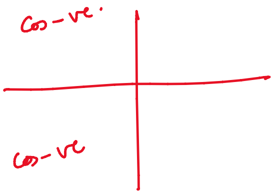
15 July 2023 09:28

Problems on Trigonometry

Number of value(s) of x, in [0, 2π] satisfying $\cos\left(\frac{\pi}{2}\sin\left(\frac{\pi}{2}\cos x\right)\right) = -\frac{1}{2}$

(A) 0 (B) 1 (C) 2 (D) 4

0



$\cos^{-1}(-1/2) = \frac{2\pi}{3}, \frac{4\pi}{3}$

$\cos\left(\frac{\pi}{2}\sin\left(\frac{\pi}{2}\cos x\right)\right) = -1/2.$

$\frac{\pi}{2}\sin\left(\frac{\pi}{2}\cos x\right) = \cos^{-1}(-1/2)$

$2\pi/3,\ 4\pi/3.$

$\frac{\cancel{\pi}}{2}\sin\left(\frac{\pi}{2}\cos x\right) = \frac{2\cancel{\pi}}{3}$

$\boxed{\sin\left(\frac{\pi}{2}\cos x\right) = \frac{4}{3}}\ \varphi$

$\frac{\cancel{\pi}}{2}\sin\left(\frac{\pi}{2}\cos x\right) = \frac{4\cancel{\pi}}{3}$

$\boxed{\sin\left(\frac{\pi}{2}\cos x\right) = \frac{8}{3}}\ \varphi$

If x, y ∈ [0, 10], then number of solutions (x,y) of inequation $3^{\sec^2 x-1}\sqrt{9y^2-6y+2} \leqslant 1$ is

(A) 2 (B) 4

(C) 6 (D) infinite

$\cos\alpha = \cos\beta$

$3^{\sec^2 x-1}\sqrt{9y^2-6y+2} \le 1.$

$x, y \in [0, 10]$

$\sec^2 x - 1 = \tan^2 x$

$\sqrt{9y^2-6y+2} \le \frac{1}{3^{\tan^2 x}}.$

$1 \le \underbrace{\sqrt{9y^2-6y+2}}_{LHS} \le \underbrace{3^{-\tan^2 x}}_{RHS}.$

$LHS = \sqrt{9y^2-6y+1+1}$

$= \sqrt{(3y-1)^2+1}$

$(3y-1)^2 \geq 0.$

$LHS \geq \sqrt{1} = 1$

$3^{-\tan^2 x} \geq 1.$

$3^{\tan^2 x} \le 1 \rightarrow \log\left(3^{\tan^2 x}\right) \le \log 1$

$= 1$

$\log = 0.4771$

$\underbrace{\boxed{\tan^2 x}}_{\geq 0}\ \underbrace{\log 3}_{>0} \le 0.$

$\Rightarrow \tan x = 0.$

$\boxed{x = n\pi}$

LHS = 1

$9y^2-6y+2 = 1$

$9y^2-6y+1 = 0 \Rightarrow (3y-1)^2 = 0 \Rightarrow \boxed{y = \frac{1}{3}}$

$x = [0, 10]$

n	x
0	0
1	π = 3.14
2	2π = 6.28
3	3π = 9.42

The value of k for which the equation $(k-2)x^2 + 8x + k + 4 = 0$ has both roots real, distinct and negative is

