = xz + yx + y^2 + yz = x(y+z) + y(y+z) = (x+y)(y+z)$$

$$(x+y)(y+z) = xz + y(x+y+z) = xz + \frac{1}{xz} \geq 2$$

From a square of side 10cm each 4 square corners are cut off and the resultant figure is folded into an open box. Find the maximum volume of the box.







$V = (10-2x)^2 x$

let $10-2x = y$

$V = xy^2$

$$\frac{4x+y+y}{3} \geq \sqrt[3]{4xy^2}$$

$$\frac{20}{3} \geq \sqrt[3]{4xy^2}$$

$$\frac{8000}{27} \geq 4xy^2$$

$$xy^2 \leq \frac{2000}{27}$$

$$V \leq 74\frac{2}{27}$$

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$$\frac{20}{3} \geq \sqrt[3]{4xy^2}$$

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$$xy^2 \leq \frac{2000}{27}$$

$$V \leq 74\frac{2}{27}$$

For the above question if x is an integer then find the max volume.

$$V = (10-2x)^2 x = y^2 x$$

$$10 - 2x = y.$$

$$\boxed{y + 2x = 10}$$

x	y	$V = xy^2$
1	8	64
2	6	72
3	4	48
4	2	16