Q. Let $X \sim N(\mu, \sigma^2)$ where μ is known. Based on a r.s of size 'n' from this popln, test $H_0 : \sigma = \sigma_0$ vs $H_1 : \sigma = \sigma_1$ $[\sigma_1 > \sigma_0]$. Find BCR using NP Lemma.

NP Lemma: $H_0 : \theta = \theta_0$ vs $H_1 : \theta = \theta_1$.

$$BCR := \left\{ x \,\middle|\, \frac{L(x, \theta_1)}{L(x, \theta_0)} \geq k \;\; \forall x \in W \right\}, \; k > 0.$$

r.s: $X_1, X_2, \cdots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$, μ = known

$$f(x_i) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x_i - \mu}{\sigma}\right)^2}$$

$$L(x, \sigma) = \prod_{i=1}^{n} f(x_i) = \frac{1}{(\sigma\sqrt{2\pi})^n} e^{-\frac{1}{2\sigma^2}\Sigma(x_i - \mu)^2}.$$

Under H_0: $L(x, \sigma_0) = \frac{1}{(\sigma_0\sqrt{2\pi})^n} e^{-\frac{1}{2\sigma_0^2}\Sigma(x_i - \mu)^2}$

Under H_1: $L(x, \sigma_1) = \frac{1}{(\sigma_1\sqrt{2\pi})^n} e^{-\frac{1}{2\sigma_1^2}\Sigma(x_i - \mu)^2}$

$$\frac{\frac{1}{(\sigma_1\cancel{\sqrt{2\pi}})^n}}{\frac{1}{(\sigma_0\cancel{\sqrt{2\pi}})^n}} = \left(\frac{\sigma_0^n}{\sigma_1^n}\right)$$

NP Lemma: $\boxed{\frac{L(x, \sigma_1)}{L(x, \sigma_0)} > k}$ solves the BCR for the test $\forall x \in W$ and $k > 0$.

$$\Rightarrow \left(\frac{\sigma_0}{\sigma_1}\right)^n e^{\Sigma(x_i - \mu)^2 \left\{-\frac{1}{2\sigma_1^2} + \frac{1}{2\sigma_0^2}\right\}} > k.$$

$$\Rightarrow e^{\Sigma(x_i - \mu)^2 \left\{\frac{1}{2\sigma_0^2} - \frac{1}{2\sigma_1^2}\right\}} > k \cdot \left(\frac{\sigma_1}{\sigma_0}\right)^n$$

$$\Rightarrow \frac{1}{2}\Sigma(x_i - \mu)^2 \left(\frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2}\right) > \ln k + n \ln\left(\frac{\sigma_1}{\sigma_0}\right).$$

$$\Rightarrow \frac{1}{2}\sum(x_i-\mu)^2\left(\frac{1}{\sigma_0^2}-\frac{1}{\sigma_1^2}\right) > \ln k + n\ln\left(\frac{\sigma_1}{\sigma_0}\right).$$

$$\Rightarrow \sum(x_i-\mu)^2 > \frac{\ln k + n\ln\left(\frac{\sigma_1}{\sigma_0}\right)}{\frac{1}{2}\left(\frac{1}{\sigma_0^2}-\frac{1}{\sigma_1^2}\right)} = \lambda_1.$$

computed from the sample.

$$\Rightarrow \sum(x_i-\mu)^2 > \lambda_1. \quad \Rightarrow \text{BCR for the test}.$$

For determing k: use the condition, $L.O.S = \alpha$.

$$L.O.S = P\left[\sum(x_i-\mu)^2 > \lambda_1 \mid H_0\right] = \alpha$$

$$= P\left[\sum(x_i-\mu)^2 > \lambda_1 \mid \sigma_0\right] = \alpha.$$

$$\Rightarrow P\left[\sum(x_i-\mu)^2 > \lambda_1 \mid \sigma_0\right] = \alpha \quad \Rightarrow \text{solve for } k$$

[Given α].

$$P\left[\frac{\sum(x_i-\mu)^2}{\sigma_0^2} > \frac{\lambda_1}{\sigma_0^2}\right] = \alpha.$$

$\hookrightarrow$ Chi-sq. variate.

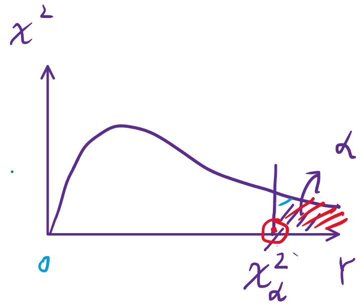
$\therefore$ Under H_0: $X_1, X_2, \cdots, X_n \overset{iid}{\sim} N(\mu, \sigma_0^2)$.

known

$\hookrightarrow$ hypothesized value.

$$Y = \sum_{i=1}^{n}\left(\frac{x_i-\mu}{\sigma_0}\right)^2 \sim \chi^2_{(n)}$$

$$\Rightarrow P\left[Y > \frac{\lambda_1}{\sigma_0^2}\right] = \alpha \quad \text{where } Y \sim \chi^2_{(n)}.$$



$\hookrightarrow$ upper α point of χ^2 distn.

$$\therefore \chi^2_{\alpha;(n)} = \frac{\lambda_1}{\sigma_0^2}.$$

Q. Consider a r.s X_1, X ... iid ...

Q. Consider a r.s $X_1, X_2, \cdots, X_n \overset{iid}{\sim} f_\theta(x)$, where $f_\theta(x) = \dfrac{1+\theta}{(x+\theta)^2}$, $x \in [1, \infty)$

Find the BCR to test $H_0 : \theta = \theta_0$ vs $H_1 : \theta = \theta_1$ $[\theta_1 > \theta_0]$

$$X_1, X_2, \cdots, X_n \overset{iid}{\sim} f_\theta(x)$$

$$f_\theta(x_i) = \frac{1+\theta}{(x_i+\theta)^2}, \quad x_i \in [1, \infty)$$

$$L(x, \theta) = \prod_{i=1}^{n} f_\theta(x_i) = \frac{(1+\theta)^n}{\prod_{i=1}^{n} (x_i+\theta)^2}$$

Under H_0: $L(x, \theta_0) = \dfrac{(1+\theta_0)^n}{\prod_{i=1}^{n} (x_i+\theta_0)^2}$

Under H_1: $L(x, \theta_1) = \dfrac{(1+\theta_1)^n}{\prod_{i=1}^{n} (x_i+\theta_1)^2}$

critical region

NP Lemma: $\dfrac{L(x, \theta_1)}{L(x, \theta_0)} > k \quad \forall x \in W$ & $k > 0$.

$$\frac{(1+\theta_1)^n}{\prod_{i=1}^{n} (x_i+\theta_1)^2} > k \cdot \frac{(1+\theta_0)^n}{\prod_{i=1}^{n} (x_i+\theta_0)^2}$$

$$\left(\frac{1+\theta_1}{1+\theta_0}\right)^n > k \prod_{i=1}^{n} \left(\frac{x_i+\theta_1}{x_i+\theta_0}\right)^2$$

$$\prod_{i=1}^{n} \left(\frac{x_i+\theta_0}{x_i+\theta_1}\right)^2 > k \left(\frac{1+\theta_0}{1+\theta_1}\right)^n$$

Take log: HW.
$$\sum_{i=1}^{n} \ln\left(\frac{x_i+\theta_0}{x_i+\theta_1}\right) > \frac{1}{2}\left\{\ln k + n\ln\left(\frac{1+\theta_0}{1+\theta_1}\right)\right\}$$