Numbers

$\mathbb{N} = \{1, 2, 3, \cdots\}$, Null

$W = \{0, 1, 2, 3, \cdots\}$

$\mathbb{Z} = \{\cdots, -1, 0, 1, \cdots\}$ $\to \mathbb{Z}^+$

$2 + \_\_ = 2$ ✗

$3 - 4 = \ ?$ ✗

$4 \times \_\_ = 1$ ✗

$x, -x$

Additive inverse : number + (additive inverse) = additive identity

number + $\underbrace{\text{additive identity}}_{\text{(zero)}}$ = number

Multiplicative identity : number × MI = number

$\mathbb{Q} = \{ p/q ,\ p, q \in \mathbb{Z},\ q \neq 0 \}$ $\longrightarrow$ $x^2 - 2 = 0$?

$\mathbb{P}, \mathbb{Q}^c = \{ \text{numbers not in the form of } p/q \}$ $\longrightarrow$ π, e ?

Transcendental numbers

Terminating $= \dfrac{p}{2^m 5^n}$ format $m, n \in \mathbb{Z}^+$

Real numbers $\to \mathbb{R} = \mathbb{Q} \cup \mathbb{Q}^c$ $\longrightarrow$ $x^2 + 2 = 0$

Complex number $\longrightarrow$ $\mathbb{C} : \{ a + bi,\ a, b \in \mathbb{R} \}$

Properties

(1) Closure $\to$ Integer closed under addition

(2) Commutative $\to$ $x + y = y + x$ / $x \cdot y = y \cdot x$

(3) Associative $\to$ $x + (y + z) = (x + y) + z$ / $x(yz) = (xy)z$

(4) Identity $<$ additive, multiplicative $\}$ zero, one

(5) Distributive $\to$ $x(y + z) = xy + xz$ | $(x + y)z = xz + yz$

Properties of divisibility

(1) $x \mid y$ and $y \mid z \Rightarrow x \mid z$

(2) $x \mid y$ and $x \mid z \Rightarrow x \mid (my + nz)$, $m, n \in \mathbb{Z}$

(3) $x \mid y$ and $y \mid x \Rightarrow x = \pm y$

(4) $x \mid y$, $x, y > 0$, then $x \leq y$

(5) $x \mid y \Rightarrow x \mid yz$, for any $z \in \mathbb{Z}$

(6) $x \mid y$ iff $nx \mid ny$ for any $n \in \mathbb{Z}, n \neq 0$

Numbers

Division algorithm $\longrightarrow$ $a = bq + r$, $0 \leq r < b$

HCF / GCD $\longrightarrow$ $c|a$ & $c|b$, $c \to$ common divisor

GCD
(1) $d|a$ & $d|b$
(2) If $c|a$ & $c|b$, then $c \mid d$

12, 18
$C = \{2, 3, 6\}$
$d = 6$
$c_1|d$, $c_2|d$

Properties

(1) If $(b,c) = g$, and d is any common divisor, then $d|g$

(2) For $m > 0$, $(mb, mc) = m(b,c)$, $m \in \mathbb{Z}$

(3) If $d|b$, $d|c$, $d > 0$ then $\left(\frac{b}{d}, \frac{c}{d}\right) = \frac{1}{d}(b,c)$

(4) If $(b,c) = g$, then $\left(\frac{b}{g}, \frac{c}{g}\right) = 1$

(5) If $(b,c) = g$, then $\exists\, m, n \in \mathbb{Z}$, $g = mb + nc$

$(12, 18) = 6$, $6 = 12m + 18n$ $m = 5, n = -3$

(6) If $(a,b) = 1$ and $(a,c) = 1$, then $(a, bc) = 1$

(7) If $(a, bc) = 1$, and $(a,b) = 1$, then $(a,c) = 1$

Prime
(1) $P > 1$
(2) P has no divisors except 1 and itself

(+) Integers $= \{1\} \cup \{\text{Composites}\} \cup \{\text{Primes}\}$

Numbers