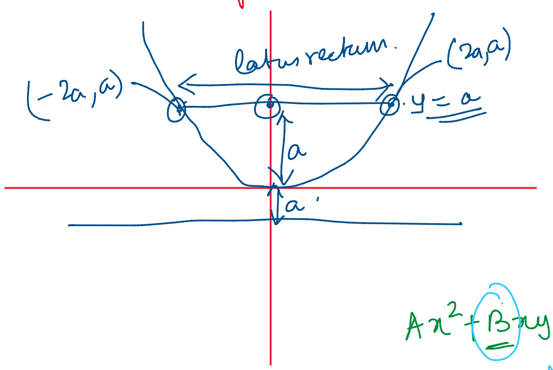
21 February 2024 17:01

$\vec{a}\cdot(\vec{b}\times\vec{c})$ → Scalar.

$\vec{a}\times(\vec{b}\times\vec{c})$ → vector.

Show that the eqn $4x^2-4xy+y^2+2x-26y+9=0$ represent a parabola whose latus rectum is $2\sqrt{5}$ units.



$x^2 = 4ay$.

$x^2 = 4a^2$

$x = \pm 2a$.

latus rectum = 4a

$A=4 \quad B=-4 \quad C=1$

$\cot 2\theta = \frac{A-C}{B} = \frac{3}{-4}$.

$Ax^2 + Bxy + Cy^2 + \dots$

Coeff of xy

$2\theta = -53^\circ$.

$\theta = -26.5^\circ$

$\sin\theta = -0.45^\circ$

$\cos\theta = 0.9$

$x = \hat{x}\cos\theta - \hat{y}\sin\theta$ and $y = \hat{x}\sin\theta + \hat{y}\cos\theta$.

$x = 0.9x' + 0.45y' \qquad y = -0.45x' + 0.9y'$

$4x^2 - 4xy + y^2 + 2x - 26y + 9 = 0$

$4(0.81x'^2 + \cancel{0.81x'y'} + 0.2025y'^2) - 4(0.405x'^2 + \cancel{0.6075x'y'} + 0.405y'^2)$

$+ (0.2025x'^2 - \cancel{0.81x'y'} + 0.81y'^2) + 0.18x' + 0.9y' + 11.7x' - 23.4y' + 9 = 0$

$1.8225x'^2 + 11.88x' - 22.5y' + 9 = 0$

$x'^2 + 6.52x' + 4.94 = 12.35y'$

$x'^2 + 2\times 3.26x' + 10.6276 - 5.6876 = 12.35y'$

$(x'+3.26)^2 = 12.35y' + 5.6876. = 12.35(y'+0.46)$

$(x'+3.26)^2 = 12.35(y'+0.46)$ Vertex $(-3.26, -0.46)$

12.35 → $4a$.

$\vec{a} = 2i - j + k \qquad \vec{b} = i + 2j - 2k. \qquad \vec{c} = -i + j + 2k.$

$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ 1 & 2 & -2 \end{vmatrix} = 5j + 5k.$

$(\vec{a}\times\vec{b})\times\vec{c} = \begin{vmatrix} i & j & k \\ 0 & 5 & 5 \\ -1 & 1 & 2 \end{vmatrix}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ 1 & 2 & -2 \end{vmatrix} = 5j + 5k.$$

$$(\vec{a}\times\vec{b})\cdot\vec{c} = 5 + 10 = 15$$

$$(\vec{a}\times\vec{b})\times\vec{c} = \begin{vmatrix} i & j & k \\ 0 & 5 & 5 \\ -1 & 1 & 2 \end{vmatrix}$$

$$= 5i - 5j + 5k.$$

$$= 5(i - j + k)$$

Tangents & Normals

Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$x = a\cos\theta \quad y = b\sin\theta.$

$$\frac{dy}{dx} = -\left(\frac{a}{b}\right)\cot\theta\left(\frac{b^2}{a^2}\right)$$

$$= -\left(\frac{b}{a}\right)\cot\theta.$$

$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\frac{2y}{b^2}\frac{dy}{dx} = -\frac{2x}{a^2}$$

$$\frac{dy}{dx} = -\left(\frac{x}{y}\right)\left(\frac{b^2}{a^2}\right) \rightarrow \text{slope of the tangent.}$$

$P(\alpha, \beta).$

$$y - \beta = \left(\frac{dy}{dx}\right)_P (x - \alpha).$$

slope of normal $\rightarrow$ $-\dfrac{1}{\frac{dy}{dx}} = \left(\frac{y}{x}\right)\left(\frac{a^2}{b^2}\right)$

$$y - \beta = -\left(\frac{\alpha}{\beta}\right)\left(\frac{b^2}{a^2}\right)(x - \alpha)$$