True Model: $Y_i = \alpha + \beta X_i + u_i$

Estimated Model: $\hat{Y}_i = \hat{\alpha} + \hat{\beta} X_i$

where $\hat{\beta} = \dfrac{\sum(Y_i - \bar{Y})(X_i - \bar{X})}{\sum(X_i - \bar{X})^2}$ and $\hat{\alpha} = \bar{Y} - \hat{\beta}\bar{X}$

Already shown: $E(\hat{\beta}) = \beta$, $\widehat{Var(\hat{\beta})} = \dfrac{\hat{\sigma}^2}{\sum(X_i - \bar{X})^2}$, where $\hat{\sigma}^2 = \dfrac{RSS}{(n-2)}$

HW Check: $E(\hat{\alpha}) = \alpha$, $\widehat{Var(\hat{\alpha})} = \dfrac{\hat{\sigma}^2 \sum X_i^2}{n\sum(X_i - \bar{X})^2}$

Testing of Hypothesis:

X causes Y.

True Model: $\boxed{Y_i} = \alpha + \beta \boxed{X_i} + u_i$ ... [Causal connection b/w Y & X].

Dependent variable.

Independent variable / Explanatory variable.

∴ We use X_i to explain variations in Y_i through the regression model.

If the explanatory variable chosen is significant, then $\beta \neq 0$.

If the explanatory variable chosen is insignificant, $\beta = 0$.

To test: $H_0: \beta = 0$ vs $H_1: \beta \neq 0$.

This test is performed to check if the explanatory variable used is a meaningful (significant) variable or not.

Steps:

(i) Construct a test statistic.

(ii) Check the distribution of test-statistic under H_0 [$\tau / \chi^2 / t / F$]

(iii) Fix a level of significance (5% / 1%) and get the corresponding critical value.

(iv) Frame the testin...

corresponding critical value.

(iv) Frame the testing rule based on the observed value of the test-statistic and the critical value.

Note: True Model: $Y_i = \alpha + \beta X_i + u_i$ → Random disturbance terms.

Assumptions: (i) $E(u_i) = 0 \quad \forall i$

(ii) $Var(u_i) = \sigma^2 \quad \forall i$

(iii) $cov(u_i, u_j) = 0 \quad \forall i \neq j$

(iv) $cov(X, u) = 0$.

(v) $u_i \overset{iid}{\sim} N(0, \sigma^2)$

$Y_1 \quad X_1 \Rightarrow Y_1 = \alpha + \beta X_1 + u_1$

$Y_2 \quad X_2 \Rightarrow Y_2 = \alpha + \beta X_2 + u_2$

$\vdots \quad \vdots$

$Y_n \quad X_n \Rightarrow Y_n = \alpha + \beta X_n + u_n$

$\{u_1, u_2, \ldots, u_n\} \overset{iid}{\sim} N(0, \sigma^2)$

Recall:- $$\hat{\beta} = \beta + \frac{\sum u_i (X_i - \bar{X})}{\sum (X_i - \bar{X})^2}$$ $\Rightarrow$ $\hat{\beta}$ is a linear combination of u_i's.

Result: Any linear combination of normal variables will also follow a normal distribution with the corresponding mean and variance.

As $\hat{\beta}$ is a linear combination of u_i's:

$$\therefore \hat{\beta} \sim N[E(\hat{\beta}), Var(\hat{\beta})].$$

$$\Rightarrow \hat{\beta} \sim N\left[\beta, \frac{\sigma^2}{\sum (X_i - \bar{X})^2}\right].$$

can this be used as a test-statistic ?

$$\therefore \frac{\hat{\beta} - \beta}{\sqrt{\sigma^2 / \sum (X_i - \bar{X})^2}} \sim N(0, 1) \Rightarrow \frac{\hat{\beta} - \beta}{\sigma / \sqrt{\sum (X_i - \bar{X})^2}} \sim N(0, 1)$$

$$\boxed{\frac{(\hat{\beta}-\beta)^2}{\sigma^2/\sum(x_i-\bar{x})^2}} \sim \chi^2_{(1)}$$

As σ is unknown, this cannot be used as a test-statistic.

$$\chi^2 = \sum_{i=1}^{n} \tau_i^2 \sim \chi^2_{(n)}$$

χ

τ

Result: $$\frac{\sum e_i^2}{\sigma^2} \sim \chi^2_{(n-2)}$$

τ, χ^2, t, F ?