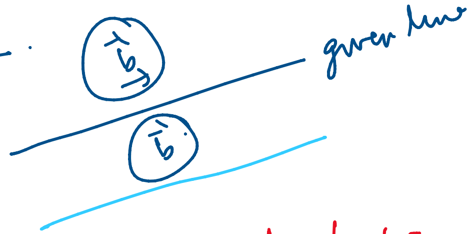
05 January 2024 17:07

$x = y-3 = \frac{20-2z}{4}$ → eqn of a line. (Given line)

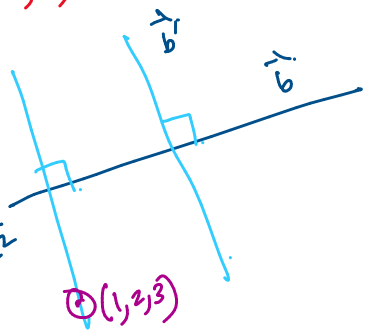
① find the starting pt and the direction vector.

(0, 3, 10)  (1, 1, −2)

② find the equation of a line parallel to the given line and passing through (1, 2, 3).




③ find the equation of a line ⊥ to the given line and passing through (1, 2, 3)




$\boxed{\vec{b}\cdot\vec{b}' = 0}$

$\vec{b} = \hat{i} + \hat{j} - 2\hat{k}$

$\vec{b}' = \alpha i + \beta j + \gamma k$

$\boxed{\alpha + \beta - 2\gamma = 0}$

$|\vec{b}\times\vec{b}'| = |\vec{b}||\vec{b}'|$

$|\vec{b}| = \sqrt{1^2+1^2+2^2}$

$= \sqrt{6}$

$|\vec{b}'| = \sqrt{x^2+y^2+z^2}$

$\vec{b}\times\vec{b}' = (i+j-2k)\times(xi+yj+zk)$

$= yk - zj - xk + zi - 2xj + 2yi$

$= (2y+z)\hat{i} - (2x+z)\hat{j} + (y-x)k$

$|\vec{b}\times\vec{b}'| = \sqrt{\quad}$

$\boxed{\alpha+\beta-2\gamma=0}$  $\vec{b}' = \alpha i + \beta j + \gamma k$

$\vec{r} = (1,2,3) + \lambda(\alpha,\beta,\gamma)$

$x = 1+\lambda\alpha \rightarrow \frac{x-1}{\alpha} = \lambda$

$y = 2+\lambda\beta \rightarrow \frac{y-2}{\beta} = \lambda$

$z = 3+\lambda\gamma \rightarrow \frac{z-3}{\gamma} = \lambda$

$\frac{x-1}{\alpha} = \frac{y-2}{\beta} = \frac{z-3}{\gamma}$

$\boxed{\alpha+\beta-2\gamma=0}$

L1 : $\vec{r} = (1,2,3) + \lambda(1,2,1)$

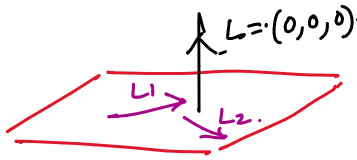
L1 : $\vec{r} = (1,2,3) + \lambda(1,2,1)$

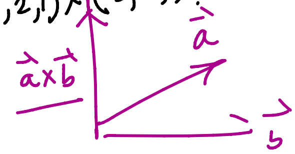
L2 : $\vec{r} = (0,1,1) + \beta(2,-1,1)$.

find the eqn of a line which is $\perp$ to both lines and passing through $(0,0,0)$

any 2 lines are always coplanar.

$L = (0,0,0) + \lambda[(1,2,1)\times(2,-1,1)]$



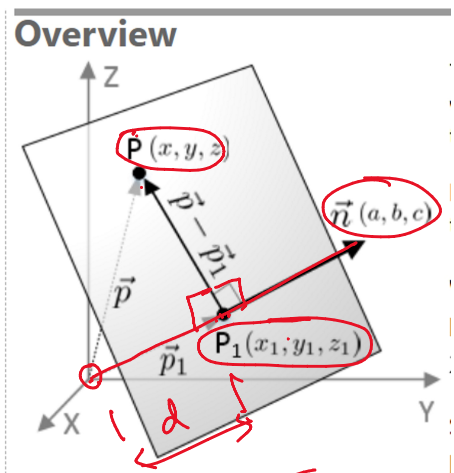




$$(1,2,1)\times(2,-1,1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 2 & -1 & 1 \end{vmatrix} = 3\hat{i} - \hat{j} - 5\hat{k}.$$

$\vec{L} = \lambda 3\hat{i} - \lambda\hat{j} - 5\lambda\hat{k}$.

$$\boxed{\frac{x}{3} = \frac{y}{-1} = \frac{z}{-5}}$$

## Equation of a plane




$d = \sqrt{x_1^2+y_1^2+z_1^2}$

$\boxed{\vec{P_1P} \text{ is } \perp \text{ to } \vec{n}}$

$\vec{P_1P} = \vec{p} - \vec{p}_1$

$\vec{p}_1 + \vec{P_1P} = \vec{p}$.

$\vec{P_1P} = \vec{p} - \vec{p}_1$.

$\vec{P_1P}\cdot\vec{n} = 0$ $\quad (\vec{p} - \vec{p}_1)\cdot\vec{n} = 0$.

$\vec{p}\cdot\vec{n} - \vec{p}_1\cdot\vec{n} = 0$

$\vec{p}\cdot\vec{n} = \vec{p}_1\cdot\vec{n} = |\vec{p}_1||\vec{n}|$.

$\vec{p}_1$ is collinear with $\vec{n}$.

$\boxed{\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}}$

$\vec{p}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$.

$\boxed{\vec{n} = a\hat{i} + b\hat{j} + c\hat{k}}$

$|\vec{p}_1| = \sqrt{x_1^2+y_1^2+z_1^2}$ $\quad |\vec{n}| = \sqrt{a^2+b^2+c^2}$.

$\hat{n}$ ... $a\hat{i}+b\hat{j}+c\hat{k}$ ... $\vec{r}\cdot\vec{n} = d|\vec{n}| \longrightarrow \vec{p}\cdot\hat{n} = d$.

$$\hat{n} = \frac{a\hat{i} + b\hat{j} + c\hat{k}}{\sqrt{a^2+b^2+c^2}}$$

$$\vec{p} \cdot \vec{n} = d|\vec{n}| \longrightarrow \vec{p} \cdot \frac{\vec{n}}{|\vec{n}|} = d$$

$$\downarrow$$

$$\vec{p} \cdot \hat{n} = d$$

$$\boxed{xa + yb + zc = d}$$