Testing of Hypothesis

Sample vs Population → the group under the statistical study

↳ subset of the population to be used to draw conclusions about the population.

Testing of Hypothesis: [for the unknown population parameters].

Eg: suppose we have a normally distributed population. Consider a random sample of size (= n) from this popln.

Suppose we want test the hypothesis:

$$H_0 : \mu = \mu_0 \quad vs \quad H_1 : \mu = \mu_1$$

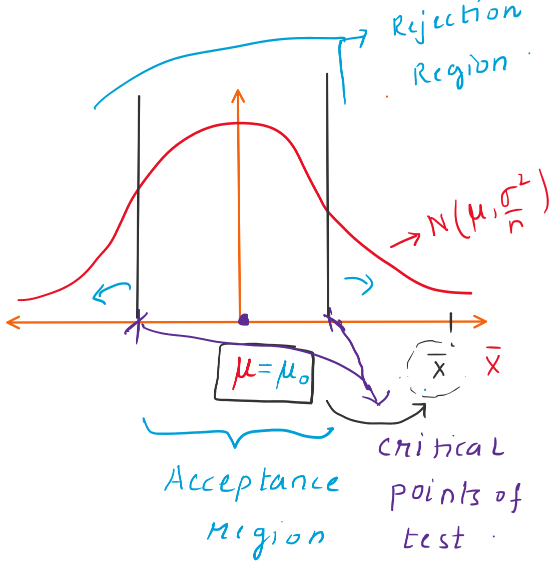
↳ Null hypothesis ↳ Alternative hypothesis.

r.s: $X_1, X_2, \dots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$.

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \Rightarrow \bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right) \quad [\text{Result}]$$

↳ sampling distribution of $\bar{X}$

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$



With the sampling distribution, we divide the values of $\bar{X}$ into 2 regions:

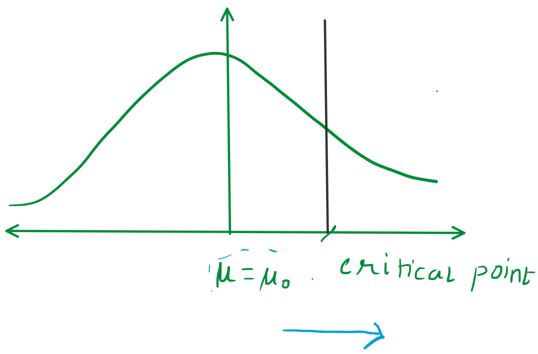
i) Acceptance Region: Given the sample if $\bar{X}$ is present here, accept H_0.

ii) Rejection Region: Given the sample and $\bar{X}$, reject H_0 or accept H_1.

To test: $H_0: \mu = \mu_0$ vs $H_1: \mu > \mu_0$.

To test: $H_0: \mu = \mu_0$ vs $H_1: \mu < \mu_0$.

→ one-tailed test.



To test : $H_0: \mu = \mu_0$ vs $H_1: \mu \neq \mu_0$.

→ Two-tailed test.

Note: In testing of Hypothesis we can have 2 types of error situations:-

actual scenario

	H_0 is true	H_0 is false
Accept H_0.	✓	Type II Error
Reject H_0.	Type I Error	✓

depends on the test outcome

Type I Error: Reject H_0 when H_0 is true.

Type II Error: Accept H_0 when H_0 is false.

In practice since the true scenario is unknown, it is not possible to know if either of these errors have been committed. Hence, since minimizing these errors is not possible, we will try to minimize the probability of occurrance of these errors.

Suppose: Ω : sample space (all possible values of $\bar{X}$).

W : Rejection Region / Critical region.

$\Omega - W = W^c$: Acceptance Region.

Type I Error: $\alpha = P[\bar{X} \in W \mid H_0]$.

Type II Error: $\beta = P[\bar{X} \in W^c \mid H_1]$.

Type II Error: $\beta = P[\bar{X} \in W^c | H_1]$.

$1-\beta = P[\bar{X} \in W | H_1] = $ Power of test.

true scenario.

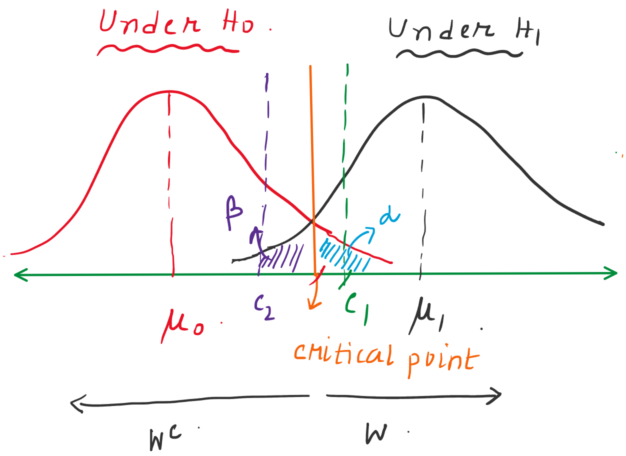
H_0 will be rejected based on the test.

To test: $H_0: \mu = \mu_0$ vs $H_1: \mu = \mu_1$ $(\mu_1 > \mu_0)$ $[\sigma^2$ is known$]$.

We know, $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$.

Under H_0: $\bar{X} \sim N(\mu_0, \frac{\sigma^2}{n})$.

Under H_1: $\bar{X} \sim N(\mu_1, \frac{\sigma^2}{n})$.



Note: The choice of critical point determines the values of Type I Error and Type II Error.

No choice of critical point is possible that minimizes both these errors.

[If we try to choose a critical point to minimize one of the errors, the other will increase]