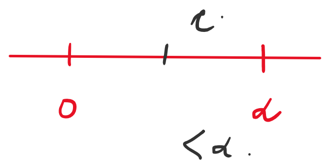
Q. If $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x = 0$, $a_1 \neq 0$, $n \geq 2$ has a positive root 'α', then, $\underbrace{n a_n x^{n-1} + (n-1) a_{n-1} x^{n-2} + \cdots + a_1}_{f'(x)} = 0$ has a positive root which is:

(a) $> \alpha$ (b) $< \alpha$ (c) $\geq \alpha$ (d) $= \alpha$.

$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_1 x$.

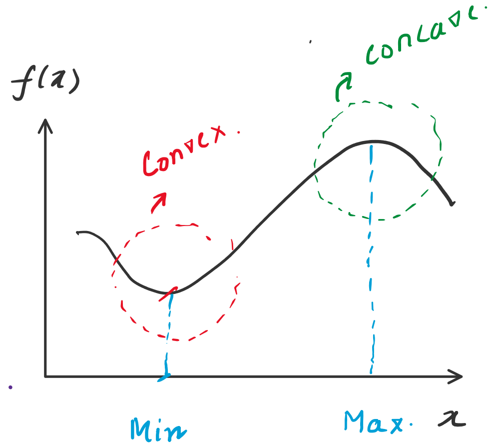


Observe: $f(0) = 0$.

& 'α' is a root $f(\alpha) = 0$.

$\therefore$ By Rolle's theorem $\exists$ a pt 'c' $\in (0, \alpha)$ s.t $f'(c) = 0$.

Maxima & Minima



Let $f(x)$ be a differentiable fn.

Maxima: value of x at which $f(x)$ is max.

Minima: value of x at which $f(x)$ is min.

Criteria for obtaining pts of maxima/minima:-

I Derivative Criteria:

For Max: $f'(x) = 0$ and $f''(x) < 0$ at pt of maxima.

For Min: $f'(x) = 0$ and $f''(x) > 0$ at pt of minima.

↳ Evaluating gives pt of extremum.

When $f''(x) = 0$. We will use a Generalized criteria:

Suppose 'c' is a pt s.t $f'(c) = f''(c) = f'''(c)$ ...

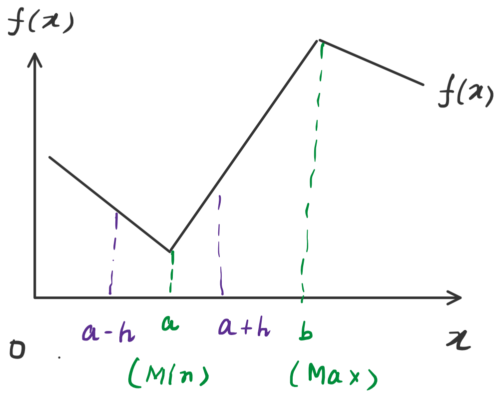
...alized criteria:

Suppose 'c' is a pt s.t $f'(c) = f''(c) = f'''(c) = \cdots = f^{(n-1)}(c) = 0$ and $f^{(n)}(c) \neq 0$.

(i) If 'n' is odd, 'c' is a pt of inflexion [neither a max, nor minima].

(ii) If 'n' is even, then $f^{(n)}(c) < 0 \Rightarrow$ Maximum at 'c' & $f^{(n)}(c) > 0 \Rightarrow$ Minimum at 'c'.

II Non-Derivative Criteria:



Check if $x = 'a'$ is a pt of minimum.

'a' is a pt of local min if

$$f(a-h) > f(a) \ \& \ f(a+h) > f(a)$$

Check if $x = 'b'$ is a pt of maximum:

'b' is a pt of local max if:

$$f(b-h) < f(b) \ \& \ f(b+h) < f(b)$$

III AM $\geq$ GM $\geq$ HM.

Eg: $f(x) = x + \frac{1}{x}$.

$f'(x) = 1 - \frac{1}{x^2}$

$f'(x) = 0 \Rightarrow 1 - \frac{1}{x^2} = 0$.

$\Rightarrow 1 = \frac{1}{x^2} \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$.

$f''(x) = \frac{2}{x^3}$.

$f''(x)\Big|_{x=1} = 2 > 0 \Rightarrow$ Minima.

2 No.s $x, \frac{1}{x}$.

$AM = \frac{x + \frac{1}{x}}{2}$.

$GM = \sqrt{x \cdot \frac{1}{x}}$.

$\therefore$ AM $\geq$ GM.

$$\frac{x + 1/x}{2} \geq \sqrt{x \cdot \frac{1}{x}}$$

$$\frac{x + 1/x}{2} \geq 1$$

$x = 1$

$f(x)$ is min at $x = 1$.

Minimum value $= 1 + 1 = 2$.

$\frac{\quad}{2} \geq 1$

$x + \frac{1}{x} \geq 2 \Rightarrow f(x) \geq 2$

$\therefore$ Min value is 2

HW.

Q. For any 2 positive no.s a, b. Find the minimum value of $\underbrace{\left(a+\frac{1}{a}\right)^2} + \underbrace{\left(b+\frac{1}{b}\right)^2}$. [Use AM $\geq$ GM].

Q. Let $f(x) = \sin^3 x + k \cdot \sin^2 x$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Find the interval in which 'k' should lie s.t $f(x)$ has exactly one max and one min in the given range.

For a pt of max & min in $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, we should have $f'(x) = 0$.

$$f'(x) = 3\sin^2 x \cos x + 2k \cdot \sin x \cos x.$$
$$= \sin x \cos x \left[3 \sin x + 2k\right].$$

For max/min, $f'(x) = 0 \Rightarrow \sin x \cdot \cos x \left[3 \sin x + 2k\right] = 0$

$\Rightarrow \sin x = 0 \qquad \cos x = 0 \qquad 3\sin x + 2k = 0$.

$\Rightarrow \boxed{x = 0} \qquad x = \pi/2 \ (\times) \qquad \sin x = -\frac{2k}{3}$.

$\therefore \boxed{\sin x = -\frac{2k}{3}}$ [If $k = 0 \Rightarrow \boxed{x = 0} \Rightarrow$ one pt $\Rightarrow x = 0$.

$k \neq 0$ ($\because x = 0$ cannot be pt of max & pt of min)

$\therefore x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ then $-1 < \sin x < 1$.

$\therefore -1 < -\frac{2k}{3} < 1 \Rightarrow 1 > \frac{2k}{3} \ldots$

$\therefore \quad -1 < -\frac{2k}{3} < 1 \Rightarrow 1 > \frac{2k}{3} > -1 \Rightarrow -\frac{3}{2} < k < \frac{3}{2}$

$\therefore$ Reqd Range of k $\left(-\frac{3}{2}, 0\right) \cup \left(0, \frac{3}{2}\right)$.

HW

Q. A rectangle has its lower left hand corner at the origin and upper right hand corner on the graph $f(x) = x^2 + \frac{1}{x^2}$. Find the value of x so that area of rectangle is minimized.

HW.

Q. Let $f(x) = x^4 - 4x^3 + 6x^2 - 4x + 1$. Determine the nature of the pt $x = 1$.