(A) 4 (B) 3

(C) 6 (D) 1

$(k-2)x^2 + 8x + (k+4) = 0$

$D > 0$

$D = 8^2 - 4(k-2)(k+4) > 0$

$64 - 4(k^2 + 2k - 8) > 0$

$16 - k^2 - 2k + 8 > 0$

$k^2 + 2k - 24 < 0$

$(k+6)(k-4) < 0$

$\boxed{-6 < k < 4}$

$2 < k < 4$

$-4 < k < 2$

$-x^2 + 8x + 5 = 0$

$x^2 - 8x - 5 = 0$

$x = \frac{8 \pm \sqrt{84}}{2}$

$x^2 + 8x + 7 = 0$

$(x+7)(x+1) = 0$

$x = -1, -7$

$x = \frac{-8 \pm \sqrt{D}}{2(k-2)}$

$x = \frac{-8 + \sqrt{D}}{2(k-2)}$, $\frac{-8 - \sqrt{D}}{2(k-2)}$

$\frac{a}{b}$ $\frac{a}{b}$

$a = \sqrt{D} - 8$

$b = k - 2$

$a < 0$, $b > 0$; $a > 0$, $b < 0$

$\sqrt{D} - 8 < 0 \Rightarrow \sqrt{D} < 8$

$-8 < \sqrt{D} < 8$; $\sqrt{D} < -8$

$\boxed{D < 64}$; $D > 64$

$a = -\sqrt{D} - 8$

$b = k - 2$

$a < 0$, $b > 0$; $a > 0$, $b < 0$

$-\sqrt{D} - 8 < 0$

$\sqrt{D} > -8$

$\sqrt{D} > 8$

$D > 64$

$-8 < \sqrt{D} < 8$

$D < 64$

$k > 2$, $k \cancel{<} 4$

$-\sqrt{D} - 8 > 0$

$\sqrt{D} < -8$

$D > 64$

$k < 2$

$\boxed{D < 64}$

$64 - 4(k^2 + 2k - 8) < 64$

$16 - (k^2 + 2k - 8) < 16$

$k^2 + 2k - 8 > 0$

$(k+4)(k-2) > 0$

$\boxed{k < -4}$ or $\boxed{k > 2}$

$\boxed{2 < k < 4}$

$b > 0$

$k - 2 > 0$

$\boxed{k > 2}$ $\Rightarrow$ & $\Leftarrow$

$b < 0$

$\boxed{k < 2}$

$\sqrt{D} - 8 > 0$

$\sqrt{D} > 8$

$D > 64$

$64 - 4(k^2 + 2k - 8) > 64$

$(k+4)(k-2) < 0$

$\boxed{-4 < k < 2}$

$\boxed{-4 < k < 2}$

If $\cos 2x - 3\cos x + 1 = \frac{\operatorname{cosec} x}{\cot x - \cot 2x}$, then which of the following is true?

(A) $x = (2n+1)\frac{\pi}{2}, n \in I$

(B) $x = 2n\pi, n \in I$

(C) $x = 2n\pi \pm \cos^{-1}\left(\frac{2}{5}\right), n \in I$

(D) no real x

$2\cos^2 x - \cancel{1} - 3\cos x + \cancel{1} = \frac{1/\sin x}{\frac{\cos x}{\sin x} - \frac{\cos 2x}{\sin 2x}}$

$2\cos^2 x - 3\cos x = \frac{1/\sin x}{\frac{\sin 2x \cos x - \cos 2x \sin x}{\sin x \cdot \sin 2x}} = \frac{\sin 2x \sin x}{\sin x \cdot \sin x}$

$2\cos^2 x - 3\cos x = \frac{2\sin x \cos x \cdot \sin x}{\sin^2 x} = \frac{2\cancel{\sin^2 x}\cos x}{\cancel{\sin^2 x}}$

$\sin x = 0 \Rightarrow \boxed{x = n\pi}$

$\cos x = 0 \Rightarrow x = (2n+1)\frac{\pi}{2}$

$2\cos^2 x - 3\cos x = 2\cos x$

$2\cos^2 x - 5\cos x = 0$

$\cos x(2\cos x - 5) = 0$

$\cos x = 0$; $\cos x = 5/2$

The number of solutions of the equation

$\cos\left(x+\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}-x\right)=\frac{1}{4}\cos^2 2x,\ x\in[-3\pi,3\pi]$

is:

(A) 8 (B) 5

(C) 6 (D) 7

$16 = k^2 + 4k - 2k$

$16 = k^2 + 2k - 8$

$k^2 + 2k - 24$

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$2\cos\left(\frac{\pi}{3}+x\right)\cos\left(\frac{\pi}{3}-x\right) = \frac{1}{2}\cos^2 2x.$

$\cos\frac{2\pi}{3} + \cos 2x = \frac{1}{2}\cos^2 2x.$

$-\frac{1}{2} + \cos 2x = \frac{1}{2}\cos^2 2x$

$-1 + 2\cos 2x = \cos^2 2x.$

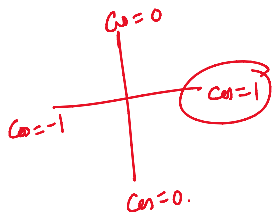
$\cos^2 2x - 2\cos 2x + 1 = 0$

$(\cos 2x - 1)^2 = 0.$

$\cos 2x = 1 = \cos 2n\pi.$

$2x = 2n\pi.$

$\boxed{x = n\pi}$



If α and β be two real roots of the equation $(k+1)\tan^2 x - \sqrt{2}\,\lambda\tan x = (1-k)$, where $k(\neq -1)$ and λ are real numbers. If $\tan^2(\alpha+\beta) = 50$, then a value of λ is:

(A) 5 (B) 10

(C) $5\sqrt{2}$ (D) $10\sqrt{2}$

$(k+1)\tan^2 x - \sqrt{2}\lambda\tan x + (k-1) = 0.$

$\tan\alpha + \tan\beta = \frac{\sqrt{2}\lambda}{k+1}$ $\tan\alpha\tan\beta = \frac{k-1}{k+1}.$

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} = \frac{\sqrt{2}\lambda/(k+1)}{1-\frac{k-1}{(k+1)}}$

$\tan(\alpha+\beta) = \frac{\sqrt{2}\lambda}{2} = \frac{\lambda}{\sqrt{2}}.$

$\tan^2(\alpha+\beta) = \frac{\lambda^2}{2} = 50.$

$\lambda^2 = 100$

$\lambda = \pm 10$

Identity the correct statements:

S_1 : Number of values of x, between 0 to 2π which satisfy $e^x \cot x = 1$ is four

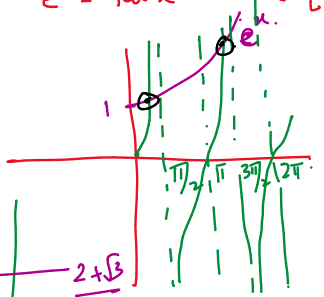
S_2: If $\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} = 2 + \sqrt{3}$ then there exist two values of θ between 0 to π which satisfy the abo

equation

S_3: If $\alpha^2 + \beta^2 + \gamma^2 = \alpha\beta + \beta\gamma + \alpha\gamma$ then $4\cos(60^\circ - \alpha)\cos\beta\ \cos(60^\circ + \gamma) = \cos(\alpha + \beta + \gamma)$

(A) TFT (B) TTF (C) FFT (D) FFF

$e^x = \tan x$ $\quad x = [0, 2\pi]$



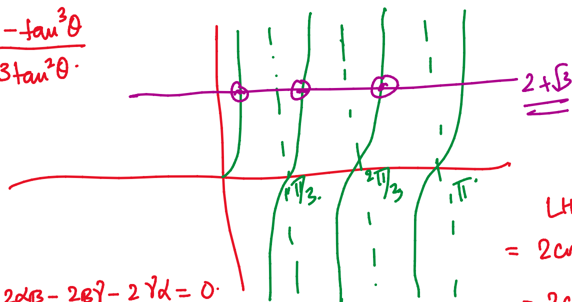
$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}.$

$\theta = 0 \quad 3\theta = 0.$

$\theta = \frac{\pi}{3} \quad 3\theta = \pi.$

$\theta = \frac{\pi}{6}$

$3\theta = \frac{\pi}{2}.$



$2\alpha^2 + 2\beta^2 + 2\gamma^2 - 2\alpha\beta - 2\beta\gamma - 2\gamma\alpha = 0.$

$(\alpha-\beta)^2 + (\beta-\gamma)^2 + (\gamma-\alpha)^2 = 0 \Rightarrow \boxed{\alpha = \beta = \gamma}$

LHS

$= 2\cos\alpha \cdot 2\cos(60+\alpha)\cos(60-\alpha)$

$= 2\cos\alpha\,[\cos 120 + \cos 2\alpha]$

$= 2\cos\alpha\,[-\tfrac{1}{2} + \cos 2\alpha]$

$= 2\cos\alpha\cos 2\alpha - \cos\alpha. = 2\cos\alpha(2\cos^2\alpha - 1) - \cos\alpha$

$= 4\cos^3\alpha - 3\cos\alpha$

RHS $= \cos 3\alpha.$

$= \cos(2\alpha+\alpha) = \cos 2\alpha\cos\alpha - \sin 2\alpha\sin\alpha.$

$= \cos 2\alpha\cos\alpha - 2\sin\alpha\cos\alpha\sin\alpha.$

$= \cos 2\alpha\cos\alpha - 2\cos\alpha(1-\cos^2\alpha)$

$= \cos 2\alpha\cos\alpha - 2\cos\alpha + 2\cos^3\alpha.$

$= 2\cos^3\alpha + \cos 2\alpha\cos\alpha - 2\cos\alpha.$

$= 2\cos^3\alpha + (2\cos^2\alpha - 1)\cos\alpha - 2\cos\alpha$

$= 2\cos^3\alpha + 2\cos^3\alpha - 3\cos\alpha = 4\cos^3\alpha - 3\cos\alpha.$

Let x,y, z be real numbers with $x \geqslant y \geqslant z \geqslant \frac{\pi}{8}$ such that $x + y + z = \frac{\pi}{2}$ and let $P = \cos x . \sin y . \cos z$

$\boxed{z = \frac{\pi}{2} - (x+y)}$

then which is/are CORRECT?

(A) Minimum value of P is $\frac{1}{8}$

(B) Minimum value of P is $\frac{1}{4}$

(C) Maximum value of P is $\frac{2+\sqrt{3}}{4}$

(D) Maximum value of P is $\frac{\sqrt{2}+1}{4\sqrt{2}}$

positive

The value of $\frac{(1+\sqrt{3}\tan 1^\circ)(1+\sqrt{3}\tan 2^\circ)(\tan 1^\circ + \tan 59^\circ)(\tan 2^\circ + \tan 58^\circ)}{(1+\tan^2 1^\circ)(1+\tan^2 2^\circ)}$ is

6. If $\cos\theta = \frac{1}{2}\left(a + \frac{1}{a}\right)$ and $\cos 3\theta = \frac{1}{2}\left(a^k + \frac{1}{a^k}\right)$ then number of natural numbers 'k' less than 50 is (given $a \in R$)